이학박사 학위논문

The implied volatility surface connection with local volatility, SABR, Heston

국소 변동성, SABR, 헤스턴 모델과 내재변동성 곡면의 커넥션

2015 년 2 월

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이 논문을 이학박사 학위논문으로 제출함

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신 명 기의 이학박사 학위논문을 인준함

2014 년 12 월

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Abstract

The implied volatility surface connection with local volatility, SABR, Heston

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There are many concepts in contemporary volatility models, but they have the same orientation. So in this paper, I want to connect each other. More particularly, we will place the implied volatility surface in the center and analyze Local volatility, SABR, Heston models. In the part of non-parametric volatility, first we collect the theories that are already in use and propose more consistent model named forward local volatility. Afterwards delta hedge is simulated and the right way of using Local volatility model is verified. In the part of parametric volatility, it begins with examining all inputs(dividend, riskless rate, security index futures) of volatility model. Finally, we propose the new methodologies to make the useful models more useful. One is bootstrapping the synthetic option and the other is making the stochastic volatility model as a Mosaic.

Keywords : Implied volatility surface; Local volatility; SABR model; Heston model; Synthetic option bootstrapping; Mosaic model

Student Number : 2011-20450

Contents

List of Figures

List of Tables

List of Algorithms

Part I

Non-Parametric Volatility

Chapter 1

An exotic option

Vanilla options on the underlying asset S are conventional instruments in the market. A nonconventional structure would then be called exotic[20].

- A non-time-homogeneous structure refers to a payoff structure that does contractually change through time.
- A barrier is a price level in the market (called a trigger) that, when reached, markedly alters the payoff of the structure.

These are nonconventional structures to be dealt with in the following section.

1.1 A Bermudan up-and-out call option

A Bermudan has an early exercisable option, at the possible future exercise dates are $t_1, ..., t_N <= T$, with strike K and time to expiry(maturity) T. In order to exercise this option at $t_1, ..., t_N$, we need to pay K_c to do so. We can represent this as a value monitoring condition, which we use to infer the value at time $t_1^-, ..., t_N^-$ from $t_1^+, ..., t_N^+$ (just before and just after $t_1, ..., t_N$, where t represent physical forward-stepping time)[6]:

$$V(S, t_i^-) = max(V(S, t_i^+) - K_c, (S_{t_i} - K)^+) \quad \text{for } i = 1, \ldots, N.$$

An up-and-out call with American style (upper) barrier U that knocks out the value of the underlying European option if a barrier level is crossed at any time during the lifetime of the option has the payout function[6]:

$$V^{UOC}(S, T) = (S_T - K)^+ \mathbf{1}_{\{S_T \geq K \cap max[S_t : 0 \leq t \leq T] < U\}}.$$

Its current valuation formula is expressed in K, T, U, spot price of the underlying asset S_0, dividend yield q, riskless rate r and implied volatility σ[7]

$$\begin{aligned} V^{UOC}(S, 0) = \; & S_0 e^{-qT}[N(d_1(S_0, K)) - N(d_1(S_0, U)) \\ & - \left(\frac{U}{S_0}\right)^{2\frac{(r-q)}{\sigma}+1} \left(N(d_1(U^2, S_0 K)) - N(d_1(U, S_0))\right)] \\ & -Ke^{-rT}[N(d_2(S_0, K)) - N(d_2(S_0, U)) \\ & + \left(\frac{U}{S_0}\right)^{2\frac{(r-q)}{\sigma}-1} \left(N(d_2(U^2, S_0 K)) - N(d_2(U, S_0))\right)] \end{aligned}$$

where $d_{1;2}(S_0, X) = \frac{ln\left(\frac{S_0}{X}\right) + \left(r - q \pm \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$, and $N(\cdot)$ is the cumulative distribution function of the standard normal distribution.

- The expected first exit time(or stopping time) is the time an asset price is expected to cross a given point in the market, conditional on an expiration date[20]

$$\tau^U = inf[t : S_t \geq U].$$

Then we can put together a Bermudan up-and-out call(BUOC) option value monitoring condition:

$$V^{BUOC}(S,t_i^-) = \quad max(V^{BUOC}(S,t_i^+) - K_c, (S_{t_i} - K)^+ \mathbf{1}_{\{S_{t_i} \geq K \cap \tau^U > t_i\}}) \qquad (1.1)$$
$$\text{for } i = 1, \ldots, N.$$

The first exit time, which is in this case the time when the barrier is hit, is uncertain. Thus it is necessary to know where it is located on the term structure for pricing purpose[20].

Chapter 2

The smoothing spline price implied volatility surface

The European call (C^{BS}) and put (P^{BS}) formula was derived for the first time by Black and Scholes[2] as follows:

$$\begin{aligned} C^{BS}(S) &= S_0 e^{-qT} N(d1) - K e^{-rT} N(d2), \\ P^{BS}(S) &= K e^{-rT} N(-d2) - S_0 e^{-qT} N(-d1), \\ where \quad d_{1;2} &= \frac{ln\left(\frac{S_0}{K}\right) + \left(r - q \pm \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}. \end{aligned} \tag{2.1}$$

- The implied volatility surface (IVS) is the representation of the values of implied volatility in the market represented in function of the strike price and the time to expiry(maturity)[20]

$$\sigma_{imp}(K, T).$$

2.1 Arbitrage-free smoothing of the IVS

As an extension of the end of the previous chapter, the IVS obtained by inverting the Black-Scholes (BS) formula serves as a key parameter for pricing and hedging exotic derivatives[10].

For the same purposes, local-volatility model is more sophisticated than the BS valuation approach. This model is calibrated to the IVS. A crucial property of the calibration data, given as an ensemble of market prices ($[C]^M$) quoted for different strikes and expiries, is the absence of arbitrage. We refer to arbitrage as to any violation of the theoretical properties of option prices, such as negative butterfly and calendar spreads. If the market data admit arbitrage, the calibration of the local volatility model can fail since imaginary local volatilities or negative transition probabilities ensue, which obstructs the convergence of the finite difference schemes solving the underlying generalized Black–Scholes partial differential equation. One may therefore obtain mispricings and noisy greeks. Unfortunately, an arbitrage-free IVS is not a natural situation in practice, since it is often computed from bid and ask prices, or derived from settlement data of poor quality[10].

Mathias R. Fengler propose an approach is based on cubic spline smoothing of option prices. One obtains an IVS well-suited for pricing and hedging via the BS formula[10]. Now the notation of an element of the IVS ($\sigma_{imp}(K,T)$) is specified.

$$
\begin{aligned}
C^{BS}(K,T;\sigma_{imp}(K,T)) &= S_0 e^{-q^T T} N(d1) - K e^{-r^T T} N(d2) \\
where \quad d_{1;2} &= \frac{ln\left(\frac{S_0}{K}\right) + \left(r^T - q^T \pm \frac{\sigma_{imp}(K,T)^2}{.} 2\right) T}{\sigma_{imp}(K,T)\sqrt{T}}
\end{aligned}
\tag{2.2}
$$

Smoothing spline call prices ($C^{SS}(Y;T)$) is set up on a regular forwardmoneyness ($Y = \frac{K}{F^T}$, $F^T = S_0 e^{(r^T - q^T)T}$: forward price of the underlying asset at time T) grid and corresponding IVS is in the same manner

$$\begin{aligned} C^{BS}(K_i, t_j; \sigma_{imp}(K_i, t_j)) &= C^{SS}(Y_i; t_j) \\ & \quad where \quad K_i = Y_i \times F^{t_j}, \\ & \quad (Y_i, t_j) \in [Y_1, \ldots, Y_m] \times [t_1, \ldots, t_n]. \end{aligned} \tag{2.3}$$

The smoothing parameter (λ=1E-7 in here) can be fixed at some small number without large impact on the esmimate. And it may be the right measure of loss in relative prices by setting ($w_{(Y_i;t_j)} = \left(\frac{Y_i}{K_i}\right)^2$). Then we can iterate smoothing spline call prices from the last to the first maturity ($t_j, j = n, \ldots, 1$), and solve the following quadratic program[10] as follows.

$$\begin{aligned} \underset{C^{SS}(Y_i;\cdot), \frac{\partial^2 C^{SS}(Y_i;\cdot)}{\partial Y_i{}^2}}{\operatorname{argmin}} \quad & \sum_{i=1}^{m} w_{(Y_i;\cdot)} \left([C(Y_i;\cdot)]^M - C^{SS}(Y_i;\cdot) \right)^2 \\ & \qquad + \lambda \sum_{i=1}^{m} \left(\frac{\partial^2 C^{SS}(Y_i;\cdot)}{\partial Y_i{}^2} \right)^2 \\ \text{s. t.} \quad & 0 \le C^{SS}(Y_m;\cdot) \\ & S_0 - Y_1 e^{-r^{t_j} t_j} \le C^{SS}(Y_1;\cdot) \\ & \begin{cases} C^{SS}(Y_1;\cdot) \le S_0 & \text{for } j = n \\ C^{SS}(Y_i;\cdot) \le C^{SS}(Y_i; t_{j+1}) e^{-\left(q^{t_{j+1}} t_{j+1} - q^{t_j} t_j\right)} & \\ & \text{for } j = n-1, \ldots, 1 \end{cases} \\ & \frac{\partial C^{SS}(Y_m;\cdot)}{\partial Y_m} \le 0 \\ & -e^{-r^{t_j} t_j} \le \frac{\partial C^{SS}(Y_1;\cdot)}{\partial Y_1} \\ & 0 \le \frac{\partial^2 C^{SS}(Y_m;\cdot)}{\partial Y_m{}^2} \end{aligned} \tag{2.4}$$

This procedure generates an arbitrage-free IVS. Since smoothing spline is a natural cubic spline, we can determine any $Y \in [0, \infty)$ by its set of function values and second-order derivatives at the knots[10]. But still the IVS can fill only on the $T \in [t_1, \ldots, t_n]$.

2.2 Flat forward volatility interpolation

If we relax the assumption that the volatility depends on the strike, we have $\sigma_{t_j} = \sigma_{imp}(\cdot, t_j)$ and $\sigma_{t_{j+1}} = \sigma_{imp}(\cdot, t_{j+1})$. The effective variance to time t_{j+1} is $\sigma^2_{t_{j+1}} t_{j+1}$ and the effective variance to t_j is $\sigma^2_{t_j} t_j$. By additivity of variance, we can write

$$\sigma^2_{t_{j+1}} t_{j+1} \quad = \quad \sigma^2_{t_j} t_j + \sigma^2_{t_j, t_{j+1}} (t_{j+1} - t_j)$$

where $\sigma_{t_j, t_{j+1}}$ is a forward volatility between times t_j and t_{j+1}. Then σ_{t_j} (or $\sigma_{t_{j+1}}$) is a spot volatility from 0 to time t_j (or t_{j+1}). We can trivially solve to get

$$\sigma_{t_j, t_{j+1}} \quad = \quad \sqrt{\frac{\sigma^2_{t_{j+1}} t_{j+1} - \sigma^2_{t_j} t_j}{t_{j+1} - t_j}}.$$

This scheme is known as flat forward volatility interpolation, and also available in a strike depending volatility situation[20][6].

$$\sigma_{fwd^{t_j, t_{j+1}}} \quad = \quad \sqrt{\frac{\sigma^2_{imp^{t_{j+1}}} t_{j+1} - \sigma^2_{imp^{t_j}} t_j}{t_{j+1} - t_j}} \tag{2.5}$$
$$where \quad \sigma_{fwd^{t_j, t_{j+1}}}(K) = \sigma_{fwd}(K, t_j, t_{j+1})$$
$$, \sigma_{imp^{t_j}}(K) = \sigma_{imp}(K, t_j).$$

Chapter 3

The forward local volatility

3.1 Local-volatility model

A local volatility is the instantaneous volatility at different possible asset price levels at different points in the future[9][20]. In local-volatility model, the risk-neutral stock price evolves according to geometric Bownian motion

$$\frac{dS_t}{S_t} = (r^T - q^T)dt + \sigma(S_t, t)dW_t$$

where W_t denotes a standard Brownian Motion, and r^T and q^T the continuously compounded riskless interest rate and dividend yield. Local-volatility $\sigma(S_t, t)$ is a non parametric, deterministic function depending on the asset price S_t and time t. A priori unknown, it must be computed numerically from option price (C) or, equivalently, from the IVS[9][10].

$$
\begin{aligned}
\frac{\partial C}{\partial T} &= \frac{1}{2}\sigma_{loc}(K,T)^2 K^2 \frac{\partial^2 C}{\partial K^2} - q^T C - (r^T - q^T) K \frac{\partial C}{\partial K} \\
\Rightarrow \sigma_{loc}(K,T)^2 &= 2\frac{\frac{\partial C}{\partial T} + (r^T - q^T)K\frac{\partial C}{\partial K} + q^T C}{K^2 \frac{\partial^2 C}{\partial K^2}}.
\end{aligned}
$$

The latter of the above equations is commonly known as the Dupire fomula, derived in this form by Derman and Kani but the method was developed by Dupire[9][8][21][14].

$$
E[\sigma_{loc}^2(S_T,T)|S_T = K, \mathcal{F}_0] = 2\frac{\frac{\partial C}{\partial T} + (r^T - q^T)K\frac{\partial C}{\partial K} + q^T C}{K^2 \frac{\partial^2 C}{\partial K^2}}.
$$

The local variance can be seen as the expected variance at maturity given that the asset price at maturity is equal to the strike price[21][14].

$$
\sigma_{loc}(K,T)^2 = \frac{\begin{pmatrix} \sigma_{imp}(K,T)^2 + 2\sigma_{imp}(K,T)T\left(\frac{\partial \sigma_{imp}(K,T)}{\partial T} \right. \\ \left. +(r^T - q^T)K\frac{\partial \sigma_{imp}(K,T)}{\partial K}\right) \end{pmatrix}}{\begin{pmatrix} \left(1 - \frac{K\ln\left(\frac{K}{F^T}\right)}{\sigma_{imp}(K,T)}\frac{\partial \sigma_{imp}(K,T)}{\partial K}\right)^2 \\ +K\sigma_{imp}(K,T)T\left(\frac{\partial \sigma_{imp}(K,T)}{\partial K}\right. \\ \left. -\frac{1}{4}K\sigma_{imp}(K,T)\left(\frac{\partial \sigma_{imp}(K,T)}{\partial K}\right)^2 + K\frac{\partial^2 \sigma_{imp}(K,T)}{\partial K^2}\right) \end{pmatrix}}. \tag{3.1}
$$

This formula is consistent with other known versions including the Dupire formula. It will be the main tool for extracting the local volatility surface(LVS) from the IVS[21][22][14].

3.2 Forward local volatility

Flat forward volatility, assuming in the previous chapter, should be applied to Local-volatility for model consistency.(i.e. (2.5) is applied.) One can express $\sigma_{imp^T}(K)$ and its derivatives at any time T between t_j and t_{j+1} as interplating values. And those things make (3.1) equation reset.

$$
\begin{aligned}
\sigma^2_{imp^T} &= \sqrt{\frac{t_{j+1}(T-t_j)\sigma^2_{imp^{t_{j+1}}} + t_j(t_{j+1}-T)\sigma^2_{imp^{t_j}}}{T(t_{j+1}-t_j)}}, \\
\frac{\partial \sigma_{imp^T}}{\partial T} &= \frac{t_{j+1}t_j(\sigma^2_{imp^{t_{j+1}}} - \sigma^2_{imp^{t_j}})}{\left(\begin{array}{c} 2T^2(t_{j+1}-t_j) \\ \times\sqrt{\dfrac{t_{j+1}(T-t_j)\sigma^2_{imp^{t_{j+1}}} + t_j(t_{j+1}-T)\sigma^2_{imp^{t_j}}}{T(t_{j+1}-t_j)}} \end{array}\right)}, \\
\frac{\partial \sigma_{imp^T}}{\partial K} &= \frac{\left(\begin{array}{c} t_{j+1}(T-t_j)\sigma_{imp^{t_{j+1}}}\dfrac{\partial \sigma_{imp^{t_{j+1}}}}{\partial K} \\ +t_j(t_{j+1}-T)\sigma_{imp^{t_j}}\dfrac{\partial \sigma_{imp^{t_j}}}{\partial K} \end{array}\right)}{\sqrt{\left(\begin{array}{c} T(t_{j+1}-t_j) \\ \times\left(t_{j+1}(T-t_j)\sigma^2_{imp^{t_{j+1}}} + t_j(t_{j+1}-T)\sigma^2_{imp^{t_j}}\right) \end{array}\right)}}, \\
\frac{\partial^2 \sigma_{imp^T}}{\partial K^2} &= \frac{\left(\begin{array}{c} t_{j+1}t_j(t_{j+1}-T)(T-t_j) \\ \times\left(\sigma_{imp^{t_{j+1}}}\dfrac{\partial \sigma_{imp^{t_j}}}{\partial K} - \sigma_{imp^{t_j}}\dfrac{\partial \sigma_{imp^{t_{j+1}}}}{\partial K}\right) \\ +T(t_{j+1}-t_j) \\ \times\left(t_{j+1}(T-t_j)\sigma^2_{imp^{t_{j+1}}} + t_j(t_{j+1}-T)\sigma^2_{imp^{t_j}}\right) \\ \times\left(t_{j+1}(T-t_j)\sigma_{imp^{t_{j+1}}}\dfrac{\partial^2 \sigma_{imp^{t_{j+1}}}}{\partial K^2}\right. \\ \left. +t_j(t_{j+1}-T)\sigma_{imp^{t_j}}\dfrac{\partial^2 \sigma_{imp^{t_j}}}{\partial K^2}\right) \end{array}\right)}{\sqrt{\left(\begin{array}{c} T(t_{j+1}-t_j) \\ \times\left(t_{j+1}(T-t_j)\sigma^2_{imp^{t_{j+1}}} + t_j(t_{j+1}-T)\sigma^2_{imp^{t_j}}\right) \end{array}\right)}^{3}}.
\end{aligned}
$$

$$
\begin{aligned}
\sigma^2_{loc^T} &= \sigma_{loc}(K,T)^2 \\
&= \frac{\left(\dfrac{t_{j+1}\sigma^2_{imp^{t_{j+1}}} + t_j\sigma^2_{imp^{t_j}}}{t_{j+1}-t_j} + 2(r^T-q^T)K \times \dfrac{\left(\begin{array}{c} t_{j+1}(T-t_j)\sigma_{imp^{t_{j+1}}}\frac{\partial \sigma_{imp^{t_{j+1}}}}{\partial K} \\ +t_j(t_{j+1}-T)\sigma_{imp^{t_j}}\frac{\partial \sigma_{imp^{t_j}}}{\partial K} \end{array} \right)}{t_{j+1}-t_j} \right)}{\left(\begin{array}{c}
\left(1-\ln\left(\frac{K}{F^T}\right) K \dfrac{\left(\begin{array}{c} t_{j+1}(T-t_j)\sigma_{imp^{t_{j+1}}}\frac{\partial \sigma_{imp^{t_{j+1}}}}{\partial K} \\ +t_j(t_{j+1}-T)\sigma_{imp^{t_j}}\frac{\partial \sigma_{imp^{t_j}}}{\partial K} \end{array} \right)}{\left(\begin{array}{c} t_{j+1}(T-t_j)\sigma^2_{imp^{t_{j+1}}} \\ +t_j(t_{j+1}-T)\sigma^2_{imp^{t_j}} \end{array} \right)} \right)^2 \\
+K \dfrac{\left(\begin{array}{c} t_{j+1}(T-t_j)\sigma_{imp^{t_{j+1}}}\frac{\partial \sigma_{imp^{t_{j+1}}}}{\partial K} \\ +t_j(t_{j+1}-T)\sigma_{imp^{t_j}}\frac{\partial \sigma_{imp^{t_j}}}{\partial K} \end{array} \right)}{t_{j+1}-t_j} \\
-\frac{1}{4}K^2 \dfrac{\left(\begin{array}{c} t_{j+1}(T-t_j)\sigma_{imp^{t_{j+1}}}\frac{\partial \sigma_{imp^{t_{j+1}}}}{\partial K} \\ +t_j(t_{j+1}-T)\sigma_{imp^{t_j}}\frac{\partial \sigma_{imp^{t_j}}}{\partial K} \end{array} \right)^2}{\left(t_{j+1}-t_j\right)^2} \\
+K^2 \dfrac{\left(\begin{array}{c} t_{j+1}t_j(t_{j+1}-T)(T-t_j) \\ \times \left(\sigma_{imp^{t_{j+1}}}\frac{\partial \sigma_{imp^{t_j}}}{\partial K} - \sigma_{imp^{t_j}}\frac{\partial \sigma_{imp^{t_{j+1}}}}{\partial K} \right) \\ +T(t_{j+1}-t_j) \\ \times \left(t_{j+1}(T-t_j)\sigma^2_{imp^{t_{j+1}}} + t_j(t_{j+1}-T)\sigma^2_{imp^{t_j}} \right) \\ \times \left(t_{j+1}(T-t_j)\sigma_{imp^{t_{j+1}}}\frac{\partial^2 \sigma_{imp^{t_{j+1}}}}{\partial K^2} \right. \\ \left. +t_j(t_{j+1}-T)\sigma_{imp^{t_j}}\frac{\partial^2 \sigma_{imp^{t_j}}}{\partial K^2} \right) \end{array} \right)}{\left(\begin{array}{c} T(t_{j+1}-t_j)^2 \\ \times \left(t_{j+1}(T-t_j)\sigma^2_{imp^{t_{j+1}}} + t_j(t_{j+1}-T)\sigma^2_{imp^{t_j}} \right) \end{array} \right)}
\end{array} \right)}.
\end{aligned}
$$

Finally we define a forward local volatility between t_j and t_{j+1}

$$\sigma_{fwdloc}(K, t_j, t_{j+1})^2 \quad = \quad \frac{\int_{t_j}^{t_{j+1}} \sigma_{loc}(K,T)^2 dT}{t_{j+1} - t_j}. \tag{3.2}$$

Its numerical approximation formula is also needed

$$\begin{aligned}
\sigma^2_{fwdloc^{t_j,t_{j+1}}} &= \sigma_{fwdloc}(K, t_j, t_{j+1})^2 \\
&= \frac{\int\limits_{t_j}^{t_{j+1}} \sigma^2_{loc^T}\left(K; \begin{matrix} \frac{\partial \sigma_{imp^{t_j}}}{\partial K}, \frac{\partial \sigma_{imp^{t_{j+1}}}}{\partial K}, \\ \frac{\partial^2 \sigma_{imp^{t_j}}}{\partial K^2}, \frac{\partial^2 \sigma_{imp^{t_{j+1}}}}{\partial K^2} \end{matrix}\right) dT}{t_{j+1} - t_j} \\
&\approx \frac{\int\limits_{t_j}^{t_{j+1}} \sigma^2_{loc^T}\left(K; \begin{matrix} \frac{\Delta \sigma_{imp^{t_j}}}{\Delta K}, \frac{\Delta \sigma_{imp^{t_{j+1}}}}{\Delta K}, \\ \frac{\Delta^2 \sigma_{imp^{t_j}}}{\Delta K^2}, \frac{\Delta^2 \sigma_{imp^{t_{j+1}}}}{\Delta K^2} \end{matrix}\right) dT}{t_{j+1} - t_j} \\
&\equiv \frac{\int_{t_j}^{t_{j+1}} \widehat{\sigma}_{loc}(K,T)^2 dT}{t_{j+1} - t_j} \\
&\approx \frac{\sum_{k=1}^{n} w_k \widehat{\sigma}_{loc}(K, \frac{t_{j+1}-t_j}{2} T_k + \frac{t_j + t_{j+1}}{2})^2}{2} \qquad (3.3) \\
&= \frac{\sum\limits_{k=1}^{n} w_k \widehat{\sigma}^2_{loc^{\frac{t_{j+1}-t_j}{2} T_k + \frac{t_j + t_{j+1}}{2}}}}{2}
\end{aligned}$$

where

$$\begin{aligned}
\frac{\Delta \sigma_{imp^T}}{\Delta K} &= \frac{\sigma_{imp^T}(K + \Delta) - \sigma_{imp^T}(K - \Delta)}{2\Delta}, \\
\frac{\Delta^2 \sigma_{imp^T}}{\Delta^2 K} &= \frac{\sigma_{imp^T}(K + \Delta) - 2\sigma_{imp^T}(K) + \sigma_{imp^T}(K - \Delta)}{2\Delta^2},
\end{aligned}$$

w_k is coefficient and $T_k \in [t_j, t_{j+1}]$ is abscissa of Gauss quadrature. The forward local volatility surface(FLVS) is generated by the operation to plug (3.3) formula into the IVS.

Chapter 4

Pricing and Hedging

4.1 Delta hedging simulation

In this chapter, we will confirm the results are calculated using actual historical data. First, generate the KOSPI 200 Exchange market IVS, FLVS sequentially by (2.3), (2.4), (2.5), (3.3). Second, calculate the price and Greeks of the BUOC using the implicit finite difference scheme on (1.1) with some added condition that K^T and U^T are predetermined according to the possible future exercise times[15]. Lastly, compare the historical delta hedge perfomance under the FLVS against the IVS in three kinds of different scenarios by changing strike K^T and upper barrier U^T. The scenarios are as follows.

- Scenario 1: S_t frequently cross K^T during the BUOC lifetime.
- Scenario 2: S_t are always bigger than K^T during the BUOC lifetime.
- Scenario 3: S_t cross U^T during the BUOC lifetime.

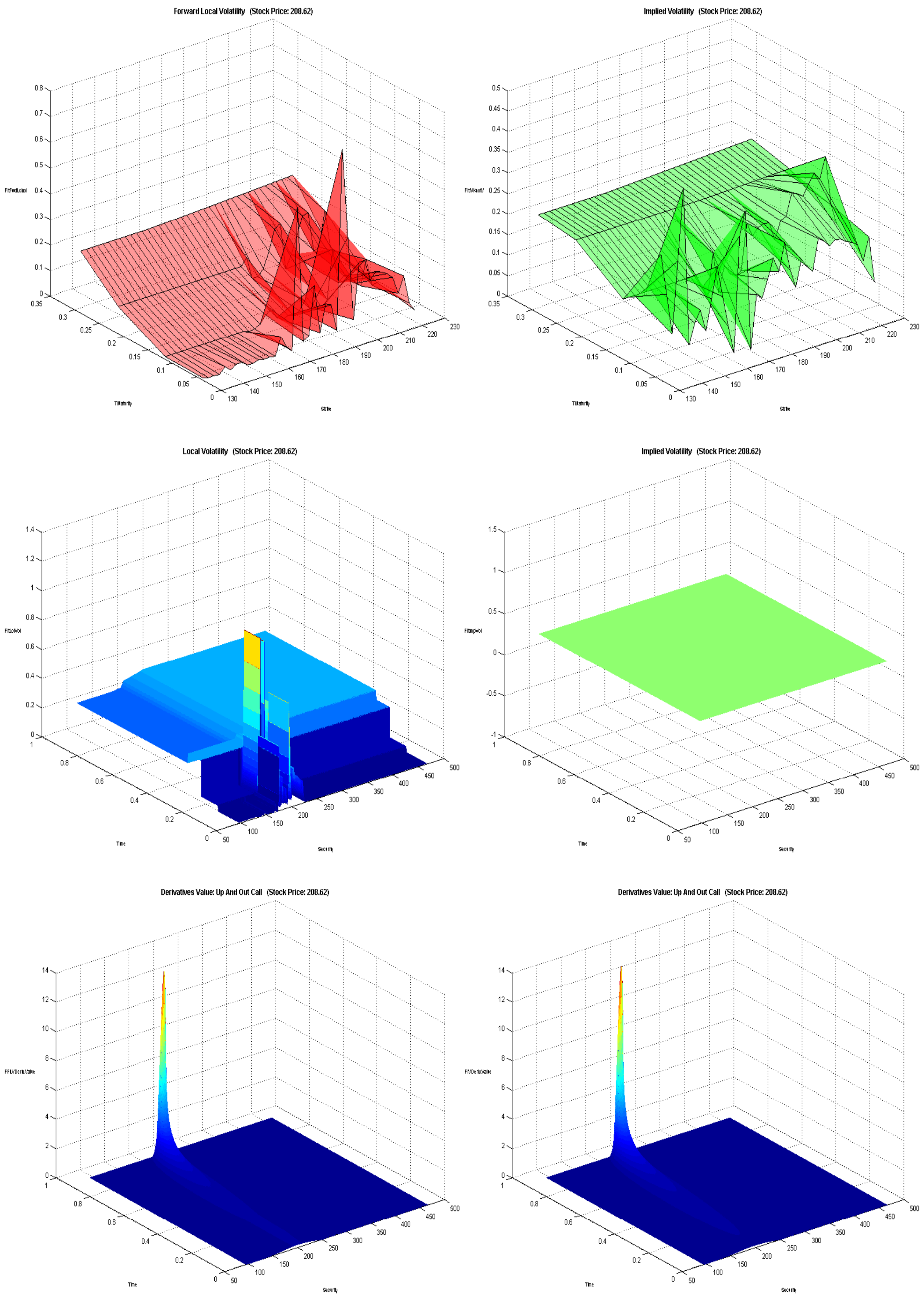


Figure 4.1: KOSPI200 Market Volatility, Volatility and Price Forward Local Volatility vs Implied Volatility ATM (27-Aug-09)

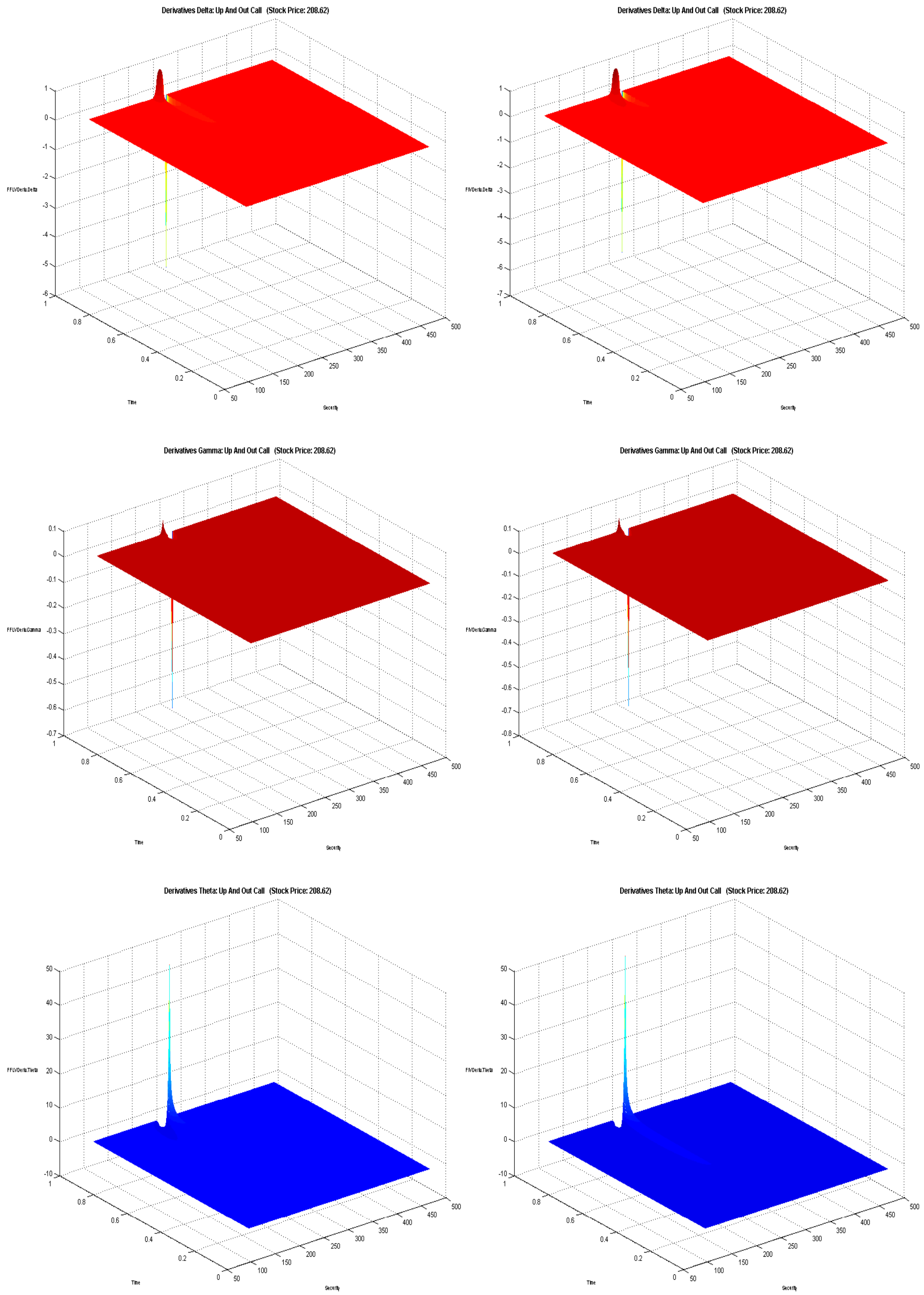


Figure 4.2: KOSPI200 Delta, Gamma, Theta Forward Local Volatility vs Implied Volatility ATM (27-Aug-09)

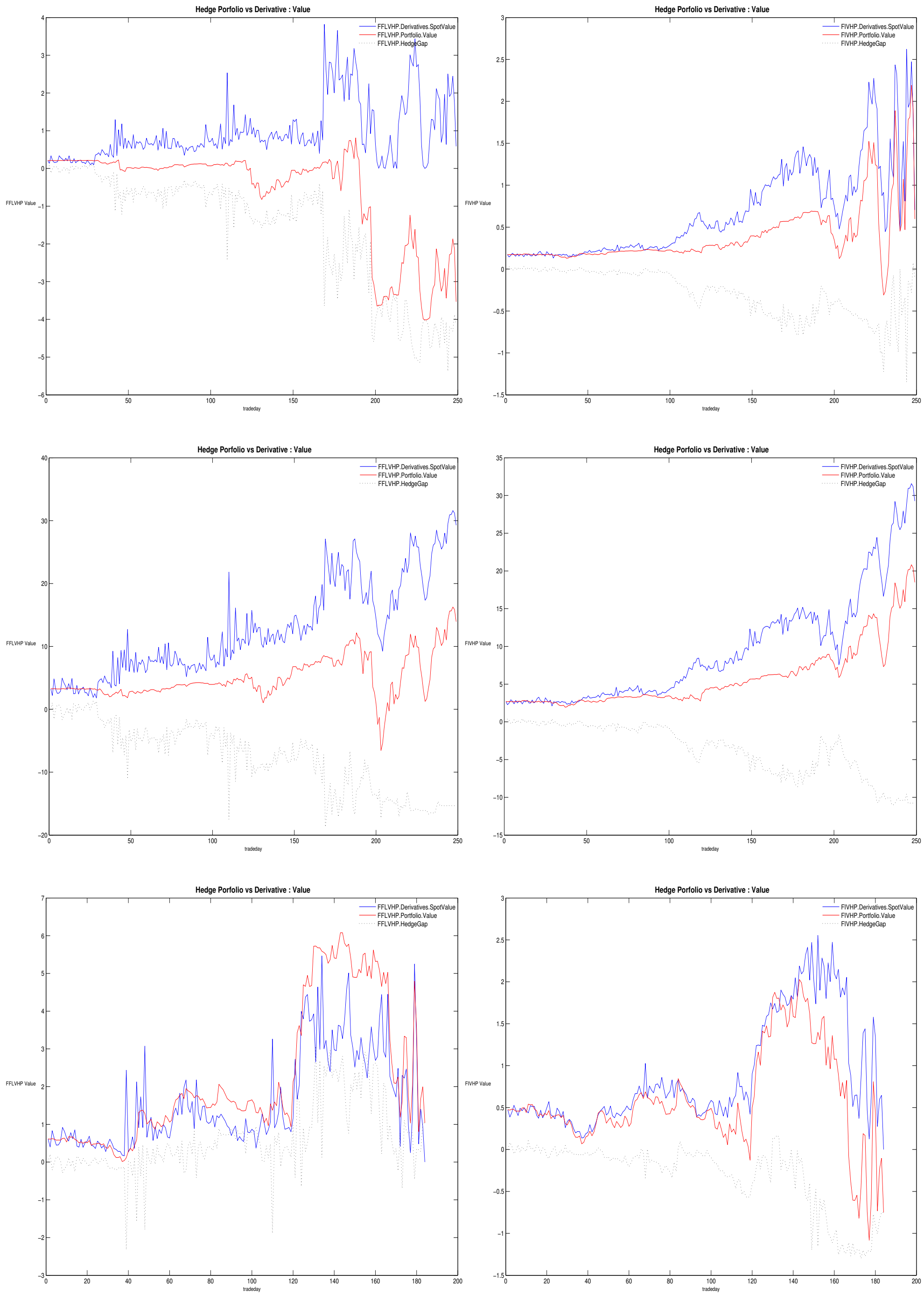


Figure 4.3: KOSPI200 Hedge Performance Forward Local Volatility vs Implied Volatility ATM, ITM, Knock Out (09-Aug-09∼30-Jul-10)

Graphs on the left column are related to the FLVS, graphs on the right column are related to the IVS in Figure 4.1, Figure 4.2 and Figure 4.3. In Figure 4.1, graphs at the top row are the FLVS and the IVS of market vanilla options, in the middle row are the FLVS and the IVS of the BUOC, at the bottom row are prices of the BUOC. In Figure 4.2, graphs at the top row are deltas of the BUOC, in the middle row are gammas of the BUOC, at the bottom row are thetas of the BUOC. Figure 4.1 and Figure 4.2 are generated from 'Scenario 1'. In Figure 4.3, graphs at the top row are the BUOC, its hedge portfolio and their difference under 'Scenario 1', graphs in the middle row are same items under 'Scenario 2' and graphs at the bottom row are same items under 'Scenario 3'. In Table 4.1, each number is root mean squared difference between the BUOC and its hedge portfolio under each scenario. 'ATM' means 'Scenario 1', 'ITM' means 'Scenario 2' and 'Knock Out' means 'Scenario 3'.

Table 4.1: Root Mean Squared KOSPI200 Forward Local Volatility vs Implied Volatility Hedge Gap (09-Aug-09∼30-Jul-10)

Forward Local Volatility			Implied Volatility		
ATM	ITM	Knock Out	ATM	ITM	Knock Out
2.198E+0	9.525E+0	1.115E+0	3.942E-1	4.973E+0	5.003E-1

The following is a part of abstract of [12].

> Market smiles and skews are usually managed by using local volatility models *a la* Dupire. We discover that the dynamics of the market smile predicted by local vol models is opposite of observed market behavior: when the price of the underlying decreases, local vol models predict that the smile shifts to higher prices; when the price increase, these models predict that the smile shifts to lower

> prices. Due to this contradiction between model and market, delta and vega hedges derived from the model can be unstable and may perform worse than naive Black-Scholes hedges. ...

The results obtained above show that the hedge on the FLVS is worse than on the IVS. And this paragraph is enough to explain the reason.

Part II

Parametric Volatility

Chapter 5

Implied dividend

So far, we have simply regarded continuously compounded dividend yield term structure(DYT: q^T) and riskless interest rate term structure(IRT: r^T) as given factors. In this chapter, DYT and IRT would be calculated precisely from the stock index markets and also stock index futures would be set up consistently.

5.1 Implied dividend yield of stock exchange and over the counter(OTC) market

In the stock index market, quote prices are as follows: stock index spot(S_0), stock index futures(F^T) and calls($C(K,T)$), puts($P(K,T)$)

$$C(K,T) + Ke^{-r^T T} = P(K,T) + F^T e^{-r^T T}. \tag{5.1}$$

(5.1) called 'Put-call parity' is a crucial property to calculate implied DYT. In the strict standard, IRT is close to reference riskless rate in interest rate market, but it is 'unclear' in the stock index market. Therefore IRT is presumed($\widehat{r}^T$). In a position of to dividend, it occurs lumpsum payment(Q_{t_k}) at dividend date(t_k) of predetermined month. lumpsum dividend could be converted to DYT.

$$\begin{aligned} F^T &= S_0 e^{\widehat{r}^T T} - \sum_{k=1}^{l} Q_{t_k} \mathbf{1}_{(t_k \leq T)}. && (5.2) \\ e^{-q^T T} &= 1 - \frac{e^{-\widehat{r}^T T} \sum_{k=1}^{l} Q_{t_k} \mathbf{1}_{(t_k \leq T)}}{S_0}. && (5.3) \end{aligned}$$

DYT can be derived from (5.1). If you are placed in a situation that any option(call or put) of the expiry T is not applicable in the market, then DYT can be obtained by (5.2), (5.3).

$$\begin{aligned} q^T &= \frac{\ln\left(\frac{P(K,T)+S_0-C(K,T)-Ke^{-\widehat{r}^T T}}{S_0} + 1\right)}{T}. && (5.4) \\ q^T &= -\frac{\ln \frac{F^T}{S_0}}{T} + \widehat{r}^T. && (5.5) \end{aligned}$$

Constraint value conversion in the bound with order preserving There is a effective computational technique that any ordered value(x) restricts in the bound with order preserving.

$$x_{LB}^{UB} = \begin{cases} LB + LC \times e^{x-LB-LC} & \text{if } x < LB + LC \\ UB - UC \times e^{-x+UB-UC} & \text{if } UB - UC \leq x \\ x & \text{if } LB + LC \leq x < UB - UC. \end{cases} \tag{5.6}$$

Algorithm 5.1 Implied dividend yield Calculation

1: Calculate dividend yield by (5.4) with (5.6)

if Input is deficient **then**
  Calculate dividend yield by (5.5)
  if Dividend yield is negative **then**
    Do not assign dividend yield.
  end if
end if

In order to merge data of stock exchange market and OTC market, the presumption IRT meditate the balancing with implied dividend and stock index futures

$$\widehat{r}^T = \frac{\ln \frac{F^T}{S_0}}{T} + q^T. \tag{5.7}$$

Algorithm 5.2 Merge Stock exchange and OTC market data

Primary market ← stock exchange
Secondary market ← OTC
Merged market ← dividend yield assigned primary market
Merged market ← merged market ∪ dividend yield assigned (secondary − primary) market
Merged market ← merged market ∪ dividend yield (not assigned primary − assigned secondary) market
Merged market ← merged market ∪ (dividend yield not assigned secondary − primary) market
Dividend yield ← merged market dividend yield
Riskless rate ← merged market riskless rate
if Dividend yield is not assigned **then**
  Calculate dividend yield by (5.3)
end if
Calculate riskless rate by (5.7)
if Riskless rate is negative **then**
  Riskless rate ← 0
  Calculate dividend yield by (5.5)
end if
Merged market dividend yield ← dividend yield
Merged market riskless rate ← riskless rate

5.2 Riskless rate and refined stock index futures

The presumption IRT play a role of buffer in the stock markets and varying value is allowed, so it is not IRT any more. It is time to input reference riskless rate in interest rate market to IRT. This yields stock index futures to be refined in order to be consitent with DYT and IRT

$$\widetilde{F}^T = S_0 e^{(r^T - q^T)T}. \tag{5.8}$$

Refining call, put and implied volatility should be performed like a chain reaction.

Algorithm 5.3 Refine Stock index futures

1: IRT $\leftarrow$ reference riskless rate in interest rate market
2: Calculate stock index futures by (5.8)

Table 5.1: KOSPI200 Stock index futures, Dividend yield, Riskless rate (16-Jan-14)

Time-to-maturity (Days)	Stock index futures (spot: 256.52)	Time-to-maturity (A/365F)	Dividend yield (%)	Riskless rate (%)
28	255.88	0.08	5.91	2.65
56	256.32	0.15	3.16	2.65
84	256.84	0.23	2.11	2.65
147	257.70	0.40	1.51	2.65
238	259.53	0.65	0.87	2.66
329	261.23	0.90	0.65	2.67
420	260.43	1.15	1.38	2.69
511	262.14	1.40	1.17	2.72
602	264.05	1.65	1.00	2.75
693	265.66	1.90	0.93	2.78
784	267.68	2.15	0.83	2.81
1057	271.32	2.90	0.96	2.90

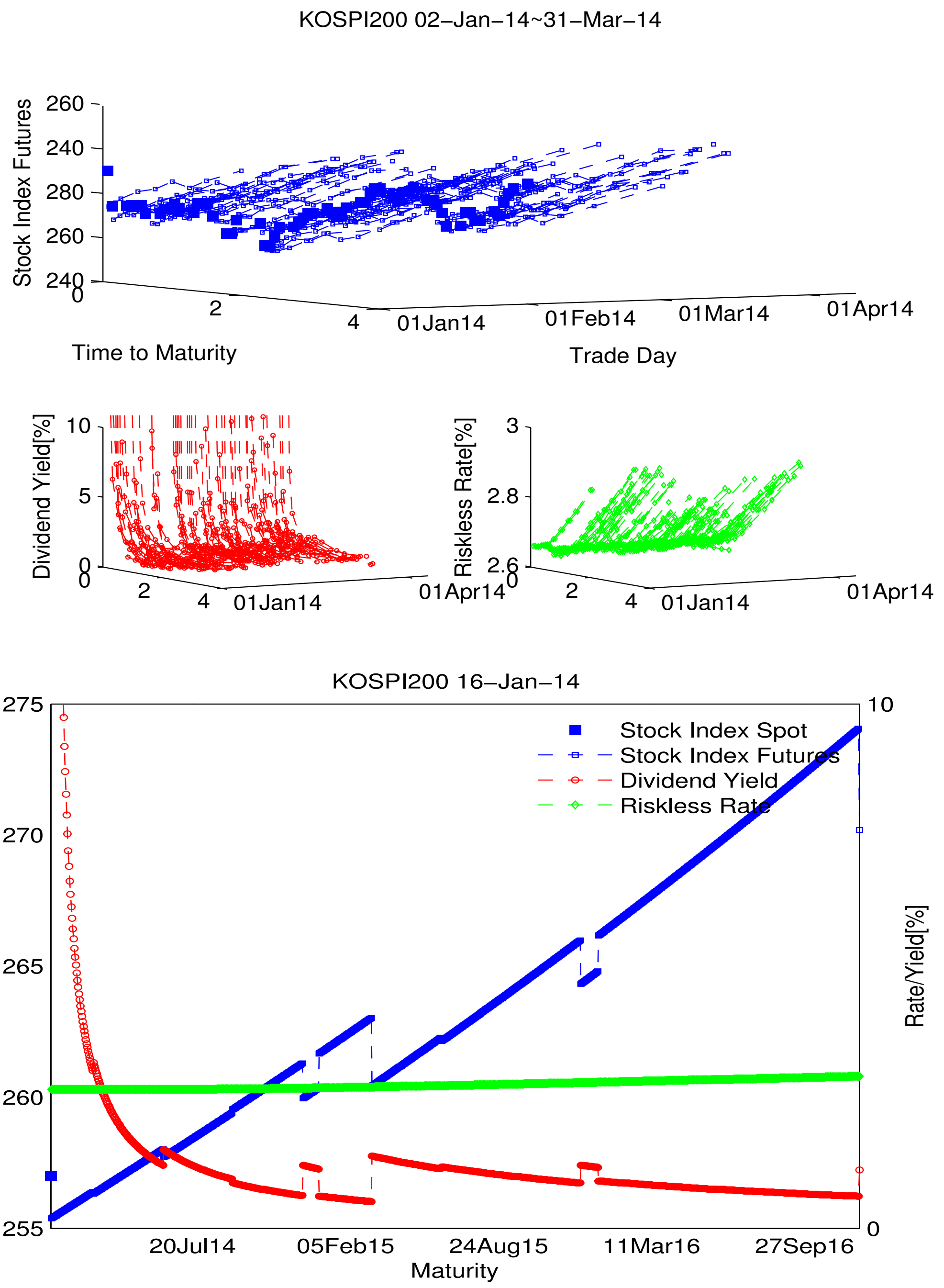


Figure 5.1: KOSPI200 Stock index futures, Dividend yield, Riskless rate (03-Jan-14∼31-Mar-14), (16-Jan-14)

In Table 5.1, there are stock index futures, DYT and IRT of KOSPI200 on January 16, 2014. And a graph of same items on January 16, 2014 is in lower half of Figure 5.1. In upper half of Figure 5.1, there are graphs of same items during January 2, 2014 to March 31, 2014.

Chapter 6

Synthetic option bootstrapping

A synthetic security is a linear combination of two or more primary instruments in the market[20]. And primary instruments of synthetic options are European options.

6.1 OTC synthetic option bootstrapping

OTC market is peering with stock exchange market in asian stock index markets. And synthetic options($SO(w_i, r_i, K_i, T_i; i = 1, \ldots, n)$) frequently trade in here.

$$SO(w_i, r_i, K_i, T_i; i = 1, \ldots, n) = \sum_{i=1}^{n} w_i \times \begin{cases} C(K_i, T_i) & \text{if } r_i \text{ is `}\mathit{Call}\text{'} \\ P(K_i, T_i) & \text{if } r_i \text{ is `}\mathit{Put}\text{'}. \end{cases} \tag{6.1}$$

For the purpose of generating the implied volatility surface or something equivalent, it is crucial that synthetic option is broken down into European options.

This procedure is named ‘synthetic option bootstrapping’ from now on.

$$C(K,T) \geq max(0,(F^T - K)e^{r^T T}), \tag{6.2a}$$

$$P(K,T) \geq max(0,(K - F^T)e^{r^T T}). \tag{6.2b}$$

Options lower bounds (6.2) are constraint of system.

Algorithm 6.1 System of linear equations for Bootstrapping

1: W ← synthetic option weight matrix with rowwise: prices of synthetic options, columnwise: unknown prices of European options in synthetic options
2: uLB ← calculate European options lower bounds in synthetic options by (6.2)
3: k ← prices of European options in stock exchange or OTC market that are matched to European options in synthetic options
4: There are the cases of same strike and expiry but different option rights(one is call, other is put) among European options in synthetic options.
   soP ← values of each European option subtract representative European option in synthetic options by (5.1)
5: s ← prices of synthetic options

Linearly dependent problem occurs ordinarily when we bootstrap synthetic options. In the linear system, constant terms are composed of synthetic option prices and unknown terms are composed of European option prices in synthetic options. System reduced methodology is the key of bootstrapping, since it is natural there are more unknown terms than constant terms.

Algorithm 6.2 Bootstrap applying System Reduced Methodology

1: Synthetic option weight matrix W and prices of synthetic options s set initial linear system
2: m ← the number of prices of synthetic options
3: n ← the number of unknown prices of European options in synthetic options and prices of European options in stock exchange or OTC market that are matched to European options in synthetic options

---

```
Reduce the system: prices of European options in stock exchange or
OTC market that are matched to European options in synthetic options
loopcount ← 0
status ← false
while loopcount < n and status = false do
    get number of nonzero column elements and sort
    for all i ∈ column indices by sorting order do
        if (rank of column 1 to i) = (rank of column 1 to i − 1) then
            Reduce the system: a linearly dependent column
            Reduce the system: linearly dependent rows
        end if
    end for
    get number of nonzero column elements and sort
    if columns that all elements are zero then
        Reduce the system
    end if
    get number of nonzero row elements and sort
    if rows that all elements are zero then
        Reduce the system
    end if
    if minimum size of system ≤ rank of the system then
        status ← true
    end if
    Solve the system by least square method
    if solution of system cross over the lower bounds then
        Reduce the system
        status ← false
    else
        if rank of the system < maximum size of system then
            Reduce the system: large residual row
            status ← false
        end if
    end if
end while
```

- Flat volatility refers a volatility whose magnitude is theoretically set the same across different delta values of an option[17].

The notion of flat volatility also can use in a synthetic option.

```
for all i ∈ European options in synthetic options do
    if 'i' is in the solution of system then
        Price of solution to add soP in Algorithm 6.1 is assigned
        Spot volatility is assigned
    else
        if 'i' overlaps at strike and expiry with other European options in
        synthetic options then
            Find the synthetic option 'i' belongs to. And does not use any more!
        else
            Flat volatility is shared as spot volatility
            Price from flat volatility is assigned
        end if
    end if
end for
```

- Option strategies refer to a combination of simultaneous buying and or selling of options[16].

All synthetic options can be categorized into option strategies.

Table 6.1: KOSPI200 OverTheCounter Synthetic Options (16-Jan-14)

Strategy	weight × rights	
Price	Bootstrapped price	
Implied volatility (%)	Forward moneyness Time-to-maturity	
Bootstrapped Type	Bootstrapped implied volatility (%)	
CallRatio	+1.0 Call	-2.0 Call
1.00E-04	2.31E+00	1.15E+00

29.94	0.99	1.11
	0.15	0.15
Spot	3.79	21.61
CallRatio	+1.0 Call	-2.0 Call
2.13E-03	2.13E-03	2.27E+00
13.21	1.01	1.08
	0.40	0.40
Spot	0.44	13.86
CallRatio	+1.0 Call	-2.0 Call
1.07E-07	2.13E-03	1.06E-03
22.45	1.01	1.16
	0.40	0.40
Spot	0.44	6.94
CallRatio	+1.0 Call	-2.0 Call
6.63E-06	6.63E-06	2.66E+00
14.92	1.00	1.19
	0.90	0.90
Spot	0.04	16.22
CallRatio	+1.0 Call	-2.0 Call
3.85E-11	3.85E-11	9.36E-38
16.14	1.00	1.26
	1.90	1.90
Flat	16.14	16.14
CallSpread	+1.0 Call	-1.0 Call
3.06E-249	1.03E+01	1.03E+01
68.81	1.03	1.23
	1.90	1.90

Spot	10.17	20.13
PutCalendarSpread	-1.0 Put	+1.0 Put
2.35E+00	9.55E-03	2.36E+00
17.13	0.96	0.95
	0.15	0.40
Flat	17.13	17.13
PutCalendarSpread	-1.0 Put	+1.0 Put
1.34E+01	2.82E+00	1.34E+01
16.92	0.98	0.97
	0.15	0.65
Spot	13.78	21.51
PutCalendarSpread	-1.0 Put	+1.0 Put
2.12E+01	3.52E+00	2.12E+01
17.37	0.85	0.84
	0.90	1.90
Spot	17.99	29.84
PutRatio	+1.0 Put	-2.0 Put
1.14E+01	1.14E+01	1.95E+00
22.56	0.97	0.89
	0.40	0.40
Spot	23.89	17.23
PutRatio	+1.0 Put	-1.5 Put
1.56E+01	4.24E+01	1.78E+01
20.14	1.02	0.92
	0.90	0.90
Flat	20.14	20.14
PutRatio	+1.0 Put	-2.0 Put

4.97E+00	8.20E+01	3.85E+01
35.51	0.86	0.69
	1.90	1.90
Flat	35.51	35.51
PutRatio	+1.0 Put	-2.0 Put
-4.66E-01	9.80E+01	4.92E+01
34.97	0.89	0.70
	2.15	2.15
Flat	34.97	34.97
PutRatio	+1.0 Put	-2.0 Put
-1.55E+01	1.10E+02	6.28E+01
36.85	0.82	0.63
	2.90	2.90
Flat	36.85	36.85
PutRatio	+1.0 Put	-2.0 Put
-1.29E+01	1.08E+02	6.02E+01
31.12	0.87	0.68
	2.90	2.90
Flat	31.12	31.12
RiskReversal	+1.0 Put	-1.0 Call
1.72E-06	1.72E-06	6.45E-20
15.94	0.94	1.07
	0.15	0.15
Flat	15.94	15.94
RiskReversal	+1.0 Put	-1.0 Call
1.53E+01	1.53E+01	7.60E-72
74.44	0.96	1.07

	0.15	0.15
Flat	74.44	74.44
RiskReversal	+1.0 Put	-1.0 Call
1.51E+01	1.51E+01	2.25E-24
18.73	0.92	1.12
	0.90	0.90
Flat	18.73	18.73
RiskReversal	+1.0 Put	-1.0 Call
4.30E+01	4.30E+01	8.68E-64
33.76	0.92	1.16
	0.90	0.90
Flat	33.76	33.76
RiskReversal	-1.0 Put	+1.0 Call
-1.03E+00	1.03E+00	1.14E+01
15.32	0.77	1.15
	1.90	1.90
Spot	12.56	17.57
RiskReversal	+1.0 Put	-1.0 Call
1.09E+01	2.12E+01	1.03E+01
14.44	0.84	1.23
	1.90	1.90
Spot	29.84	20.13
Straddle	+1.0 Put	+1.0 Call
1.66E+00	3.46E-01	1.31E+00
12.80	1.00	1.00
	0.08	0.08
Spot	2.59	2.59

Straddle	+1.0 Put	+1.0 Call
3.22E+00	9.11E-01	2.31E+00
13.63	0.99	0.99
	0.15	0.15
Spot	3.79	3.79
Straddle	+1.0 Put	+1.0 Call
9.09E+00	2.92E+00	6.18E+00
14.94	0.99	0.99
	0.40	0.40
Spot	6.79	6.79
Straddle	+1.0 Put	+1.0 Call
1.71E+01	6.72E+00	1.04E+01
15.73	0.99	0.99
	0.65	0.65
Spot	10.35	10.35
Straddle	+1.0 Put	+1.0 Call
4.10E+01	1.85E+01	2.25E+01
16.74	0.98	0.98
	1.40	1.40
Spot	17.32	17.32
Straddle	+1.0 Put	+1.0 Call
4.79E+01	2.15E+01	2.64E+01
16.97	0.98	0.98
	1.65	1.65
Spot	18.69	18.69
Straddle	+1.0 Put	+1.0 Call

5.48E+01	2.45E+01	3.02E+01		
17.22	0.98	0.98		
	1.90	1.90		
Spot	19.98	19.98		
Straddle	+1.0 Put	+1.0 Call		
7.52E+01	3.12E+01	4.40E+01		
17.94	0.95	0.95		
	2.90	2.90		
Spot	22.73	22.73		
Strangle	+1.0 Put	+1.0 Call		
5.12E-07	3.15E-07	1.97E-07		
13.72	0.94	1.03		
	0.15	0.15		
Flat	13.72	13.72		
Strangle	+1.0 Put	+1.0 Call		
2.88E-01	2.87E-01	8.75E-04		
13.57	0.98	1.01		
	0.15	0.15		
Flat	13.57	13.57		
StraddleCalendar	-1.0 Call	-1.0 Put	+1.0 Call	+1.0 Put
1.70E+01	6.18E+00	2.92E+00	1.54E+01	1.08E+01
18.72	0.99	0.99	0.98	0.98
	0.40	0.40	0.90	0.90
Spot	6.79	6.79	13.52	13.52

In Table 6.1, there are OTC synthetic options of KOSPI200 on January 16,

2014. 'Strategy' means name of option strategy. 'weight' means w_i in (6.1), 'rights' means r_i in (6.1), 'Forward moneyness' means $Y_i = \frac{K_i}{F_{T_i}}$ in (6.1), 'Time-to-maturity' means T_i in (6.1), 'Bootstrapped Type' is spot when Line 39 and Line 40 in Algorithm 6.2 is performed or flat when Line 45 and Line 46 in Algorithm 6.2 is performed. 'Price' means s in Algorithm 6.1, 'Bootstrapped Price' means price from Line 39 or Line 46 in Algorithm 6.2, 'Implied volatility' means flat volatility of 'Price', and 'Bootstrapped implied volatility' means spot volatility from Line 40 or Line 45 in Algorithm 6.2.

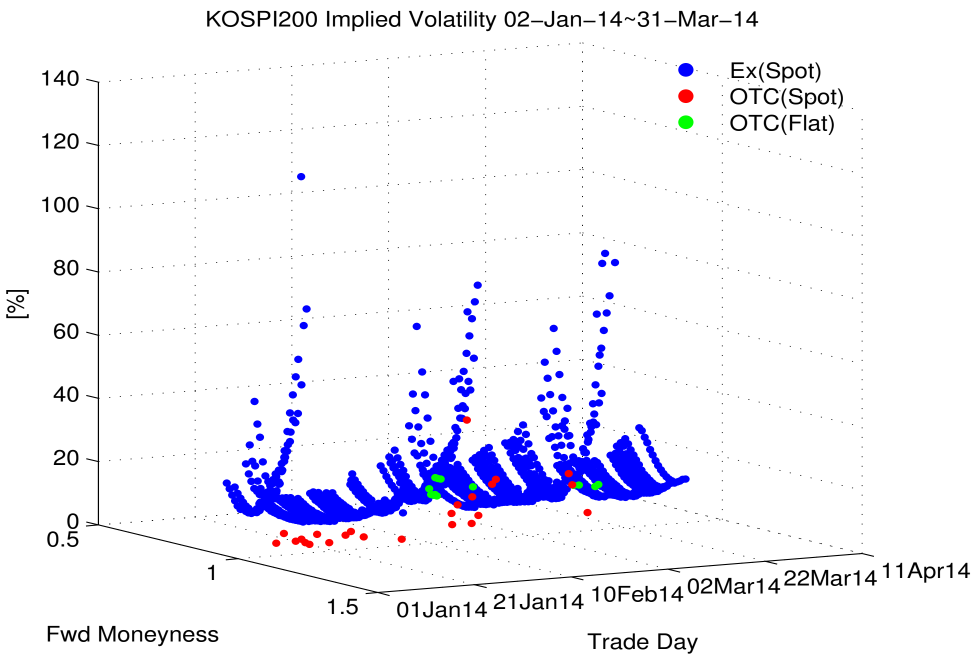


Figure 6.1: KOSPI200 Implied Volatility Nearest (03-Jan-14∼31-Mar-14)

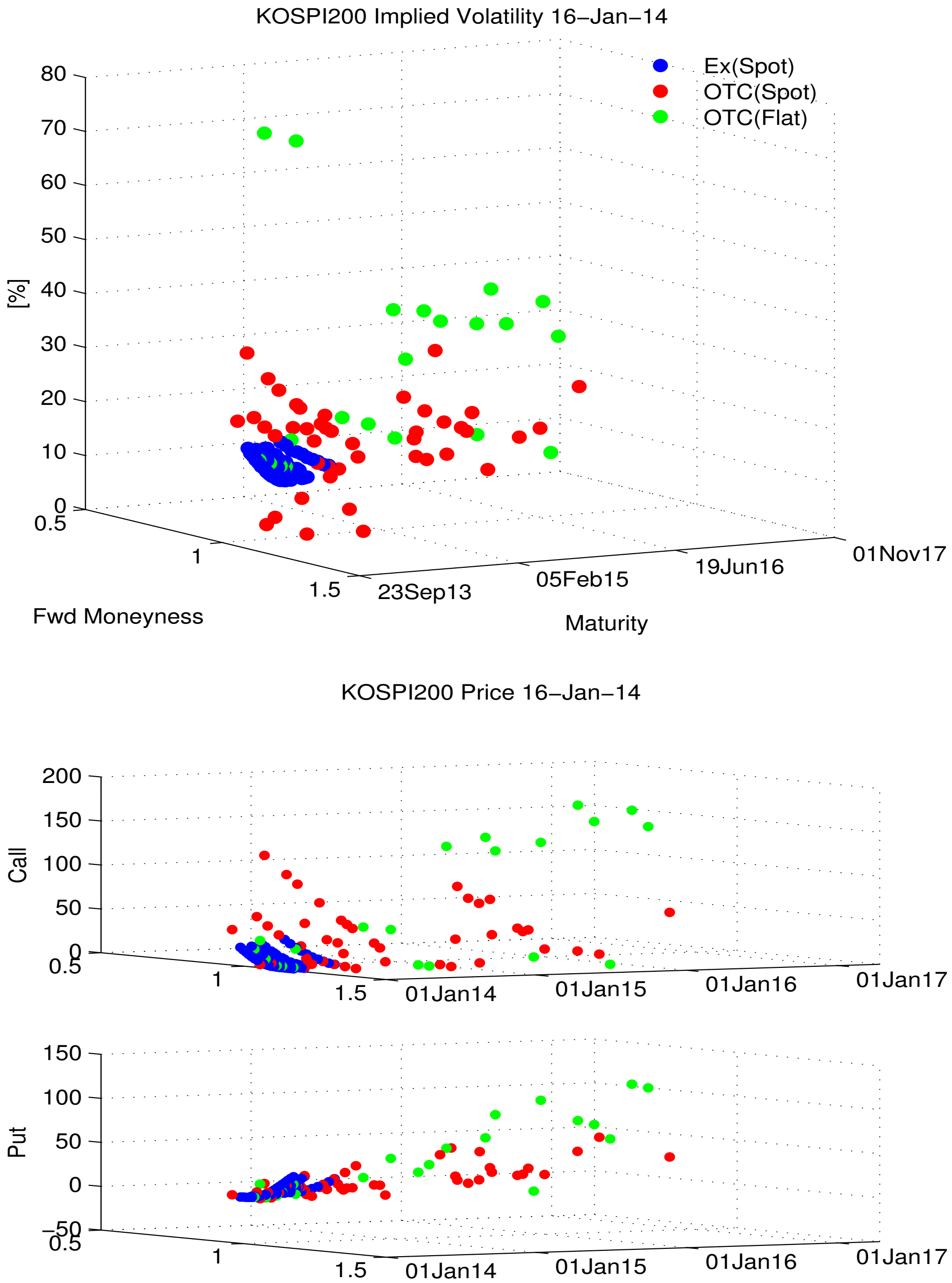


Figure 6.2: KOSPI200 Implied Volatility, Price (16-Jan-14)

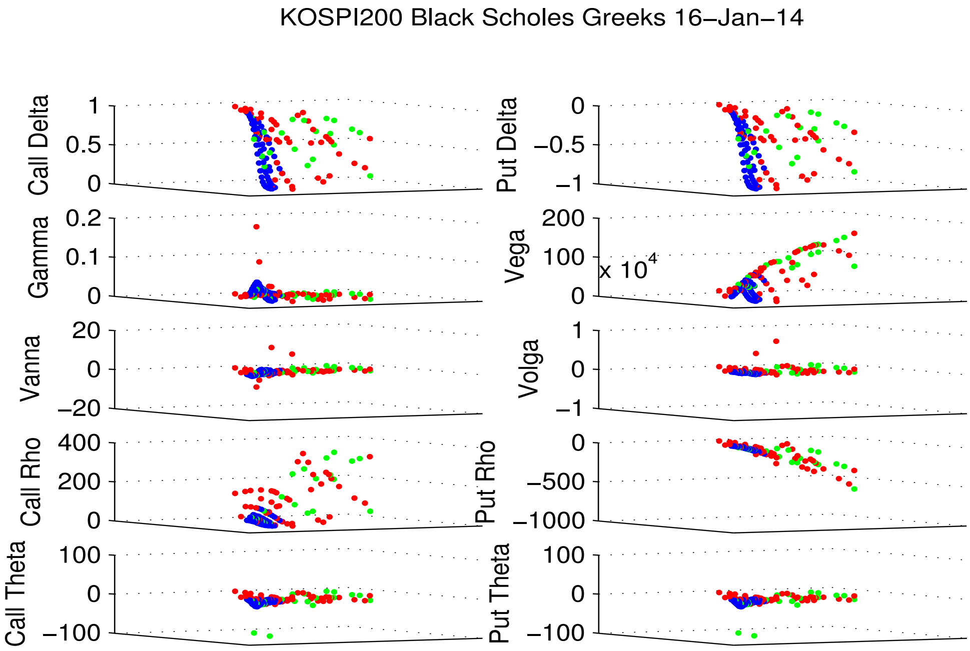


Figure 6.3: KOSPI200 Black Scholes Greeks (16-Jan-14)

There is a graph of KOSPI200 nearest futures implied volatilities during January 2, 2014 to March 31, 2014 in Figure 6.1. There are graphs of KOSPI200 implied volatility and price on January 16, 2014 in Figure 6.2 And there are graphs of KOSPI200 Black Scholes delta, gamma, vega, vanna, volga, rho, theta on January 16, 2014 in Figure 6.3

Chapter 7

The SABR/Heston Mosaic model

7.1 SABR model

In the SABR model of Hagan *et al*[12], the underlying(in our case, a stock index future F_t^T, of expiry T) follows the dynamics:

$$\begin{aligned} dF_t^T &= \alpha_t^T \left(F_t^T\right)^{\beta} dZ_t^T \\ d\alpha_t^T &= \nu^T \alpha_t^T dW_t^T \\ E^{\mathbb{Q}^T}[dZ_t^T dW_t^T] &= \rho^T dt. \end{aligned} \tag{7.1}$$

The increments dZ_t^T and dW_t^T are the increments of standard Brownian motions under the measure $\mathbb{Q}^T$. Therefore both the forward price(F_t^T) and its volatility(α_t^T) are martingales. The model is fully specified once we add to the equations above the initial conditions $F_0^T(= F^T)$ and $\alpha_0^T(= \alpha^T)$. And the parameters ρ^T and ν^T are specific to a particular expiry T[17].

- α^T: the initial volatility

- β: the exponent for the forward price

- ρ^T: the correlation between the Brownian motions

- ν^T: the volatility of volatility

The price of European call, put in the SABR model are given by Black's model(C^B, P^B)[3] as follows:

$$\begin{aligned} C^B(K,T;\sigma_{imp}(K,T;F^T)) &= e^{-r^T T}[F^T N(d1) - KN(d2)] \\ P^B(K,T;\sigma_{imp}(K,T;F^T)) &= e^{-r^T T}[KN(-d2) - F^T N(-d1)] \\ where \quad d_{1;2} &= \frac{ln\left(\frac{F^T}{K}\right) \pm \frac{\sigma_{imp}(K,T;F^T)^2}{2}T}{\sigma_{imp}(K,T;F^T)\sqrt{T}}. \end{aligned} \tag{7.2}$$

Hagan *et al*[11] use singular perturbation techniques to obtain the plain-vanilla option prices implied by the SABR model, and from these the associated implied volatilities. There is no great fundamental meaning in obtaining implied volatilities rather than prices. However, for very good reasons, these have become the common metric in the market place to communicate the prices of options. The volatility parameter $\sigma_{imp_{SABR}}$ is provided by the SABR model with α^T, β, ρ^T and ν^T

- At the money(ATM): $K = F^T$

- In the money(ITM): $K > F^T$ at call, $\quad K < F^T$ at put

- Out the money(OTM): $K < F^T$ at call, $\quad K > F^T$ at put

$$
\begin{aligned}
\sigma_{imp}(K,T;F^T) &= \sigma_{imp_{SABR}}(K,T;F^T,\alpha^T,\rho^T,\nu^T) \\
&= A\left(\frac{z}{\chi(z)}\right)B \qquad\qquad \text{if } K \neq F^T
\end{aligned} \tag{7.3a}
$$

$$
\begin{aligned}
\sigma_{imp^{ATM}}(T;F^T) &= \sigma_{imp}(F^T,T;F^T) \\
&= \sigma_{imp^{ATM}_{SABR}}(T;F^T,\alpha^T,\rho^T,\nu^T) \\
&= \sigma_{imp_{SABR}}(F^T,T;F^T,\alpha^T,\rho^T,\nu^T) \\
&= AB
\end{aligned} \tag{7.3b}
$$

where

$$
\begin{aligned}
A &= \frac{\alpha^T}{(F^TK)^{\frac{1-\beta}{2}}[1+\frac{(1-\beta)^2}{24}\ln^2\frac{F^T}{K}+\frac{(1-\beta)^4}{1920}\ln^4\frac{F^T}{K}+\dots]}, \\
B &= \left[1+\left(\frac{(1-\beta)^2}{24}\frac{(\alpha^T)^2}{(F^TK)^{1-\beta}}+\frac{\rho^T\beta\nu^T\alpha^T}{4(F^TK)^{\frac{1-\beta}{2}}}+\frac{2-3\rho^2}{24}\nu^T\right)T+\dots\right], \\
z &= \frac{\nu^T}{\alpha^T}(F^TK)^{\frac{1-\beta}{2}}\ln\frac{F^T}{K}, \\
\chi(z) &= \ln\left(\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right).
\end{aligned}
$$

This approximate expression is very accurate as long as one does not look at strikes that are too out-of-money or expiries that are too long. What 'too-out-of-the-money' or 'too long' means depends on the volatility and the volatility of volatility[17].

Estimation: β

$$
\ln\sigma_{imp^{ATM}_{SABR}}(T) \approx \ln\alpha^T-(1-\beta)\ln F^T.
$$

β can be estimated from a linear regression on a time series of logs of ATM volatilities and logs of forward prices. Alternatively, β can be chosen from prior beliefs about which model ($\beta = 0$: stochastic normal, $\beta = 1$: stochastic lognormal, $\beta = \frac{1}{2}$: stochastic CIR) is appropriate[19].

$$\begin{aligned}
[C(K,T)]^M &= C^B(K,T;[\sigma_{imp}(K,T)]^M),\\
[P(K,T)]^M &= P^B(K,T;[\sigma_{imp}(K,T)]^M).
\end{aligned}$$

Once β is set, it remains calibrate other parameters to market implied volatilities ($[\sigma_{imp}(K,T)]^M$) under $T(= t_j, j = m, \ldots, 1)$[19]. Other notations are shared with (2.3), (2.4).

Calibration method 1: α^T, ρ^T, ν^T

$$\underset{\alpha^T,\rho^T,\nu^T}{\operatorname{argmin}} \quad \sum_{i=1}^{n} \left\| \begin{aligned} &[\sigma_{imp}(K_i,T)]^M \\ &- \sigma_{imp_{SABR}}(K_i,T;F^T,\alpha^T,\rho^T,\nu^T) \end{aligned} \right\|_p \tag{7.4}$$

Calibration method 2: ρ^T, ν^T

$$\begin{aligned}
\underset{\rho^T,\nu^T}{\operatorname{argmin}} \quad & \sum_{i=1}^{n} \left\| \begin{aligned} &[\sigma_{imp}(K_i,T)]^M \\ &- \sigma_{imp_{SABR}}(K_i,T;F^T,\alpha^T,\rho^T,\nu^T) \end{aligned} \right\|_p \\
\text{s. t.} \quad & \frac{(1-\beta)^2 T}{24(F^T)^{2-2\beta}}\left(\alpha^T\right)^3 + \frac{\rho^T\beta\nu^T T}{4(F^T)^{1-\beta}}\left(\alpha^T\right)^2 \\
& + \left(1 + \frac{2-3\left(\rho^T\right)^2}{24}\nu^T T\right)\left(\alpha^T\right) \\
& - \left[\sigma_{imp^{ATM}}(T)\right]^M \left(F^T\right)^{1-\beta} = 0
\end{aligned} \tag{7.5}$$

7.2 Heston model

Heston model is represented by the bivariate system of stochastic differential equations(SDEs)

$$
\begin{aligned}
dS_t &= (r^T - q^T)S_t dt + \sqrt{v_t} S_t dW_{1,t} \\
dv_t &= \kappa(\theta^T - v_t)dt + \sigma^T \sqrt{v_t} dW_{2,t}
\end{aligned}
\tag{7.6}
$$

where $E^{\mathbb{Q}}[dW_{1,t}, dW_{2,t}] = \rho^T dt$.

S_t follows a Black-Scholes-type stochatic process, but with a stochastic variance v_t that follows a Cox, Ingersol and Ross(1985) Process. If the conditioin $2\kappa\theta > \sigma^2$ holds, then the drift is sufficiently large for the variance process to be guaranteed positive and not reach zero[18]. This condition is known as the Feller condition. We assume the time dependent parameters of model[1].

- κ the mean reversion speed for the variance
- θ^T the mean reversion level for the variance
- σ^T the volatility of the variance
- v_0 the initial level of the variance
- ρ^T the correlation between the Brownian motions

The fast Fourier transform(FFT) was applied by Carr and Madan(1999) to speed up the computation of option prices[4]. To implement the FFT on the call price, first create the integration grid $v_j, j = 1, ..., N$ and the log-strike grid $k_u, u = 1, ..., N$. Define the points x_j for $j = 1, ..., N$ as

$$
\begin{aligned}
x_j \quad = \quad & e^{i(b - \ln S_t)v_j} \psi(v_j) w_j \\
& with \quad v_j = (j-1)\eta, \\
& \qquad k_u = -b + (u-1)\lambda + \ln S_t, \\
& \qquad b = \frac{N\lambda}{2},
\end{aligned}
$$

$$
\begin{aligned}
\lambda\eta &= \frac{2\pi}{N},\\
\psi(v_j) &= \frac{e^{r\tau} f_2(v_j-(\alpha+1)i)}{\alpha^2+\alpha-v_j^2+iv_j(2\alpha+1)},\\
f_2(\phi) &= \exp\left(i\phi\left[\ln S_t + \left(r-\frac{1}{2}\sigma_i mp^2\right)\tau\right] - \frac{1}{2}\phi^2\sigma_i mp^2\tau\right).
\end{aligned}
$$

Define $\widehat{x}_u$, the call price evaluated at the log-strike point k_u. Each call price $\widehat{x}_u$ can be obtained from the set $x_j, j=1,...,N$ via the fast Fourier transform as the sum

$$
\begin{aligned}
\widehat{x}_u \quad &= \quad \frac{\eta e^{-\alpha k_u}}{\pi}\sum_{j=1}^{N} Re\left[e^{i\frac{2\pi}{N}(j-1)(u-1)}x_j\right] \qquad \text{for } u=1,\ldots,N \qquad (7.7)\\
&\qquad where \quad k_u = \ln K_u, \quad \alpha \text{ is a damping parameter.}
\end{aligned}
$$

Heston call prices(C^{Heston}) are obtained from (7.7) directly[18], and Heston put prices(P^{Heston}) are obtained by (5.1)

$$
\begin{aligned}
P^{Heston}(K;T,\kappa,\theta^T,\sigma^T,v_0,\rho^T,\alpha) &= C^{Heston}(K;T,\kappa,\theta^T,\sigma^T,v_0,\rho^T,\alpha)\\
&\quad - e^{-r^T T}[F^T-K].
\end{aligned} \tag{7.8}
$$

Calibrate parameters to market calls($[C(K,T)]^M$) and puts($[P(K,T)]^M$) using reciprocal of the Black Scholes vega ($Vega^{BS}$) as weight under $T(=t_j, j=m,\ldots,1)$[18]. Other notations are shared with (2.3), (2.4).

Calibration method: κ, θ^T, σ^T, v_0, ρ^T

$$
\begin{aligned}
\underset{\kappa,\theta^T,\sigma^T,v_0,\rho^T}{\text{argmin}} \quad & \sum_{i=1}^{n_c}\left\|\frac{[C(K_i,T)]^M - C^{Heston}(K_i;T,\kappa,\theta^T,\sigma^T,v_0,\rho^T,\alpha)}{Vega^{BS}(S;K_i,T)}\right\|_p\\
&+\sum_{j=1}^{n_p}\left\|\frac{[P(K_j,T)]^M - P^{Heston}(K_j;T,\kappa,\theta^T,\sigma^T,v_0,\rho^T,\alpha)}{Vega^{BS}(S;K_j,T)}\right\|_p\\
&\qquad where \quad Vega^{BS}(S;K,T) = S_0 e^{-q^T T}N(d1)\sqrt{T}
\end{aligned} \tag{7.9}
$$

7.3 Stochastic volatility Mosaic model

Stochastic volatility(SV) Mosaic model concept is simple. Calibrate the parameters of stochastic volatility model(in our case SABR and Heston model) 'forward moneyness ($Y = \frac{K}{F^T}$) dependent' as well as 'expiry (T) dependent' on (7.4), (7.5), (7.9)

$$\begin{aligned} \text{SABR}: \quad & \alpha^T, \rho^T, \nu^T \Rightarrow \quad \alpha^{(Y,T)}, \rho^{(Y,T)}, \nu^{(Y,T)} \\ \text{Heston}: \quad & \theta^T, \sigma^T, \rho^T \Rightarrow \quad \theta^{(Y,T)}, \sigma^{(Y,T)}, \rho^{(Y,T)}. \end{aligned} \tag{7.10}$$

But implementation method is quite complicated.

Mosaic Component: They are generated on forward moneyness Y axis at a single expiry T

- Bone: The section that parameters target a group of 'market option' and calibrated straight forward.
- Joint: The section that parameters target two options that changes 'market or model option' in condition. They could be located on the part of system Bone-to-Bone, Bone-to-Joint, Bone-to-Wing, Joint-to-Joint, Joint-to-Wing, Wing-to-Wing. They target 'model option' in border of Bone and target 'market option' in border of Wing. In Joint case, also target 'market option'.
- Wing: The section that parameters target a 'model option' on one of both ends of Y and a 'supposed option' much far from ATM than the former for numerical pricing.

Mosaic System: Its a system that Y domain is completely composed of nonoverlaped Mosaic Components set.

- Bone Wing: It is the same with expiry dependent parameters system, except we 'suppose' the option in the section market data does not exist on the basis of historical time series data. First calibrate Bone, afterwards calibrate Wing.

- Bone Joint Wing: It is divided by 'baseline' and re-connected by Joint that always over the baseline. First calibrate Bone, afterwards calibrate Joint and Wing.

- Joint Wing: It divided by 'all market points'. An interval of Joint is between two close-in market datas. The number of parameters set is the number of market points + 1. Joint sections are the number of market points - 1, Wing sections are 2.

Algorithm 7.1 Mosaic Model Design

1: set Mosaic base line:
   To divide (Y, T) domain of model into 'Mosaic'.
2: set Mosaic component:
   + set bound and interval type(open or closed).
3: set target pointer:
   informatioin of options to target per unit Mosaic component.

Algorithm 7.2 Mosaic Model Calibration

1: set Mosaic model target:
   set target depending on Mosaic System.
2: calibrate Mosaic model paramters:
3: **if** SABR **then**
4:   $\sigma_{imp} \leftarrow$ target
5:   calibrate $\Rightarrow \alpha^{(Y,T)}, \beta, \rho^{(Y,T)}, \nu^{(Y,T)}$
6: **else if** Heston **then**
7:   $C, P, Vega \leftarrow$ target
8:   calibrate $\Rightarrow \kappa, \theta^{(Y,T)}, \sigma^{(Y,T)}, v_0, \rho^{(Y,T)}$
9: **end if**

Table 7.1: SABR BoneWing KOSPI200 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.93)		1.84E+00	2.69E-01	4.62E-01
Bone	[0.93, 1.11]	(0.00, 0.08]	1.94E+00	6.64E-01	3.05E+00
Wing	(1.11,+∞)		3.87E+00	-1.28E-01	2.45E-01
Wing	(-∞, 0.86)		7.84E+00	7.23E-01	6.65E-01
Bone	[0.86, 1.11]	(0.08, 0.15]	1.28E+00	-3.52E-01	7.91E+00
Wing	(1.11,+∞)		6.55E+00	-1.51E-01	2.15E-01
Wing	(-∞, 0.92)		2.58E+00	3.91E-01	3.57E-01
Bone	[0.92, 1.05]	(0.15, 0.23]	2.23E+00	-4.74E-01	7.79E-01
Wing	(1.05,+∞)		2.13E+00	-1.07E-01	3.24E-01
Wing	(-∞, 0.81)		5.56E+00	6.85E-01	4.70E-01
Bone	[0.81, 1.16]	(0.23, 0.40]	4.29E-02	-6.28E-01	6.79E+00
Wing	(1.16,+∞)		3.62E+00	-1.36E-01	2.06E-01
Wing	(-∞, 0.85)		3.82E+00	5.22E-01	3.80E-01
Bone	[0.85, 0.99]	(0.40, 0.65]	2.13E+00	3.75E-01	2.41E+00
Wing	(0.99,+∞)		1.65E+00	-9.16E-02	3.88E-01
Wing	(-∞, 0.58)		5.02E+00	-1.09E-01	4.36E-01
Bone	[0.58, 1.23]	(0.65, 0.90]	3.57E-03	6.80E-01	5.50E+00
Wing	(1.23,+∞)		3.70E+00	-9.38E-02	4.23E-01
Wing	(-∞, 1.20]	(0.90, 1.15]	2.62E+00	6.68E-01	1.67E-01
Wing	(1.20,+∞)		2.72E+00	-1.22E-01	3.46E-01
Wing	(-∞, 0.98)	(1.15, 1.40]	2.67E+00	6.25E-01	2.80E-01
Wing	[0.98,+∞)		2.78E+00	-3.51E-01	3.86E-01
Wing	(-∞, 0.98)	(1.40, 1.65]	2.91E+00	6.38E-01	2.98E-01

Wing	[0.98,+∞)		2.99E+00	-1.07E-01	3.62E-01
Wing	(-∞, 0.69)		3.64E+00	1.70E-01	9.11E-01
Bone	[0.69, 1.26]	(1.65, 1.90]	2.09E+00	-6.36E-02	1.29E+00
Wing	(1.26,+∞)		3.86E+00	-1.15E-01	3.06E-01
Wing	(-∞, 0.70)		5.46E+00	6.46E-01	4.40E-01
Bone	[0.70, 0.89]	(1.90, 2.15]	5.37E+00	5.68E-01	8.86E-01
Wing	(0.89,+∞)		5.50E+00	-5.36E-02	4.70E-01
Wing	(-∞, 0.63)		4.33E+00	1.35E-01	7.42E-01
Bone	[0.63, 0.95]	(2.15,+∞]	3.78E+00	1.04E-01	9.51E-01
Wing	(0.95,+∞)		3.92E+00	-8.12E-01	6.71E-01

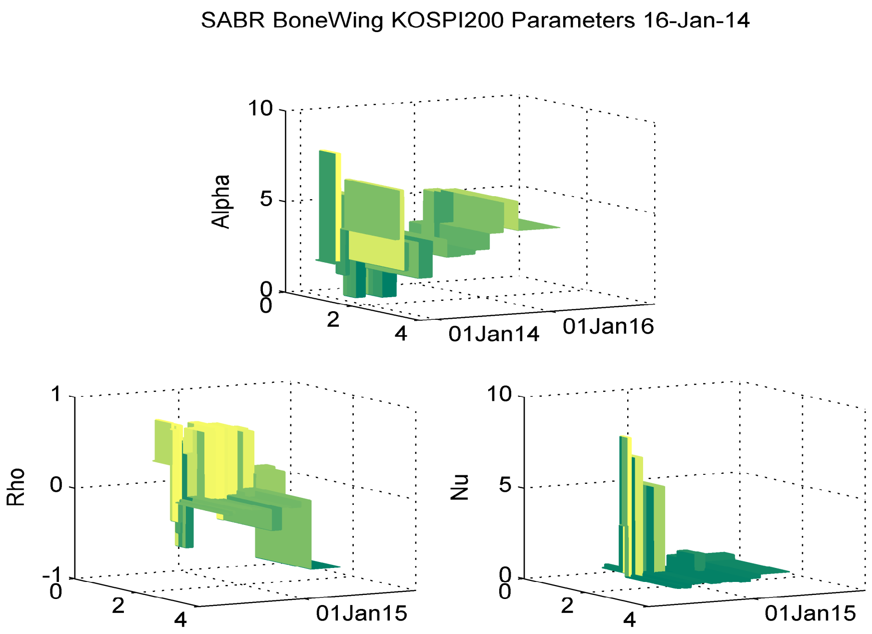


Figure 7.1: SABR BoneWing KOSPI200 Parameters

Table 7.2: Heston BoneWing KOSPI200 Parameters (κ = 1.15E+00, v0 = 1.92E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.93)		4.80E-02	8.24E-01	-9.80E-01
Bone	[0.93, 1.11]	(0.00, 0.08]	3.93E-02	9.99E-01	-5.02E-01
Wing	(1.11,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.86)		4.80E-02	8.24E-01	-9.80E-01
Bone	[0.86, 1.11]	(0.08, 0.15]	1.00E-05	6.03E-01	-9.99E-01
Wing	(1.11,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.92)		4.80E-02	8.24E-01	-9.80E-01
Bone	[0.92, 1.05]	(0.15, 0.23]	1.27E-01	2.60E+00	1.70E-02
Wing	(1.05,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.81)		1.96E-02	4.13E-03	4.07E-01
Bone	[0.81, 1.16]	(0.23, 0.40]	5.94E-05	9.99E+00	-9.85E-01
Wing	(1.16,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.85)		2.05E-02	4.65E-03	4.34E-01
Bone	[0.85, 0.99]	(0.40, 0.65]	3.75E-01	6.16E+00	-3.96E-01
Wing	(0.99,+∞)		5.13E-02	1.37E+00	-9.80E-01
Wing	(-∞, 0.58)		2.82E-01	9.36E-01	3.15E-01
Bone	[0.58, 1.23]	(0.65, 0.90]	1.56E-01	1.00E+01	-9.99E-01
Wing	(1.23,+∞)		5.04E-02	1.09E+00	-9.80E-01
Wing	(-∞, 1.20]	(0.90, 1.15]	4.11E-02	1.00E-05	-6.66E-01
Wing	(1.20,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.98)	(1.15, 1.40]	3.56E-05	2.28E-01	8.07E-01
Wing	[0.98,+∞)		4.94E-02	7.27E-01	-9.84E-01
Wing	(-∞, 0.98)	(1.40, 1.65]	1.35E-02	4.39E-02	2.00E-01

Wing	[0.98,+∞)		4.68E-02	4.39E+00	9.89E-01
Wing	(-∞, 0.69)		7.98E-02	3.61E+00	-6.16E-01
Bone	[0.69, 1.26]	(1.65, 1.90]	4.80E-02	8.24E-01	-9.80E-01
Wing	(1.26,+∞)		1.57E-01	5.52E-01	-8.54E-01
Wing	(-∞, 0.70)		2.57E-01	2.15E-03	4.45E-01
Bone	[0.70, 0.89]	(1.90, 2.15]	1.02E+00	1.13E+00	-5.20E-01
Wing	(0.89,+∞)		1.44E-01	3.66E-03	-9.80E-01
Wing	(-∞, 0.63)		4.69E-01	9.82E-01	-5.25E-01
Bone	[0.63, 0.95]	(2.15,+∞]	9.95E-01	1.14E+00	5.06E-02
Wing	(0.95,+∞)		1.45E-01	1.02E-05	-9.34E-01

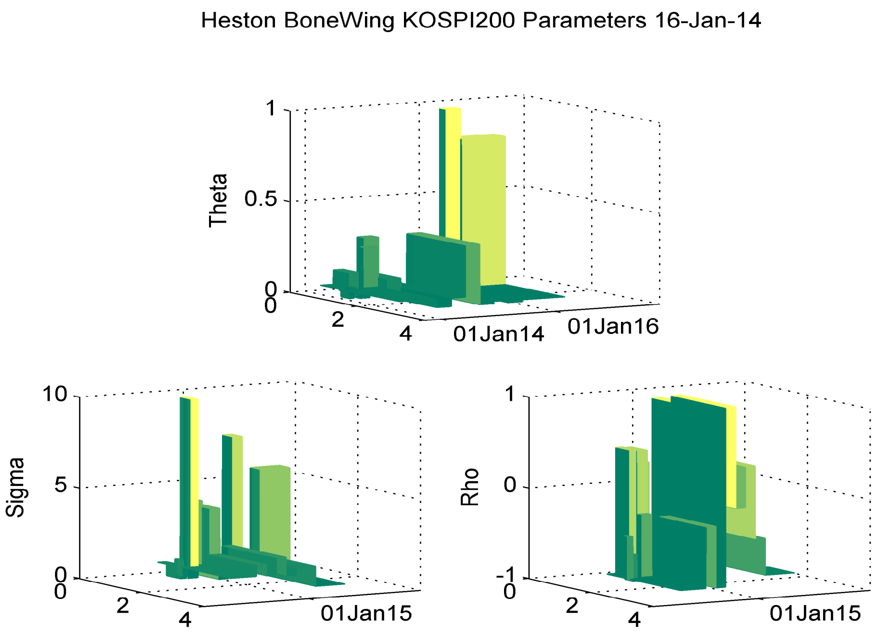


Figure 7.2: Heston BoneWing KOSPI200 Parameters

Table 7.3: SABR BoneJointWing KOSPI200 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.93)		2.56E+00	4.10E-01	3.53E-01
Bone	[0.93, 0.95]		2.07E+00	2.38E-01	2.74E+00
Joint	(0.95, 0.96)		4.34E-01	1.46E-01	6.67E+00
Bone	[0.96, 1.05]	(0.00, 0.08]	1.91E+00	7.27E-01	5.00E+00
Joint	(1.05, 1.06)		3.01E+00	-7.02E-01	1.90E+00
Bone	[1.06, 1.11]		1.80E+00	-4.45E-01	2.55E+00
Wing	(1.11,+∞)		2.27E+00	-1.19E-01	2.92E-01
Wing	(-∞, 0.86)		3.05E+00	2.75E-01	5.53E-01
Bone	[0.86, 0.95]		2.15E+00	3.82E-01	2.41E+00
Joint	(0.95, 0.96)		4.73E+00	6.78E-01	3.73E+00
Bone	[0.96, 1.04]	(0.08, 0.15]	9.73E-01	-2.42E-01	1.00E+01
Joint	(1.04, 1.05)		1.36E+00	-2.01E-01	9.69E+00
Bone	[1.05, 1.11]		6.84E+00	-9.27E-02	1.62E+00
Wing	(1.11,+∞)		7.14E+00	-1.52E-01	2.06E-01
Wing	(-∞, 0.92)		2.51E+00	3.79E-01	3.68E-01
Bone	[0.92, 0.93]		3.01E+00	7.01E-01	2.15E+00
Joint	(0.93, 0.95)	(0.15, 0.23]	2.43E+00	4.40E-01	2.32E+00
Bone	[0.95, 1.05]		2.22E+00	-4.10E-01	9.68E-01
Wing	(1.05,+∞)		2.13E+00	-1.10E-01	3.24E-01
Wing	(-∞, 0.81)		3.16E+00	2.57E-01	5.87E-01
Bone	[0.81, 0.95]		2.72E+00	5.74E-01	1.97E+00
Joint	(0.95, 0.97)		3.53E+00	7.95E-01	3.49E+00
Bone	[0.97, 1.05]	(0.23, 0.40]	1.88E-02	-3.09E-01	9.75E+00

Joint	(1.05, 1.07)		3.52E+00	-7.38E-01	2.59E+00
Bone	[1.07, 1.16]		2.10E+00	-7.09E-01	5.79E-01
Wing	(1.16,+∞)		1.69E+00	-1.32E-01	2.64E-01
Wing	(-∞, 0.85)		2.81E+00	2.92E-01	4.81E-01
Bone	[0.85, 0.93]		2.81E+00	6.24E-01	1.79E+00
Joint	(0.93, 0.97)	(0.40, 0.65]	3.64E+00	8.04E-01	2.76E+00
Bone	[0.97, 0.99]		3.70E-02	-3.84E-01	8.23E+00
Wing	(0.99,+∞)		1.66E+00	-1.28E-01	3.10E-01
Wing	(-∞, 0.58)		2.66E+00	2.18E-01	9.54E-01
Bone	[0.58, 0.92]		2.90E+00	3.83E-01	1.03E+00
Joint	(0.92, 0.96)		3.90E+00	8.04E-01	2.77E+00
Bone	[0.96, 1.02]	(0.65, 0.90]	9.86E-04	3.70E-01	9.29E+00
Joint	(1.02, 1.12)		1.23E+00	8.10E-02	2.91E+00
Bone	[1.12, 1.23]		3.99E+00	-2.68E-01	1.01E+00
Wing	(1.23,+∞)		4.23E+00	-1.14E-01	4.08E-01
Wing	(-∞, 1.20]	(0.90, 1.15]	2.62E+00	6.68E-01	1.67E-01
Wing	(1.20,+∞)		2.72E+00	-1.22E-01	3.46E-01
Wing	(-∞, 0.98)	(1.15, 1.40]	2.67E+00	6.25E-01	2.80E-01
Wing	[0.98,+∞)		2.78E+00	-3.51E-01	3.86E-01
Wing	(-∞, 0.98)	(1.40, 1.65]	2.91E+00	6.38E-01	2.98E-01
Wing	[0.98,+∞)		2.99E+00	-1.07E-01	3.62E-01
Wing	(-∞, 0.69)		1.86E+00	2.37E-01	1.29E+00
Bone	[0.69, 0.94]		3.59E+00	4.07E-01	8.17E-01
Joint	(0.94, 0.96)		6.06E-01	1.96E-01	3.51E+00
Bone	[0.96, 1.03]	(1.65, 1.90]	2.23E+00	-5.62E-01	1.59E+00
Joint	(1.03, 1.15)		1.71E+00	-1.83E-01	1.64E+00
Bone	[1.15, 1.26]		2.73E+00	-2.33E-01	1.04E+00

Wing	(1.26,+∞)		3.56E+00	-1.24E-01	3.15E-01
Wing	(-∞, 0.70)		5.46E+00	6.46E-01	4.40E-01
Bone	[0.70, 0.89]	(1.90, 2.15]	5.37E+00	5.68E-01	8.86E-01
Wing	(0.89,+∞)		5.50E+00	-5.36E-02	4.70E-01
Wing	(-∞, 0.63)		4.33E+00	1.35E-01	7.42E-01
Bone	[0.63, 0.95]	(2.15,+∞]	3.78E+00	1.04E-01	9.51E-01
Wing	(0.95,+∞)		3.92E+00	-8.12E-01	6.71E-01

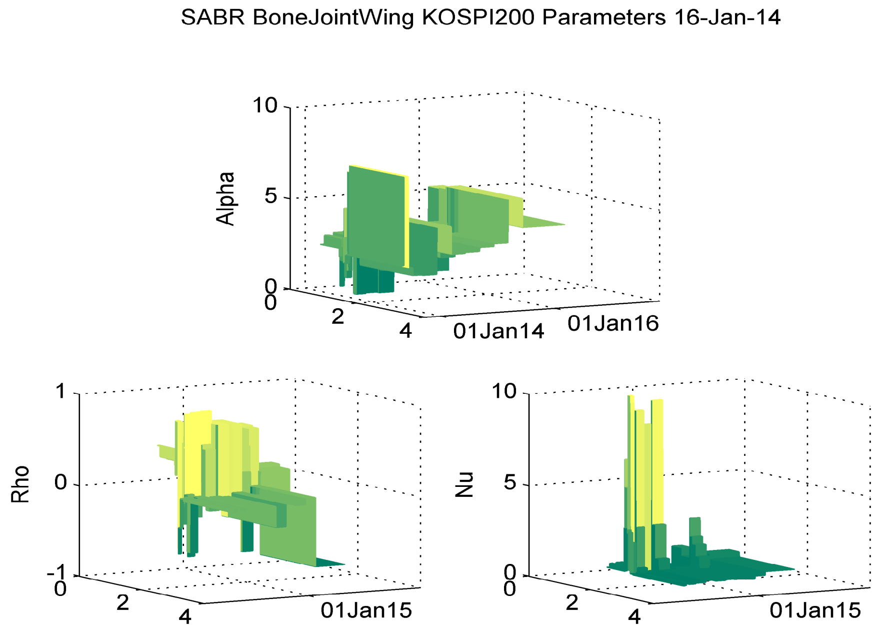


Figure 7.3: SABR BoneJointWing KOSPI200 Parameters

Table 7.4: Heston BoneJointWing KOSPI200 Parameters (κ = 1.15E+00, v0 = 1.92E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.93)		4.80E-02	8.24E-01	-9.80E-01
Bone	[0.93, 0.95]		1.22E+00	4.42E+00	8.35E-01
Joint	(0.95, 0.96)		1.09E+00	5.26E+00	8.02E-01
Bone	[0.96, 1.05]	(0.00, 0.08]	6.08E-02	6.19E+00	-1.67E-01
Joint	(1.05, 1.06)		5.29E-01	1.03E+00	-9.94E-01
Bone	[1.06, 1.11]		1.29E-01	1.17E+00	-7.20E-01
Wing	(1.11,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.86)		4.80E-02	8.24E-01	-9.80E-01
Bone	[0.86, 0.95]		1.01E-05	4.49E-01	9.99E-01
Joint	(0.95, 0.96)		1.30E-05	1.01E-05	-7.59E-01
Bone	[0.96, 1.04]	(0.08, 0.15]	1.10E-05	3.45E+00	-9.99E-01
Joint	(1.04, 1.05)		1.28E+00	7.92E+00	-9.99E-01
Bone	[1.05, 1.11]		1.00E-05	6.03E-01	-9.99E-01
Wing	(1.11,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.92)		4.80E-02	8.24E-01	-9.80E-01
Bone	[0.92, 0.93]		2.01E-01	8.73E-01	6.62E-01
Joint	(0.93, 0.95)	(0.15, 0.23]	1.38E-01	1.88E+00	1.96E-01
Bone	[0.95, 1.05]		4.88E-01	7.78E+00	-1.06E-01
Wing	(1.05,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.81)		1.96E-02	4.13E-03	4.07E-01
Bone	[0.81, 0.95]		9.82E-02	2.90E+00	1.59E-01
Joint	(0.95, 0.97)		8.51E-04	2.88E+00	-9.92E-01
Bone	[0.97, 1.05]	(0.23, 0.40]	2.04E-03	9.00E+00	-9.57E-01

Joint	(1.05, 1.07)		4.95E-02	1.07E+00	-9.93E-01
Bone	[1.07, 1.16]		7.44E-01	9.49E+00	-9.99E-01
Wing	(1.16,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.85)	(0.40, 0.65]	2.05E-02	4.65E-03	4.34E-01
Bone	[0.85, 0.93]		8.13E-02	8.43E-01	2.81E-01
Joint	(0.93, 0.97)		1.14E-01	1.82E+00	8.74E-02
Bone	[0.97, 0.99]		3.75E-01	6.16E+00	-3.96E-01
Wing	(0.99,+∞)		5.13E-02	1.37E+00	-9.80E-01
Wing	(-∞, 0.58)	(0.65, 0.90]	2.14E-01	9.88E-01	3.13E-01
Bone	[0.58, 0.92]		1.91E-01	8.60E-01	9.62E-01
Joint	(0.92, 0.96)		1.38E-02	2.05E+00	-9.99E-01
Bone	[0.96, 1.02]		1.09E-05	1.00E+01	-9.99E-01
Joint	(1.02, 1.12)		1.11E-01	7.47E+00	-9.99E-01
Bone	[1.12, 1.23]		1.56E-01	1.00E+01	-9.99E-01
Wing	(1.23,+∞)		5.04E-02	1.09E+00	-9.80E-01
Wing	(-∞, 1.20]	(0.90, 1.15]	4.11E-02	1.00E-05	-6.66E-01
Wing	(1.20,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.98)	(1.15, 1.40]	3.56E-05	2.28E-01	8.07E-01
Wing	[0.98,+∞)		4.94E-02	7.27E-01	-9.84E-01
Wing	(-∞, 0.98)	(1.40, 1.65]	1.35E-02	4.39E-02	2.00E-01
Wing	[0.98,+∞)		4.68E-02	4.39E+00	9.89E-01
Wing	(-∞, 0.69)	(1.65, 1.90]	2.72E-02	1.40E+00	-2.35E-01
Bone	[0.69, 0.94]		3.36E-02	1.70E+00	-4.11E-01
Joint	(0.94, 0.96)		2.60E-02	7.72E-01	4.13E-01
Bone	[0.96, 1.03]		4.80E-02	8.24E-01	-9.80E-01
Joint	(1.03, 1.15)		2.10E-02	9.82E+00	-9.29E-01
Bone	[1.15, 1.26]		1.97E-02	6.34E-01	-9.91E-01

Wing	(1.26,+∞)		1.57E-01	5.52E-01	-8.54E-01
Wing	(-∞, 0.70)		2.57E-01	2.15E-03	4.45E-01
Bone	[0.70, 0.89]	(1.90, 2.15]	1.02E+00	1.13E+00	-5.20E-01
Wing	(0.89,+∞)		1.44E-01	3.66E-03	-9.80E-01
Wing	(-∞, 0.63)		4.69E-01	9.82E-01	-5.25E-01
Bone	[0.63, 0.95]	(2.15,+∞]	9.95E-01	1.14E+00	5.06E-02
Wing	(0.95,+∞)		1.45E-01	1.02E-05	-9.34E-01

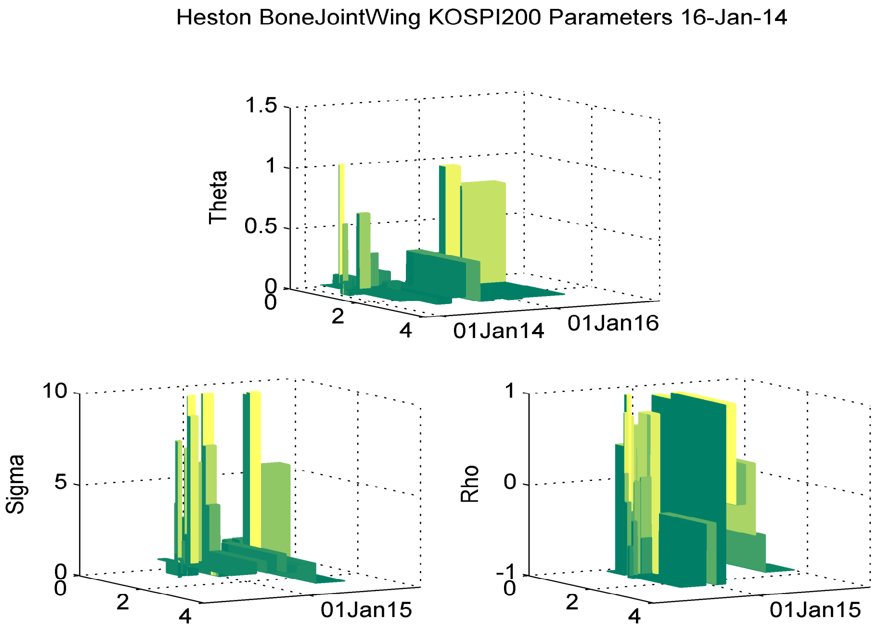


Figure 7.4: Heston BoneJointWing KOSPI200 Parameters

Table 7.5: SABR JointWing KOSPI200 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.93)	(0.00, 0.08]	2.57E+00	4.11E-01	3.51E-01
Joint	[0.93, 0.94)		1.90E+00	1.72E-01	2.94E+00
Joint	[0.94, 0.95)		2.19E+00	2.91E-01	2.57E+00
Joint	[0.95, 0.96)		2.09E+00	2.35E-01	2.71E+00
Joint	[0.96, 0.97)		2.03E+00	1.71E-01	2.70E+00
Joint	[0.97, 0.98)		1.94E+00	6.59E-02	2.78E+00
Joint	[0.98, 0.99)		1.96E+00	7.23E-02	2.65E+00
Joint	[0.99, 1.00)		4.55E-01	-2.26E-01	9.67E+00
Joint	[1.00, 1.00)		1.09E+00	1.58E-01	4.27E+00
Joint	[1.00, 1.01]		1.96E+00	-4.02E-01	1.48E+00
Joint	(1.01, 1.02]		1.84E+00	-3.50E-01	3.99E+00
Joint	(1.02, 1.03]		1.93E+00	-3.71E-01	2.35E+00
Joint	(1.03, 1.04]		2.02E+00	-4.99E-01	2.25E+00
Joint	(1.04, 1.05]		2.03E+00	-5.20E-01	2.21E+00
Joint	(1.05, 1.06]		1.98E+00	-4.79E-01	2.27E+00
Joint	(1.06, 1.06]		2.11E+00	-5.63E-01	2.16E+00
Joint	(1.06, 1.07]		1.59E+00	-2.71E-01	2.50E+00
Joint	(1.07, 1.08]		2.57E+00	-7.40E-01	1.90E+00
Joint	(1.08, 1.09]		8.87E-01	-2.31E-01	3.60E+00
Joint	(1.09, 1.10]		2.32E+00	-4.97E-01	1.60E+00
Joint	(1.10, 1.11]		7.99E-01	-3.60E-01	3.96E+00
Wing	(1.11,+∞)		2.35E+00	-1.20E-01	2.89E-01
Wing	(-∞, 0.86)		3.17E+00	2.99E-01	5.38E-01

Joint	[0.86, 0.93)	2.24E+00	3.59E-01	2.40E+00
Joint	[0.93, 0.94)	4.96E-01	2.20E-01	5.26E+00
Joint	[0.94, 0.94)	1.79E+00	2.69E-01	2.87E+00
Joint	[0.94, 0.95)	2.27E+00	4.04E-01	2.53E+00
Joint	[0.95, 0.96)	6.94E+00	-3.43E-01	6.16E-01
Joint	[0.96, 0.96)	4.25E+00	-2.44E-01	5.68E+00
Joint	[0.96, 0.97)	1.95E+00	1.68E-01	2.79E+00
Joint	[0.97, 0.98)	1.98E+00	1.56E-01	2.62E+00
Joint	[0.98, 0.98)	1.94E+00	7.98E-02	2.86E+00
Joint	[0.98, 0.98)	1.83E+00	-3.97E-02	3.28E+00
Joint	[0.98, 0.99)	2.48E-01	-1.00E-01	9.67E+00
Joint	[0.99, 0.99)	1.14E+00	9.46E-02	3.88E+00
Joint	[0.99, 1.00]	2.10E+00	-7.01E-02	3.50E-01
Joint	(1.00, 1.01]	1.74E+00	-1.15E-02	4.03E+00
Joint	(1.01, 1.01]	1.81E+00	-1.60E-01	3.74E+00
Joint	(1.01, 1.02]	1.91E+00	-2.91E-01	3.12E+00
Joint	(1.02, 1.03]	1.54E+00	1.32E-01	3.55E+00
Joint	(1.03, 1.03]	1.84E+00	-6.83E-02	2.55E+00
Joint	(1.03, 1.04]	1.93E+00	-2.98E-01	2.37E+00
Joint	(1.04, 1.05]	2.33E+00	-6.24E-01	2.02E+00
Joint	(1.05, 1.06]	2.18E+00	-5.20E-01	2.10E+00
Joint	(1.06, 1.07]	1.13E+00	-4.98E-02	9.24E+00
Joint	(1.07, 1.07]	4.21E+00	2.77E-01	4.84E+00
Joint	(1.07, 1.08]	2.53E+00	-6.94E-01	1.90E+00
Joint	(1.08, 1.09]	2.48E+00	-6.88E-01	1.45E+00
Joint	(1.09, 1.10]	2.08E+00	-5.00E-01	1.78E+00
Joint	(1.10, 1.11]	7.73E-01	-1.97E-01	4.28E+00

Joint	(1.11, 1.11]		2.09E+00	-2.09E-01	2.39E+00
Wing	(1.11,+∞)		2.09E+00	-1.20E-01	3.21E-01
Wing	(-∞, 0.92)	(0.15, 0.23]	2.51E+00	3.81E-01	3.69E-01
Joint	[0.92, 0.93)		3.01E+00	7.01E-01	2.15E+00
Joint	[0.93, 0.95)		2.20E+00	3.29E-01	2.43E+00
Joint	[0.95, 0.96)		2.38E+00	4.68E-01	2.47E+00
Joint	[0.96, 0.97)		1.93E+00	1.32E-01	2.84E+00
Joint	[0.97, 0.99)		1.94E+00	2.17E-01	3.07E+00
Joint	[0.99, 1.00]		2.25E+00	8.01E-01	1.54E-01
Joint	(1.00, 1.02]		2.06E+00	-4.27E-01	2.80E+00
Joint	(1.02, 1.04]		2.02E+00	-3.25E-01	2.42E+00
Joint	(1.04, 1.05]		2.37E+00	-5.78E-01	2.04E+00
Wing	(1.05,+∞)		2.12E+00	-1.07E-01	3.24E-01
Wing	(-∞, 0.81)	(0.23, 0.40]	2.98E+00	2.46E-01	6.21E-01
Joint	[0.81, 0.85)		2.88E+00	5.05E-01	1.46E+00
Joint	[0.85, 0.89)		2.74E+00	4.72E-01	1.52E+00
Joint	[0.89, 0.91)		2.38E+00	4.63E-01	2.10E+00
Joint	[0.91, 0.93)		2.80E+00	6.37E-01	2.34E+00
Joint	[0.93, 0.95)		3.24E+00	7.78E-01	2.08E+00
Joint	[0.95, 0.95)		1.26E+00	2.66E-02	3.48E+00
Joint	[0.95, 0.97)		3.73E+00	7.80E-01	3.75E+00
Joint	[0.97, 0.97)		1.33E+00	-1.74E-01	4.57E+00
Joint	[0.97, 0.99)		2.66E-02	-4.39E-02	8.21E+00
Joint	[0.99, 0.99)		1.52E+00	3.17E-01	2.54E+00
Joint	[0.99, 1.01]		1.62E-02	-8.93E-02	1.00E+01
Joint	(1.01, 1.01]		3.93E-01	5.33E-02	6.15E+00
Joint	(1.01, 1.03]		1.96E+00	-2.80E-01	2.77E+00

Joint	(1.03, 1.05]		2.35E+00	-5.32E-01	2.29E+00
Joint	(1.05, 1.05]		1.06E+00	1.36E-01	3.46E+00
Joint	(1.05, 1.07]		2.50E+00	-5.66E-01	1.85E+00
Joint	(1.07, 1.08]		2.55E+00	-5.83E-01	1.66E+00
Joint	(1.08, 1.09]		1.56E+00	-2.23E-01	2.34E+00
Joint	(1.09, 1.12]		2.67E+00	-6.19E-01	1.40E+00
Joint	(1.12, 1.16]		2.00E+00	-6.70E-01	4.30E-01
Wing	(1.16,+∞)		1.13E+00	-1.47E-01	2.94E-01
Wing	(-∞, 0.85)		2.80E+00	2.83E-01	4.77E-01
Joint	[0.85, 0.93)		2.81E+00	6.24E-01	1.79E+00
Joint	[0.93, 0.97)	(0.40, 0.65]	3.68E+00	8.04E-01	2.79E+00
Joint	[0.97, 0.99]		3.70E-02	-3.84E-01	8.23E+00
Wing	(0.99,+∞)		1.66E+00	2.23E-02	2.92E-01
Wing	(-∞, 0.58)		1.94E+00	2.69E-01	1.15E+00
Joint	[0.58, 0.65)		2.14E+00	2.20E-01	1.05E+00
Joint	[0.65, 0.69)		2.36E+00	2.16E-01	9.62E-01
Joint	[0.69, 0.77)		2.67E+00	2.50E-01	8.48E-01
Joint	[0.77, 0.85)		2.61E+00	2.82E-01	9.32E-01
Joint	[0.85, 0.87)		2.71E+00	5.17E-01	1.39E+00
Joint	[0.87, 0.89)		3.17E+00	7.09E-01	1.82E+00
Joint	[0.89, 0.92)		3.58E+00	7.67E-01	1.53E+00
Joint	[0.92, 0.96)	(0.65, 0.90]	1.48E+00	2.26E-01	2.88E+00
Joint	[0.96, 0.98)		1.24E+00	-4.77E-02	2.89E+00
Joint	[0.98, 1.00]		9.44E-04	-2.48E-01	8.92E+00
Joint	(1.00, 1.02]		9.14E-03	2.24E-01	8.80E+00
Joint	(1.02, 1.12]		3.06E+00	-4.91E-01	1.63E+00
Joint	(1.12, 1.16]		2.83E-01	-2.75E-01	4.22E+00

Joint	(1.16, 1.19]		3.52E+00	-2.28E-01	1.41E+00
Joint	(1.19, 1.23]		2.77E+00	-2.48E-01	4.41E-01
Wing	(1.23,+∞)		2.65E+00	-1.30E-01	4.69E-01
Wing	(-∞, 1.20]	(0.90, 1.15]	2.62E+00	6.68E-01	1.67E-01
Wing	(1.20,+∞)		2.72E+00	-1.22E-01	3.46E-01
Wing	(-∞, 0.98)	(1.15, 1.40]	2.67E+00	6.25E-01	2.80E-01
Wing	[0.98,+∞)		2.78E+00	-3.51E-01	3.86E-01
Wing	(-∞, 0.98)	(1.40, 1.65]	2.91E+00	6.38E-01	2.98E-01
Wing	[0.98,+∞)		2.99E+00	-1.07E-01	3.62E-01
Wing	(-∞, 0.69)	(1.65, 1.90]	4.61E+00	1.64E-01	7.33E-01
Joint	[0.69, 0.73)		2.00E+00	2.36E-01	1.36E+00
Joint	[0.73, 0.77)		7.79E-01	7.08E-01	2.03E+00
Joint	[0.77, 0.80)		2.77E+00	7.08E-01	3.92E-01
Joint	[0.80, 0.84)		3.36E+00	2.31E-01	1.08E+00
Joint	[0.84, 0.86)		3.22E+00	1.34E-01	1.74E+00
Joint	[0.86, 0.94)		7.29E-01	3.39E-01	3.23E+00
Joint	[0.94, 0.96)		1.99E+00	2.69E-01	1.56E+00
Joint	[0.96, 0.98)		3.03E+00	7.95E-01	3.79E+00
Joint	[0.98, 1.00)		8.93E-01	-2.27E-03	3.48E+00
Joint	[1.00, 1.03]		2.60E+00	-7.35E-01	1.20E+00
Joint	(1.03, 1.15]		1.85E+00	-2.96E-01	1.56E+00
Joint	(1.15, 1.23]		1.94E-01	-4.24E-01	3.04E+00
Joint	(1.23, 1.26]		2.73E+00	-2.33E-01	1.04E+00
Wing	(1.26,+∞)		2.72E+00	-1.35E-01	3.43E-01
Wing	(-∞, 0.70)	(1.90, 2.15]	5.30E+00	5.81E-01	3.66E-01
Joint	[0.70, 0.89]		5.37E+00	5.68E-01	8.86E-01
Wing	(0.89,+∞)		5.50E+00	-5.36E-02	4.70E-01

Wing	(-∞, 0.63)		4.22E+00	1.39E-01	7.26E-01
Joint	[0.63, 0.68)		1.99E+00	2.24E-01	1.33E+00
Joint	[0.68, 0.82)	(2.15,+∞]	4.91E+00	4.29E-01	7.39E-01
Joint	[0.82, 0.87)		2.36E+00	1.45E-01	1.76E+00
Joint	[0.87, 0.95]		1.72E+00	1.54E-01	2.05E+00
Wing	(0.95,+∞)		3.64E+00	-2.70E-01	4.09E-01

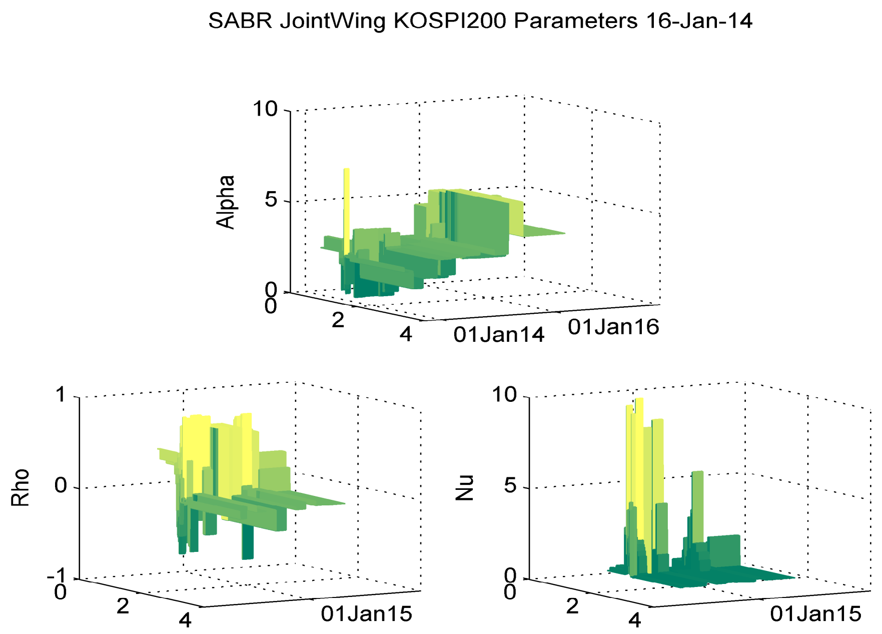


Figure 7.5: SABR JointWing KOSPI200 Parameters

Table 7.6: Heston JointWing KOSPI200 Parameters (κ = 1.15E+00, v0 = 1.92E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.93)	(0.00, 0.08]	4.80E-02	8.24E-01	-9.80E-01
Joint	[0.93, 0.94)		1.18E+00	4.89E+00	8.31E-01
Joint	[0.94, 0.95)		8.52E-01	3.07E+00	7.75E-01
Joint	[0.95, 0.96)		1.10E+00	5.27E+00	7.94E-01
Joint	[0.96, 0.97)		1.87E-02	7.38E-01	3.28E-02
Joint	[0.97, 0.98)		1.16E-01	1.23E+00	2.57E-01
Joint	[0.98, 0.99)		3.24E-01	2.46E+00	4.27E-01
Joint	[0.99, 1.00)		1.67E-01	8.43E+00	-6.46E-01
Joint	[1.00, 1.00)		1.20E-01	5.82E+00	5.45E-01
Joint	[1.00, 1.01]		5.48E-02	1.14E+00	-2.10E-01
Joint	(1.01, 1.02]		8.14E-01	6.35E+00	-5.88E-01
Joint	(1.02, 1.03]		1.05E+00	6.05E+00	-7.79E-01
Joint	(1.03, 1.04]		1.06E+00	4.64E+00	-8.62E-01
Joint	(1.04, 1.05]		1.01E-01	8.44E-01	-6.63E-01
Joint	(1.05, 1.06]		1.75E-01	1.07E+00	-7.13E-01
Joint	(1.06, 1.06]		8.64E-02	6.85E-01	-6.63E-01
Joint	(1.06, 1.07]		3.47E-01	2.04E+00	-8.05E-01
Joint	(1.07, 1.08]		2.78E-01	5.82E-01	-8.69E-01
Joint	(1.08, 1.09]		2.10E-02	1.50E+00	-7.17E-01
Joint	(1.09, 1.10]		5.37E-01	7.58E-01	-9.09E-01
Joint	(1.10, 1.11]		7.66E-02	2.26E+00	-8.18E-01
Wing	(1.11,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.86)		4.80E-02	8.24E-01	-9.80E-01

Joint	[0.86, 0.93)	6.45E-02	1.22E+00	1.59E-01
Joint	[0.93, 0.94)	1.01E-05	4.49E-01	9.99E-01
Joint	[0.94, 0.94)	1.01E-05	4.49E-01	9.99E-01
Joint	[0.94, 0.95)	7.07E-03	4.45E-01	-3.31E-01
Joint	[0.95, 0.96)	9.10E-01	1.70E+00	2.29E-01
Joint	[0.96, 0.96)	1.72E+00	5.94E+00	1.87E-01
Joint	[0.96, 0.97)	1.65E-01	1.73E+00	2.82E-01
Joint	[0.97, 0.98)	1.58E-02	9.19E+00	-9.72E-01
Joint	[0.98, 0.98)	1.20E-02	9.20E+00	-9.23E-01
Joint	[0.98, 0.98)	1.15E-01	1.46E+00	-1.78E-01
Joint	[0.98, 0.99)	3.12E-01	7.28E+00	-9.64E-01
Joint	[0.99, 0.99)	2.01E-01	6.78E+00	-5.24E-01
Joint	[0.99, 1.00]	5.12E-01	6.42E+00	-9.55E-03
Joint	(1.00, 1.01]	4.84E-01	8.68E+00	-9.99E-01
Joint	(1.01, 1.01]	3.25E-01	6.76E+00	-9.98E-01
Joint	(1.01, 1.02]	6.11E-01	6.02E+00	-5.72E-01
Joint	(1.02, 1.03]	1.10E-05	3.45E+00	-9.99E-01
Joint	(1.03, 1.03]	1.10E-05	3.45E+00	-9.99E-01
Joint	(1.03, 1.04]	8.33E-01	6.00E+00	-7.92E-01
Joint	(1.04, 1.05]	8.80E-01	4.42E+00	-8.85E-01
Joint	(1.05, 1.06]	1.24E-01	9.15E-01	-6.37E-01
Joint	(1.06, 1.07]	1.00E-05	6.03E-01	-9.99E-01
Joint	(1.07, 1.07]	1.00E-05	6.03E-01	-9.99E-01
Joint	(1.07, 1.08]	1.80E-01	8.12E-01	-7.67E-01
Joint	(1.08, 1.09]	2.94E-01	1.05E+00	-8.53E-01
Joint	(1.09, 1.10]	1.80E-01	1.09E+00	-7.52E-01
Joint	(1.10, 1.11]	4.80E-02	8.24E-01	-9.80E-01

Joint	(1.11, 1.11]		4.80E-02	8.24E-01	-9.80E-01
Wing	(1.11,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.92)		4.80E-02	8.24E-01	-9.80E-01
Joint	[0.92, 0.93)		2.01E-01	8.73E-01	6.62E-01
Joint	[0.93, 0.95)		6.82E-02	7.90E-01	-2.36E-02
Joint	[0.95, 0.96)		7.87E-02	5.47E-01	2.42E-01
Joint	[0.96, 0.97)		3.18E-01	3.99E+00	3.23E-01
Joint	[0.97, 0.99)	(0.15, 0.23]	3.47E-01	4.52E+00	3.58E-01
Joint	[0.99, 1.00]		3.88E-01	5.31E+00	7.57E-03
Joint	(1.00, 1.02]		5.55E-01	6.38E+00	-5.69E-01
Joint	(1.02, 1.04]		6.06E-01	6.24E+00	-6.65E-01
Joint	(1.04, 1.05]		1.00E+01	1.00E+01	9.99E-01
Wing	(1.05,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.81)		1.96E-02	4.13E-03	4.07E-01
Joint	[0.81, 0.85)		2.42E-02	3.97E-01	-5.87E-01
Joint	[0.85, 0.89)		5.74E-02	4.91E-01	-1.23E-01
Joint	[0.89, 0.91)		6.34E-02	9.77E-01	-2.46E-02
Joint	[0.91, 0.93)		4.39E-02	2.98E-01	-2.69E-01
Joint	[0.93, 0.95)		6.12E-02	2.46E+00	-9.73E-01
Joint	[0.95, 0.95)		9.82E-02	2.90E+00	1.59E-01
Joint	[0.95, 0.97)		2.56E-01	2.10E+00	4.52E-01
Joint	[0.97, 0.97)		4.70E-01	5.76E+00	-4.05E-01
Joint	[0.97, 0.99)		3.59E-01	7.62E+00	-9.89E-01
Joint	[0.99, 0.99)	(0.23, 0.40]	9.33E-02	2.62E+00	3.44E-01
Joint	[0.99, 1.01]		9.83E-04	3.56E+00	-9.98E-01
Joint	(1.01, 1.01]		1.37E-03	9.01E+00	-9.75E-01
Joint	(1.01, 1.03]		4.89E-01	6.71E+00	-1.61E-01

Joint	(1.03, 1.05]		4.52E-01	6.74E+00	-1.84E-01
Joint	(1.05, 1.05]		4.31E-01	6.44E+00	-5.04E-01
Joint	(1.05, 1.07]		7.23E-02	5.26E-01	-5.59E-01
Joint	(1.07, 1.08]		9.87E-02	7.58E-01	-6.07E-01
Joint	(1.08, 1.09]		5.60E-02	2.73E+00	1.41E-01
Joint	(1.09, 1.12]		1.24E-01	8.08E-01	-6.93E-01
Joint	(1.12, 1.16]		7.44E-01	9.49E+00	-9.99E-01
Wing	(1.16,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.85)		2.05E-02	4.65E-03	4.34E-01
Joint	[0.85, 0.93)		8.13E-02	8.43E-01	2.81E-01
Joint	[0.93, 0.97)	(0.40, 0.65]	1.55E-01	1.41E+00	3.17E-01
Joint	[0.97, 0.99]		3.75E-01	6.16E+00	-3.96E-01
Wing	(0.99,+∞)		5.13E-02	1.37E+00	-9.80E-01
Wing	(-∞, 0.58)		1.24E-02	9.03E-01	-9.06E-01
Joint	[0.58, 0.65)		4.56E-02	1.25E+00	2.06E-01
Joint	[0.65, 0.69)		4.32E-02	1.15E+00	1.08E-01
Joint	[0.69, 0.77)		5.73E-02	7.53E-01	5.80E-02
Joint	[0.77, 0.85)		5.11E-02	5.84E-01	-7.84E-02
Joint	[0.85, 0.87)		9.00E-02	1.20E+00	3.75E-01
Joint	[0.87, 0.89)		9.51E-02	1.12E+00	4.30E-01
Joint	[0.89, 0.92)		1.91E-01	9.40E-01	9.98E-01
Joint	[0.92, 0.96)	(0.65, 0.90]	5.58E-01	6.58E+00	-9.02E-01
Joint	[0.96, 0.98)		2.78E-01	4.83E+00	-1.44E-01
Joint	[0.98, 1.00]		1.09E-05	1.00E+01	-9.99E-01
Joint	(1.00, 1.02]		1.09E-05	1.00E+01	-9.99E-01
Joint	(1.02, 1.12]		9.45E-02	1.00E+01	-9.99E-01
Joint	(1.12, 1.16]		1.56E-01	1.00E+01	-9.99E-01

Joint	(1.16, 1.19]		1.56E-01	1.00E+01	-9.99E-01
Joint	(1.19, 1.23]		4.80E-02	8.24E-01	-9.80E-01
Wing	(1.23,+∞)		5.04E-02	1.09E+00	-9.80E-01
Wing	(-∞, 1.20]	(0.90, 1.15]	4.11E-02	1.00E-05	-6.66E-01
Wing	(1.20,+∞)		4.80E-02	8.24E-01	-9.80E-01
Wing	(-∞, 0.98)	(1.15, 1.40]	3.56E-05	2.28E-01	8.07E-01
Wing	[0.98,+∞)		4.94E-02	7.27E-01	-9.84E-01
Wing	(-∞, 0.98)	(1.40, 1.65]	1.35E-02	4.39E-02	2.00E-01
Wing	[0.98,+∞)		4.68E-02	4.39E+00	9.89E-01
Wing	(-∞, 0.69)	(1.65, 1.90]	4.27E-01	6.43E-01	-1.39E-01
Joint	[0.69, 0.73)		3.55E-01	3.77E+00	-6.11E-01
Joint	[0.73, 0.77)		1.41E-02	3.80E+00	-6.56E-01
Joint	[0.77, 0.80)		5.75E-03	1.69E+00	-9.26E-01
Joint	[0.80, 0.84)		1.41E-01	4.03E-01	8.83E-01
Joint	[0.84, 0.86)		3.76E-01	1.15E+00	-1.96E-01
Joint	[0.86, 0.94)		8.28E-01	6.04E+00	-7.04E-01
Joint	[0.94, 0.96)		8.76E-02	1.90E+00	-4.76E-01
Joint	[0.96, 0.98)		2.62E+00	6.27E+00	-4.83E-01
Joint	[0.98, 1.00)		4.80E-02	8.24E-01	-9.80E-01
Joint	[1.00, 1.03]		4.80E-02	8.24E-01	-9.80E-01
Joint	(1.03, 1.15]		5.31E-01	5.41E+00	-5.89E-01
Joint	(1.15, 1.23]		2.56E-03	6.97E-01	7.27E-01
Joint	(1.23, 1.26]		1.97E-02	6.34E-01	-9.91E-01
Wing	(1.26,+∞)		1.97E-02	6.34E-01	-9.91E-01
Wing	(-∞, 0.70)	(1.90, 2.15]	2.13E-01	4.67E-01	1.28E-01
Joint	[0.70, 0.89]		1.02E+00	1.13E+00	-5.20E-01
Wing	(0.89,+∞)		1.44E-01	3.66E-03	-9.80E-01

Wing	(-∞, 0.63)	(2.15,+∞]	1.38E-01	9.82E-01	-2.05E-01
Joint	[0.63, 0.68)		1.64E+00	1.38E+00	1.97E-01
Joint	[0.68, 0.82)		1.98E+00	1.39E+00	1.15E-01
Joint	[0.82, 0.87)		2.72E+00	4.73E+00	-7.63E-01
Joint	[0.87, 0.95]		7.14E-01	5.83E+00	-5.14E-01
Wing	(0.95,+∞)		1.45E-01	1.02E-05	-9.34E-01

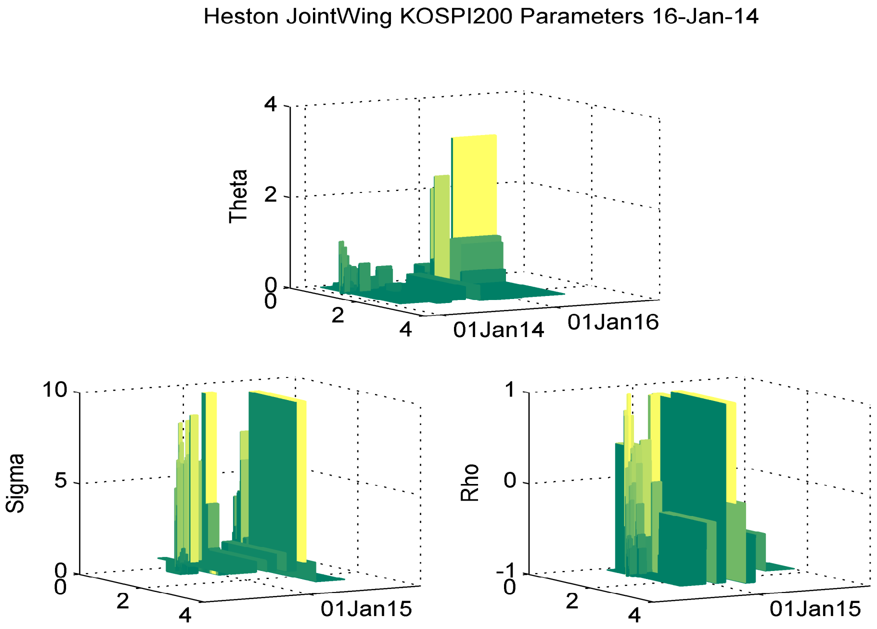


Figure 7.6: Heston JointWing KOSPI200 Parameters

Table 7.7: Elapsed Seconds KOSPI200 Calibration (02-Jan-14∼31-Mar-14)

SABR BoneWing	SABR BoneJoint Wing	SABR JointWing	Heston BoneWing	Heston BoneJoint Wing	Heston JointWing
7.295E+0	1.313E+1	2.993E+1	4.394E+2	8.400E+2	2.026E+3

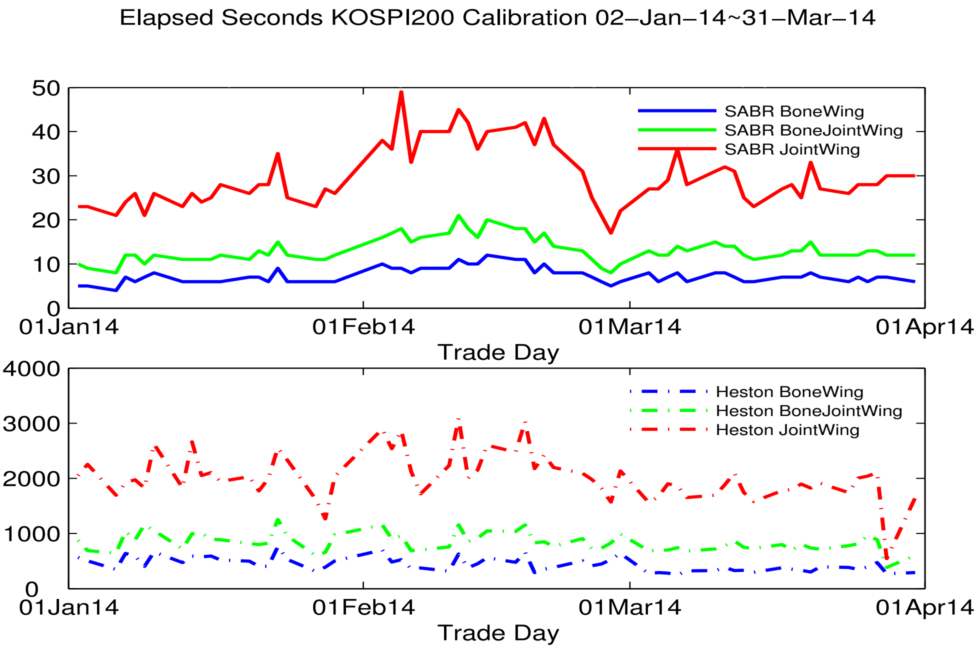


Figure 7.7: Elapsed Seconds KOSPI200 Calibration (02-Jan-14∼31-Mar-14)

There are SV Mosaic model parameters of KOSPI200 on January 16, 2014 that 'Forward moneyness', 'Time-to-maturity' and parameter values of SABR $(\alpha^{(Y,T)}, \beta, \rho^{(Y,T)}, \nu^{(Y,T)})$ or Heston $(\kappa, \theta^{(Y,T)}, \sigma^{(Y,T)}, v_0, \rho^{(Y,T)})$ specify the Mosaic components of Mosaic system in Table 7.1 , Table 7.2 , Table 7.3 , Table 7.4 , Table 7.5 and Table 7.6. And graphs of same items on January 16, 2014 are in Figure 7.1 , Figure 7.2 , Figure 7.3 , Figure 7.4 , Figure 7.5 and Figure 7.6.

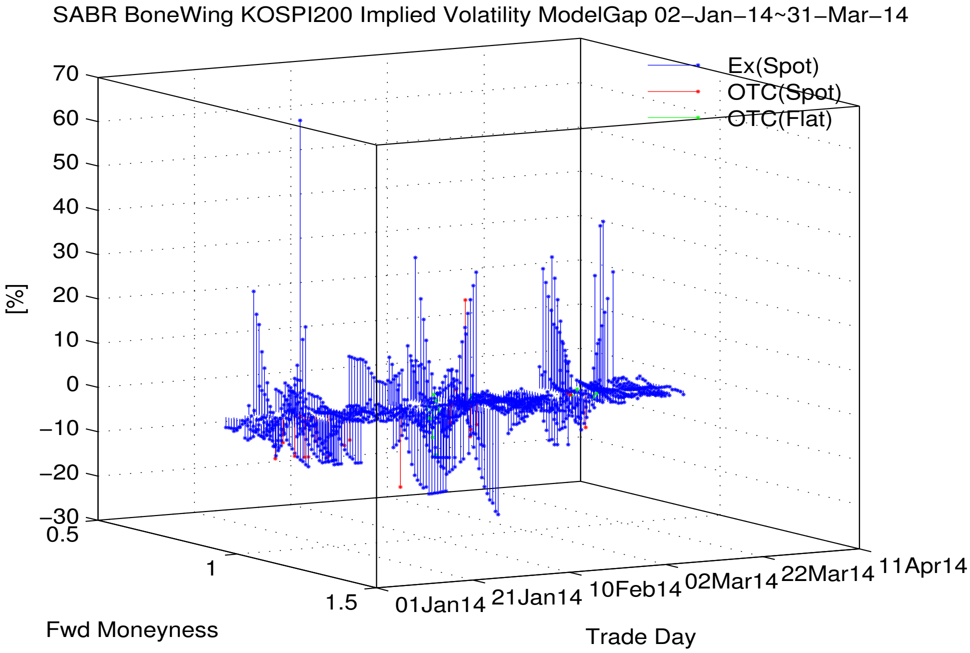
SABR model with ‘Bone Wing’ Mosaic system is implemented in Table 7.1 and Figure 7.1. Heston model with ‘Bone Wing’ Mosaic system is implemented in Table 7.2 and Figure 7.2. SABR model with ‘Bone Joint Wing’ Mosaic system is implemented in Table 7.3 and Figure 7.3. Heston model with ‘Bone Joint Wing’ Mosaic system is implemented in Table 7.4 and Figure 7.4. SABR model with ‘Joint Wing’ Mosaic system is implemented in Table 7.5 and Figure 7.5. Heston model with ‘Joint Wing’ Mosaic system is implemented in Table 7.6 and Figure 7.6. And there are mean and time series of elapsed seconds to calibrate Mosaic model of KOSPI200 during January 2, 2014 to March 31, 2014 in Table 7.7 and Figure B.40. In the result, calculation time of Heston model it takes 50 to 100 times more than that of SABR model. The main reason of the result is relatively more complicated structure of the formula.

Chapter 8

The SABR/Heston Mosaic implied volatility surface

8.1 SV Mosaic IVS

(7.3), (7.8) perform a role connecting the paremeter sets of the SV model to the IVS rooted in (2.2) or (7.2). Therefore, SV Mosaic IVS is generated by (7.10). Meanwhile there is a conspicuous difference between SABR Mosaic IVS and Heston Mosaic IVS. The approximate implied volatility form($\sigma_{imp} \approx \frac{C^B - C^{Heston}}{Vega^{BS}}$) in (7.9) works properly around ATM, but it can not fulfill the purpose around deep ITM and deep OTM, as Vega is close to zero. Thus Heston mosaic IVS has difficulties to target the implied volatility ITM and OTM sections. This problem will be called 'Inaccurate target implied volatility (IV)' in here.



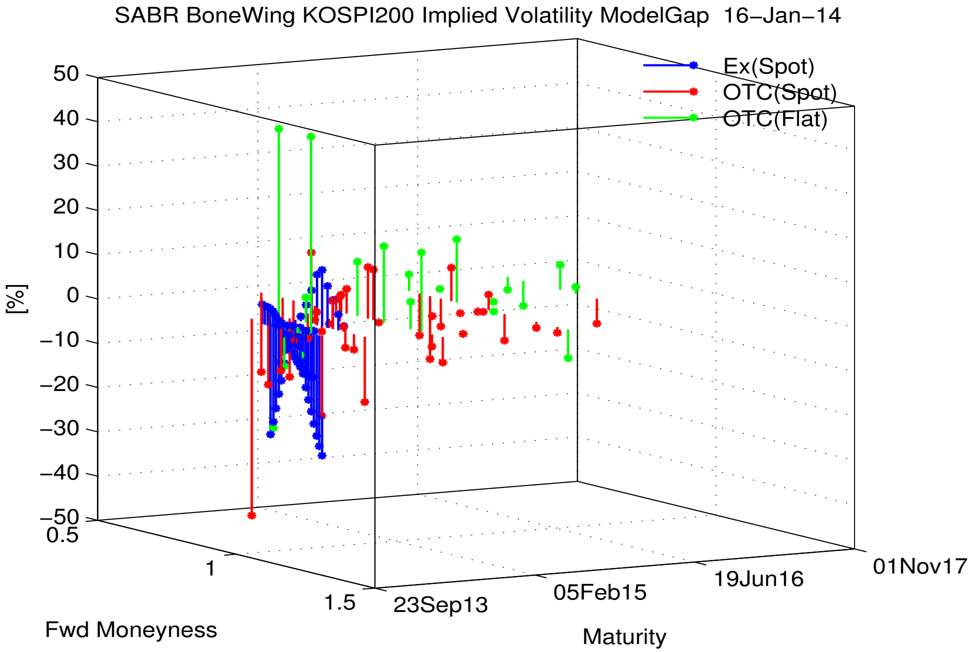


Figure 8.1: SABR BoneWing KOSPI200 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)

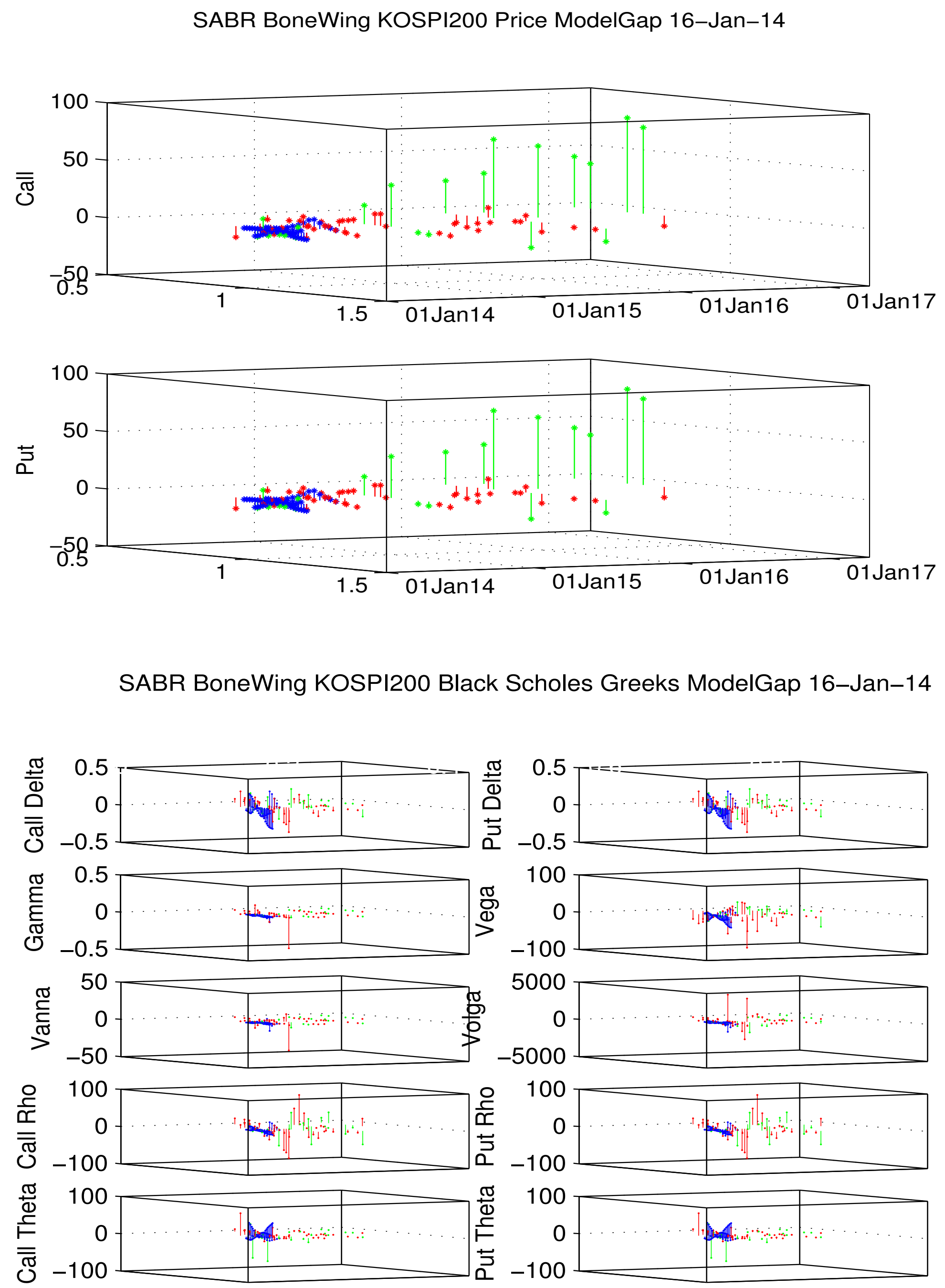
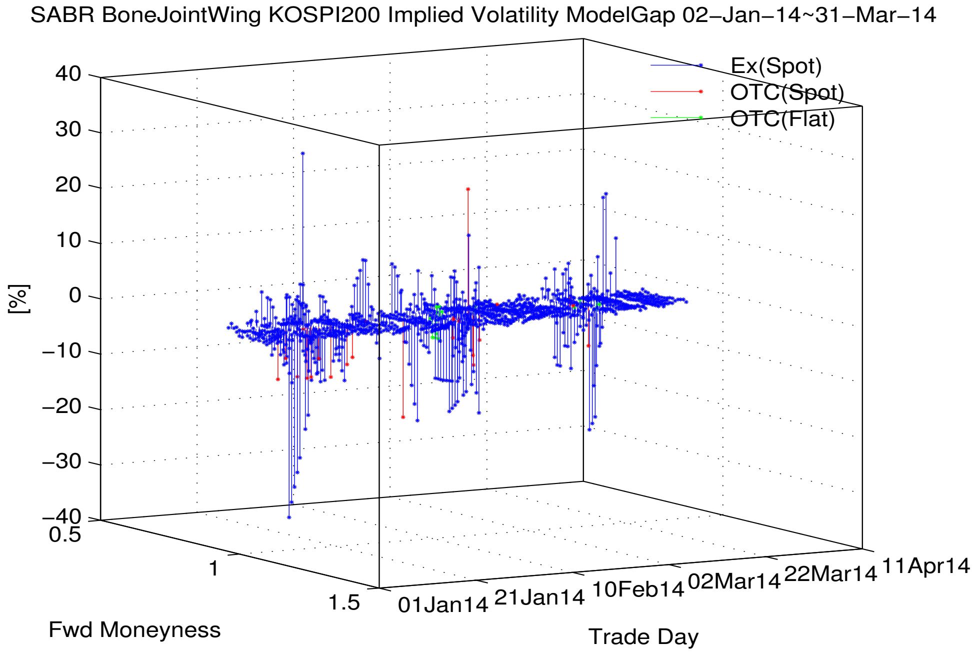


Figure 8.2: SABR BoneWing KOSPI200 Price, Black Scholes Greeks Model-Gap (16-Jan-14)



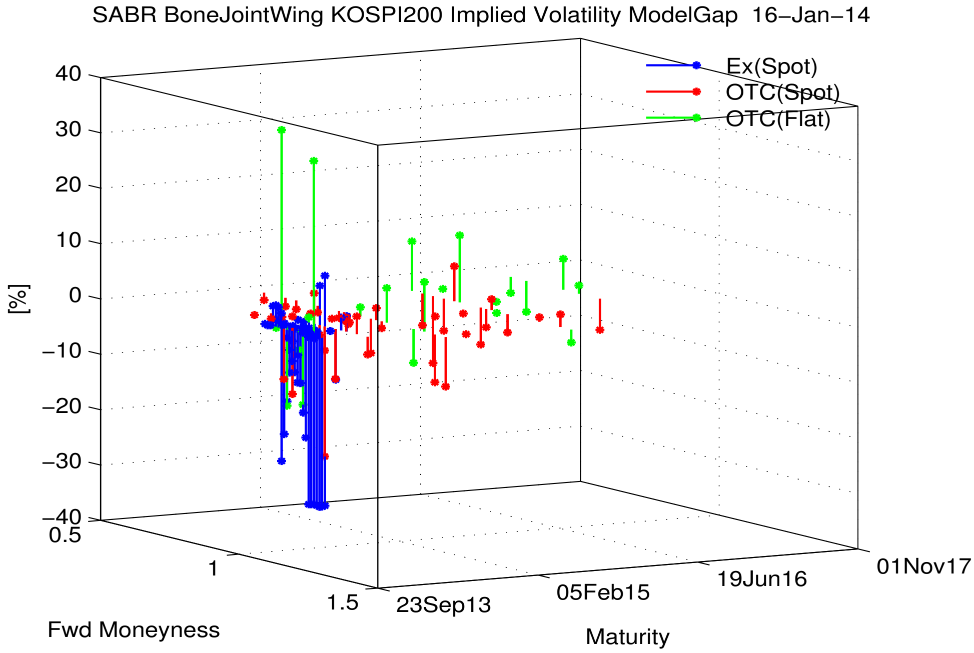


Figure 8.3: SABR BoneWing KOSPI200 Implied Volatility (Nearest) Model-Gap (03-Jan-14∼31-Mar-14), (16-Jan-14)

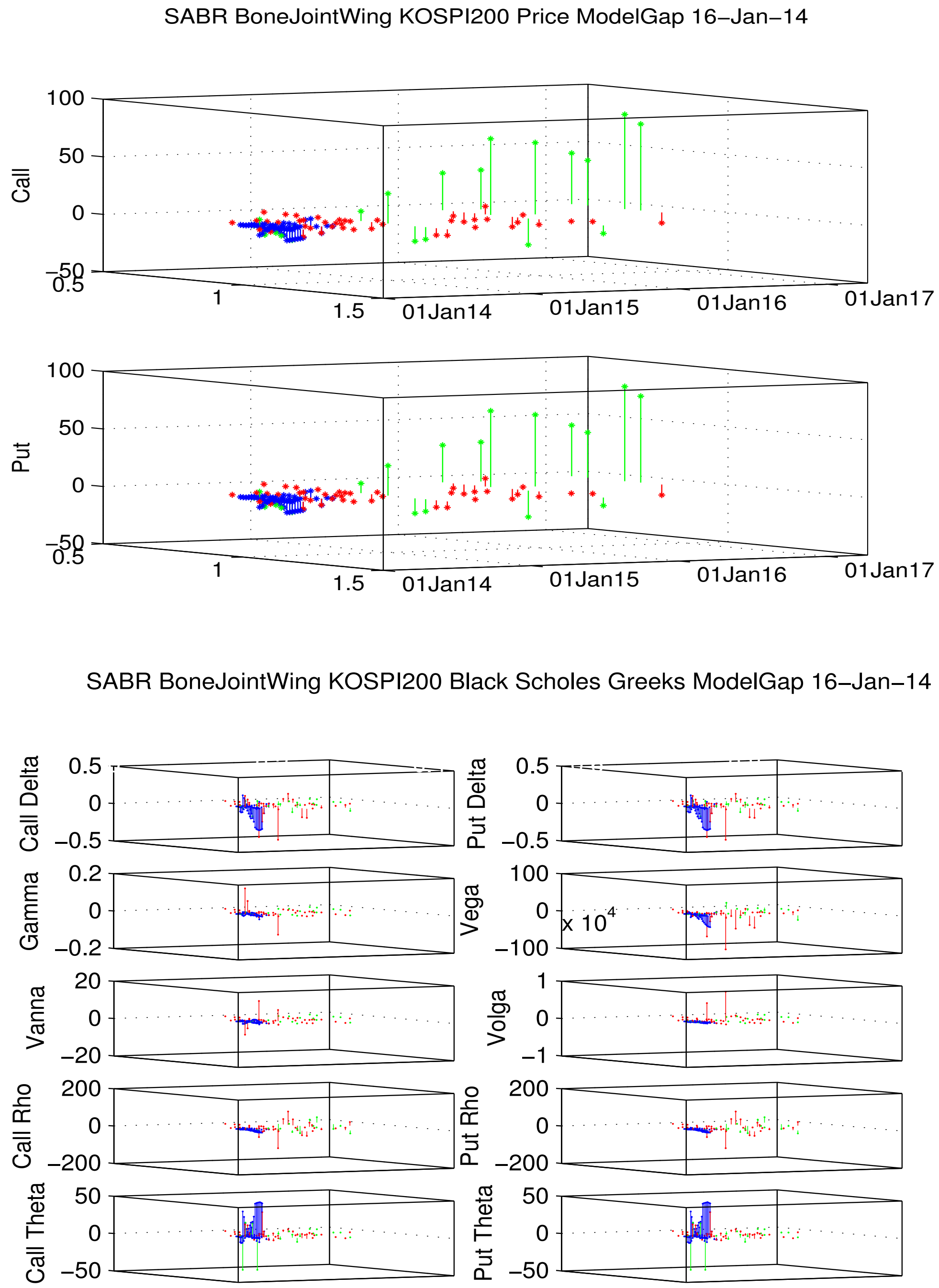
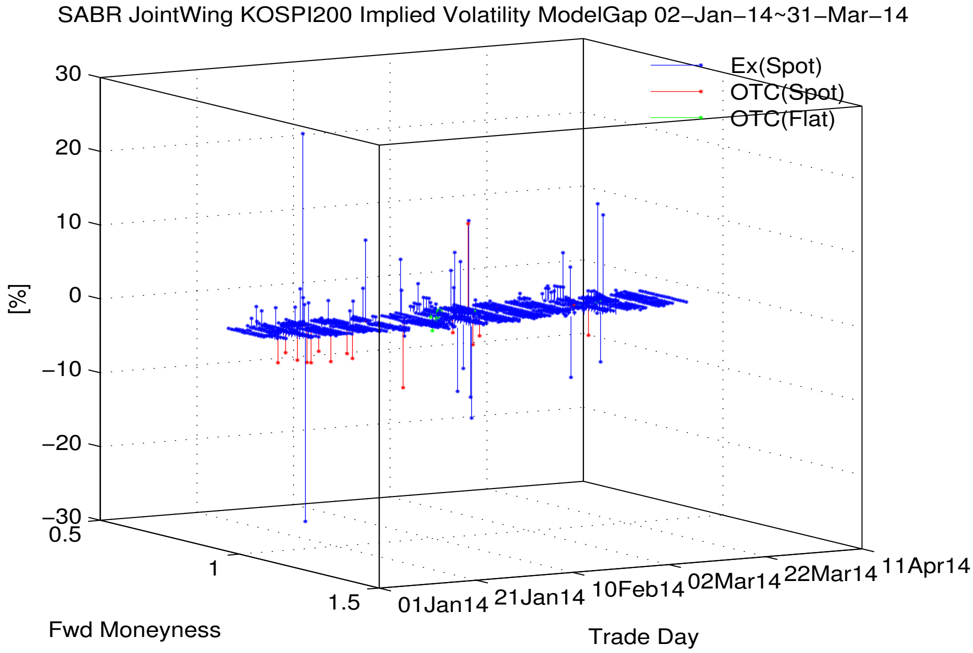


Figure 8.4: SABR BoneWing KOSPI200 Price, Black Scholes Greeks Model-Gap (16-Jan-14)



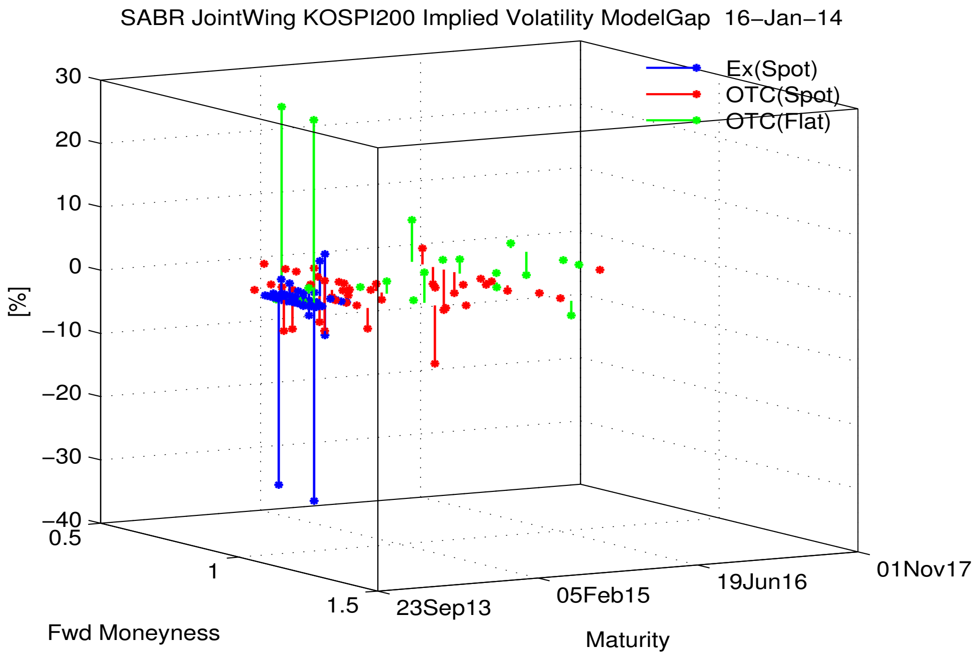
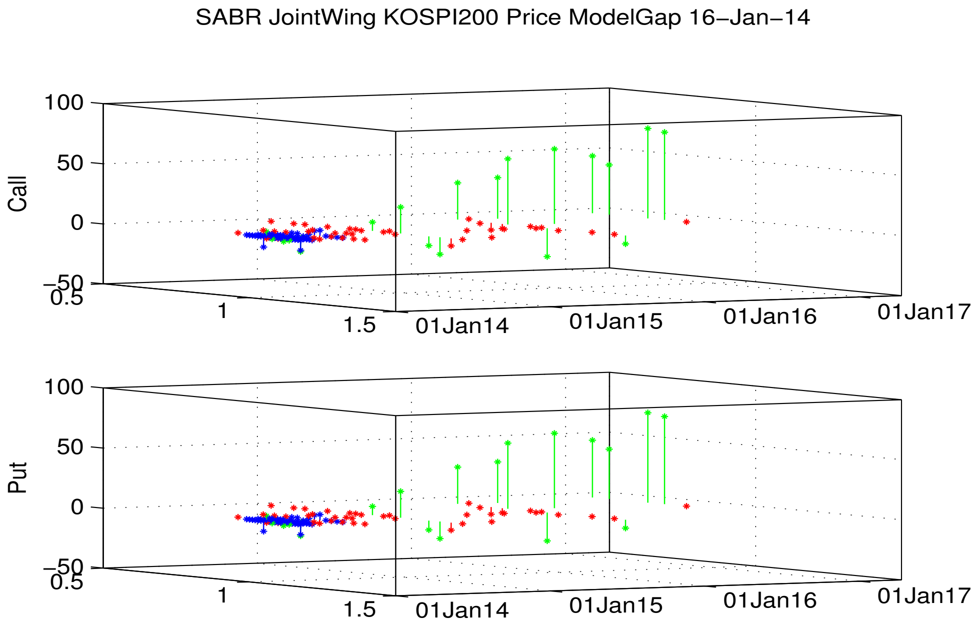


Figure 8.5: SABR BoneWing KOSPI200 Implied Volatility (Nearest) Model-Gap (03-Jan-14∼31-Mar-14), (16-Jan-14)



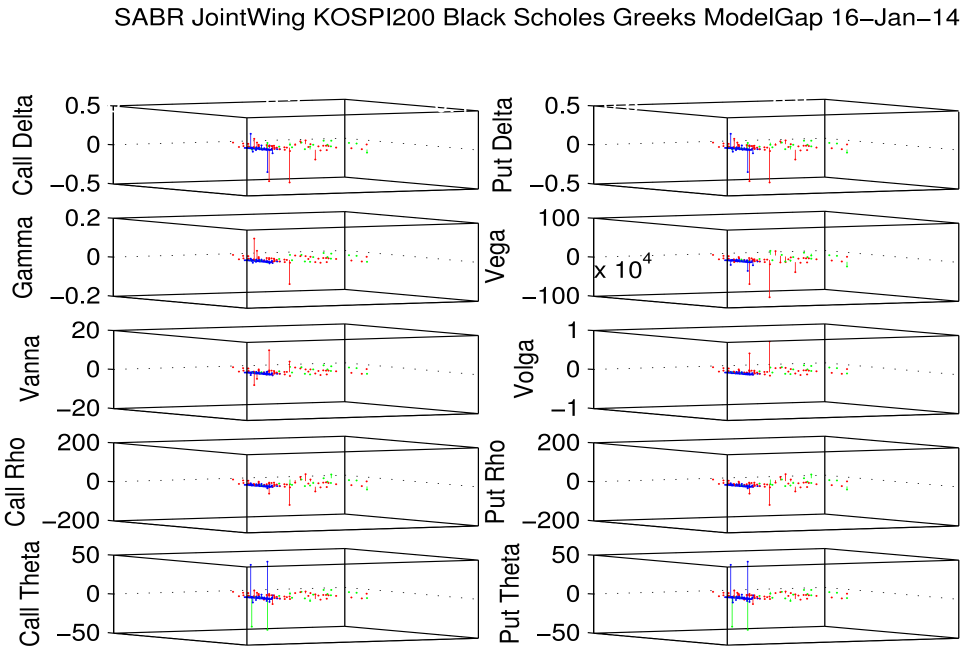


Figure 8.6: SABR BoneWing KOSPI200 Price, Black Scholes Greeks ModelGap (16-Jan-14)

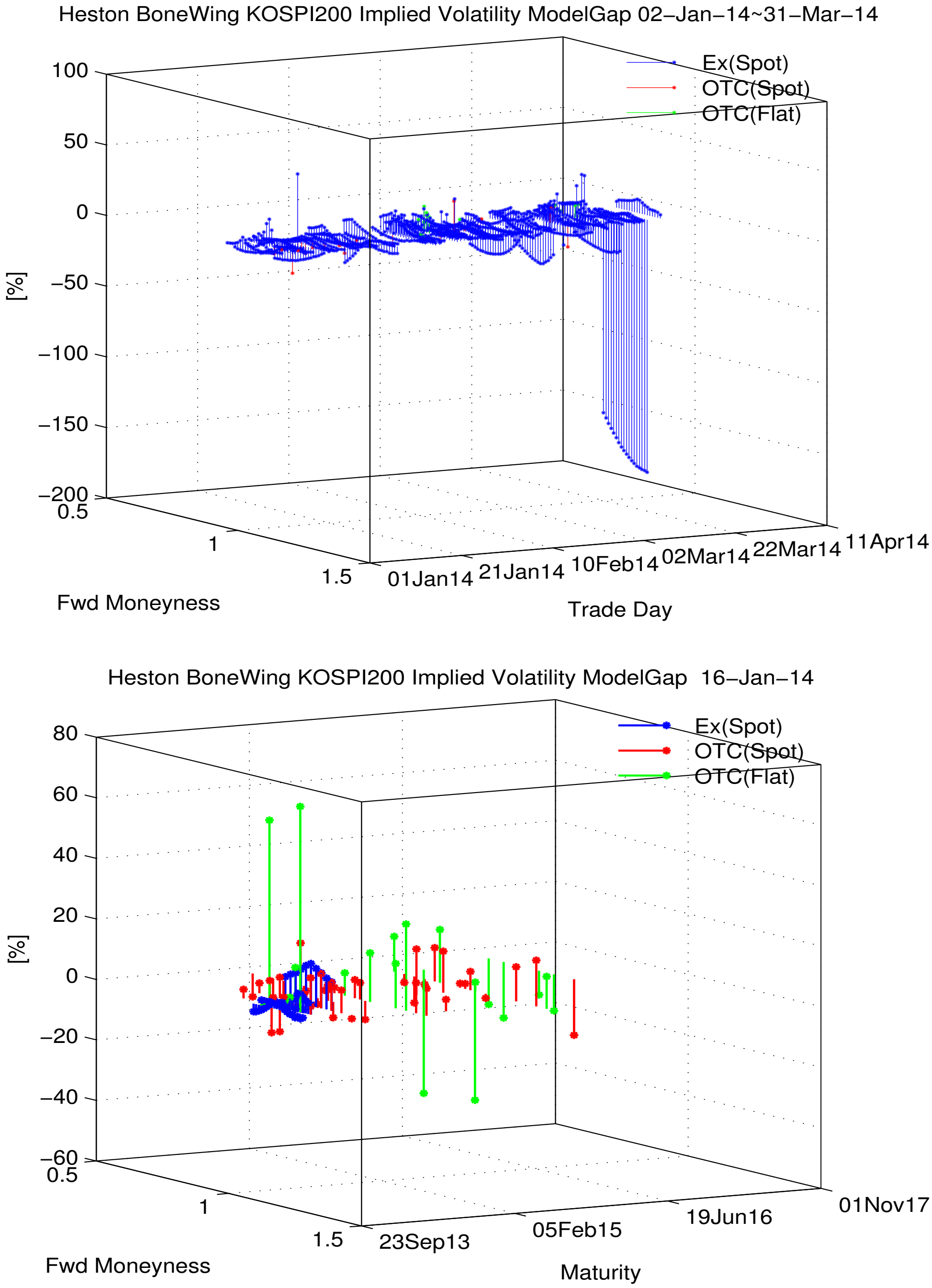


Figure 8.7: Heston BoneWing KOSPI200 Implied Volatility (Nearest) Model-Gap (03-Jan-14∼31-Mar-14), (16-Jan-14)

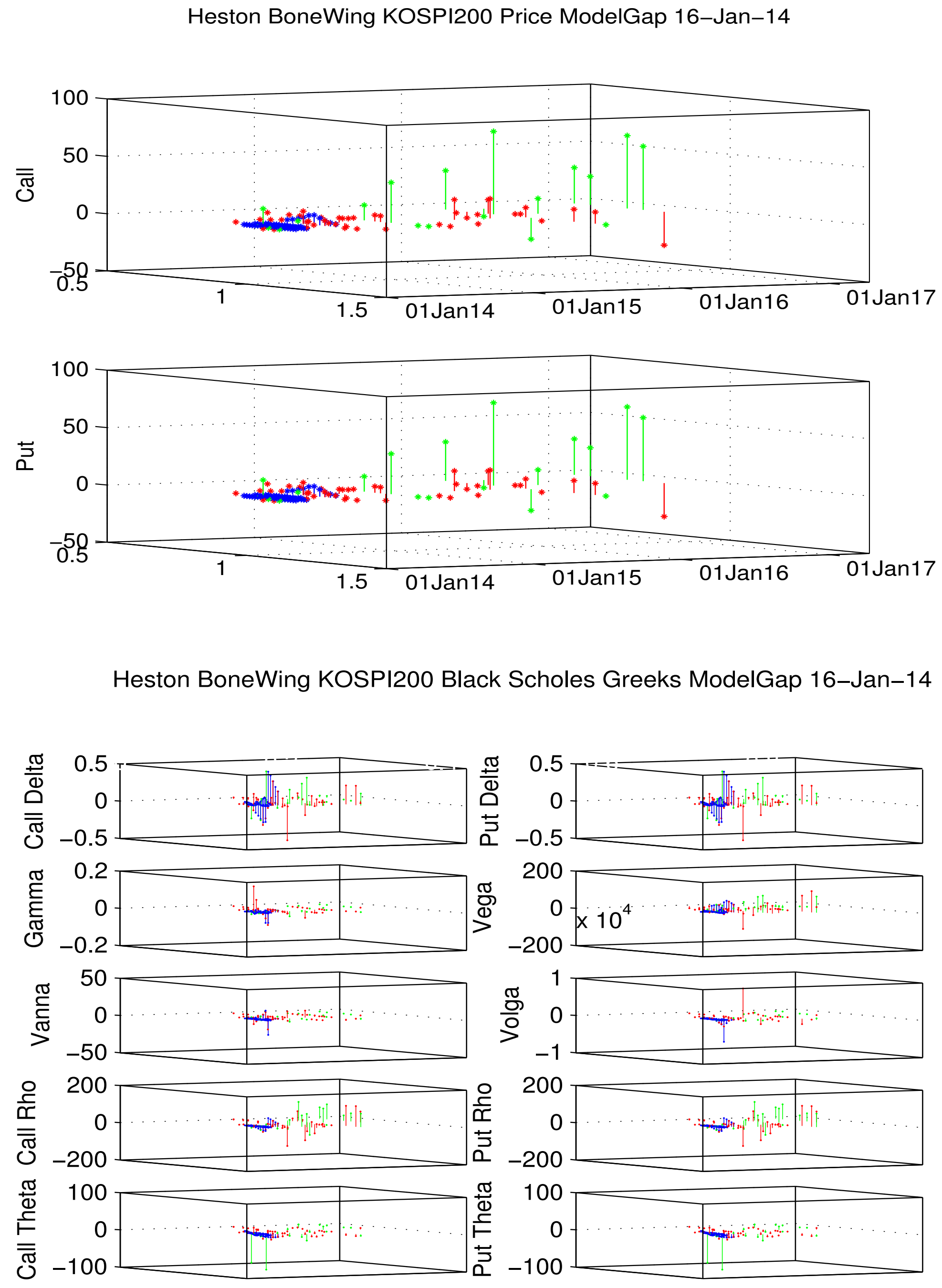
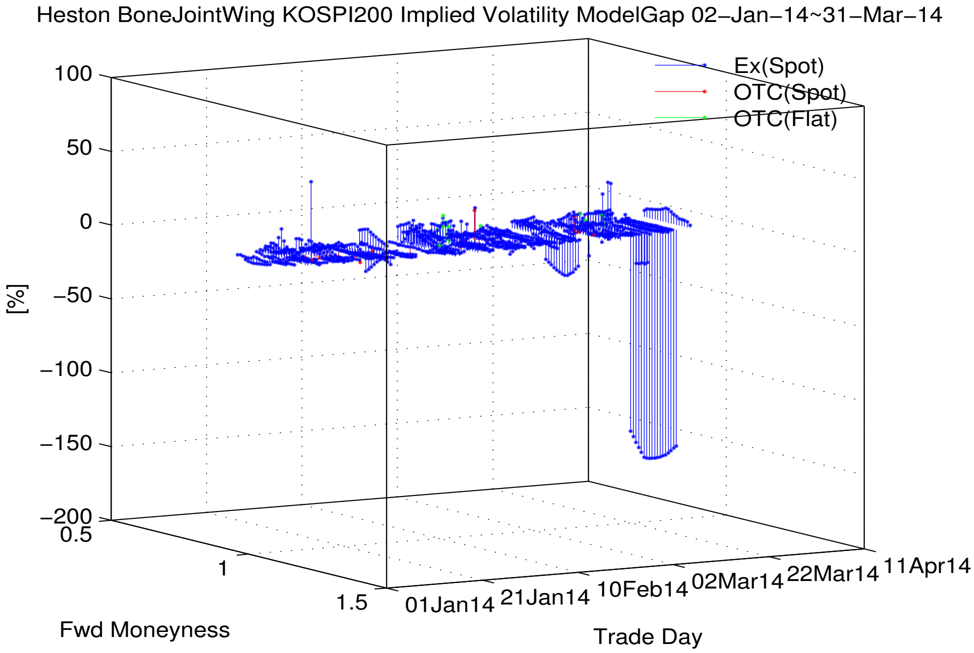


Figure 8.8: Heston BoneWing KOSPI200 Price, Black Scholes Greeks Model-Gap (16-Jan-14)



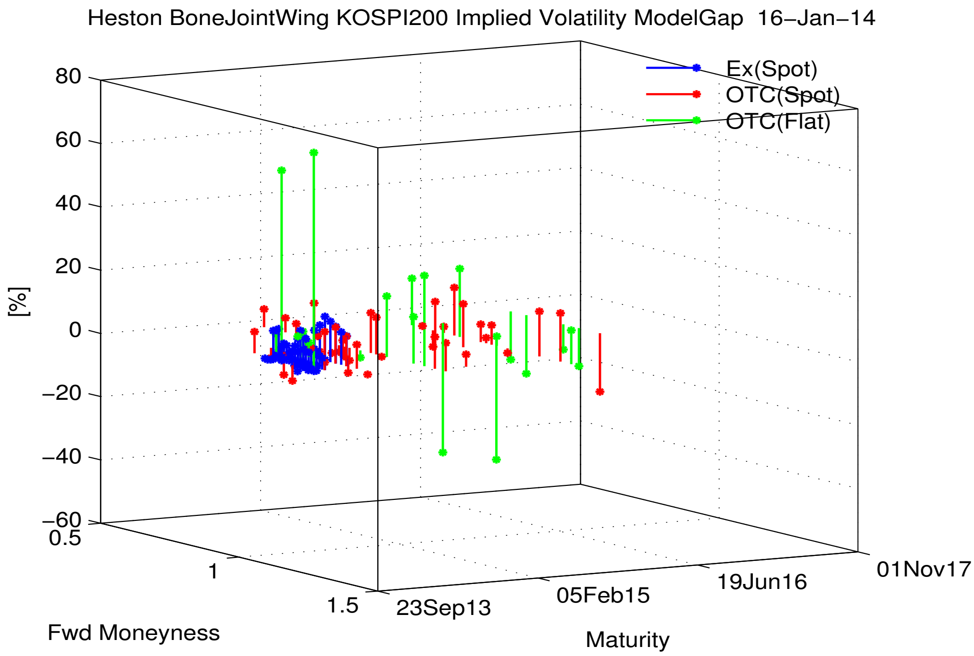


Figure 8.9: Heston BoneWing KOSPI200 Implied Volatility (Nearest) Model-Gap (03-Jan-14∼31-Mar-14), (16-Jan-14)

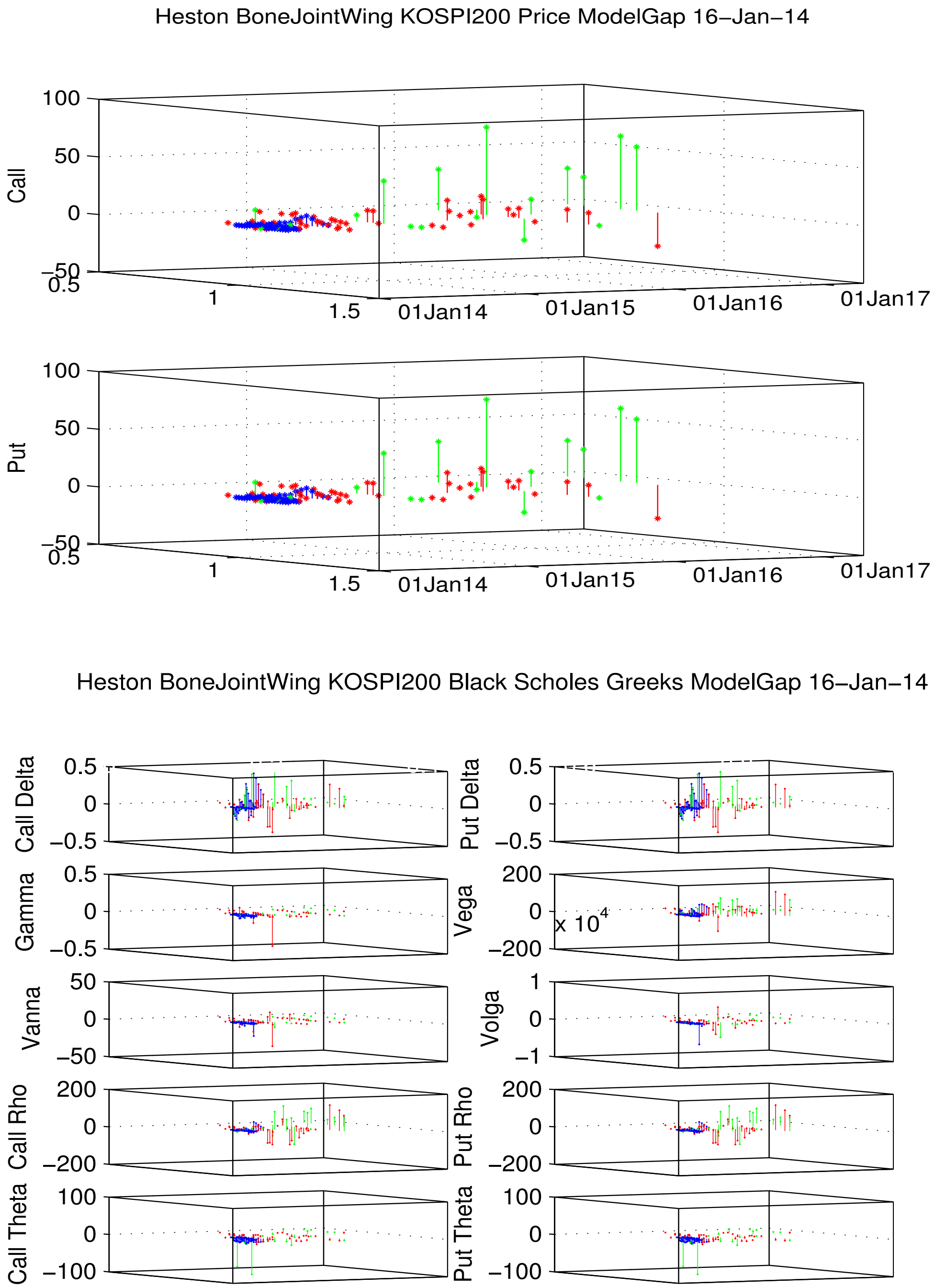


Figure 8.10: Heston BoneWing KOSPI200 Price, Black Scholes Greeks ModelGap (16-Jan-14)

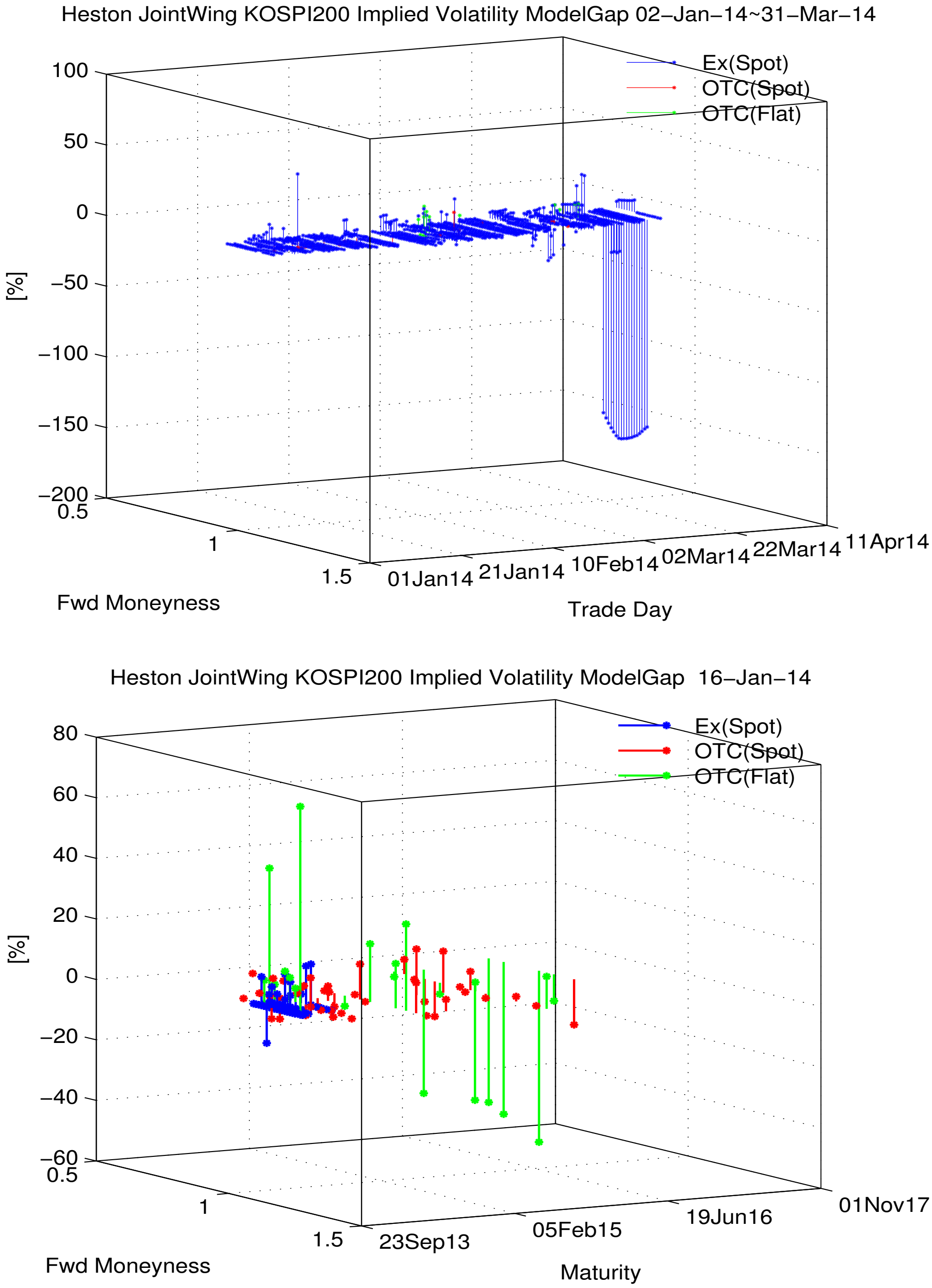


Figure 8.11: Heston BoneWing KOSPI200 Implied Volatility (Nearest) ModelGap (03-Jan-14~31-Mar-14), (16-Jan-14)

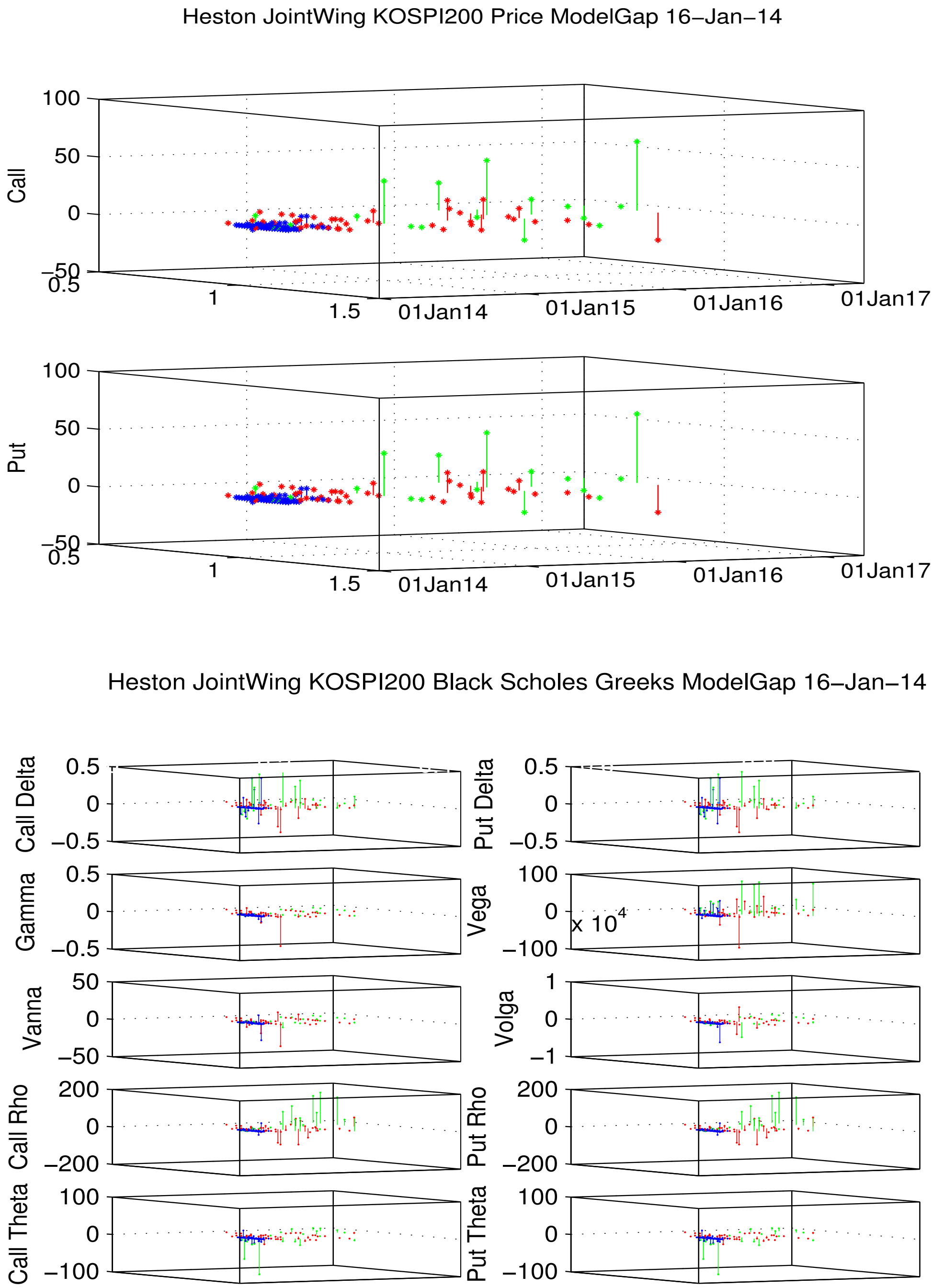


Figure 8.12: Heston BoneWing KOSPI200 Price, Black Scholes Greeks ModelGap (16-Jan-14)

Table 8.1: Root Mean Squared KOSPI200 Implied Volatility ModelGap (02-Jan-14∼31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
1.701E-01	1.430E-01	8.704E-02	1.797E-01	1.724E-01	1.600E-01

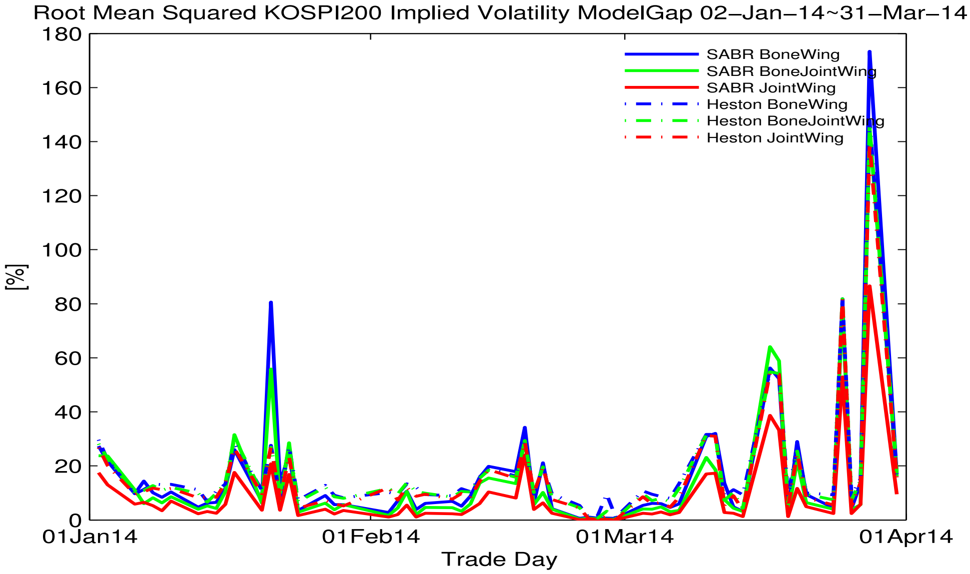


Figure 8.13: Root Mean Squared KOSPI200 Implied Volatility ModelGap (02-Jan-14∼31-Mar-14)

Graphs of the differences between nearest futures SV Mosaic implied volatilities for the KOSPI200 market data during January 2, 2014 to March 31, 2014 are in upper half of Figure B.119 , Figure B.121 , Figure B.123 , Figure B.125 , Figure B.127 and Figure B.129. In lower half of Figure B.119 , Figure B.121 , Figure B.123 , Figure B.125 , Figure B.127 and Figure B.129, there are graphs of the difference between SV Mosaic implied volatility for the KOSPI200 market data on January 16, 2014. Graphs of the difference between SV Mosaic price for the KOSPI200 market data on January 16, 2014 are in upper half of Figure B.120 , Figure B.122 , Figure B.124 , Figure B.126 , Figure B.128 and Figure B.130. In lower half of Figure B.120 , Figure B.122 , Figure B.124 , Figure B.126 , Figure B.128 and Figure B.130, there are graphs of the differences between SV Mosaic Black Scholes delta, gamma, vega, vanna, volga, rho, theta for the KOSPI200 market data on January 16, 2014. The main purpose of SV Mosaic model is to reduce the gap between model values for the actual data. We can find the improvement of that in order of 'Joint Wing'-'Bone Joint Wing'-'Bone Wing'. But still noticeable gap exists in 'Joint Wing' the best system for the purpose. This is because of that actual data admit arbitrage, whereas a unit SV Mosaic component do not allow arbitrage. And there are root mean squared difference and time series of differences between SV Mosaic implied volatility for the KOSPI200 market data during January 2, 2014 to March 31, 2014 in Table 8.1 and Figure B.131. This result is ambiguous to interpret, because abovementioned noticeable gap is dominant for the result. Fortunately, however, we can recognize that Mosaic system affects more than SV model in reducing the gap.

Graphs on the left column are related to SABR model, graphs on the right column are related to Heston model in Figure B.132. Graphs at the top row are 'Bone Wing', in the middle row are 'Bone Joint Wing', at the bottom row are 'Joint Wing' in upper half of Figure B.132 and lower half of Figure B.132. 'Market Grid' means IVS domain $(Y = \frac{K}{F_T}, T)$ is composed of market data points. 'Standard Grid' means IVS domain is composed of standardized data points that could be inside or outside of the market data. We could observe the different IVS between SABR Mosaic model and Heston Mosaic model in

‘Market Grid’. This is due to structure of the formula and method of targeting the implied volatility in SABR model and Heston model.

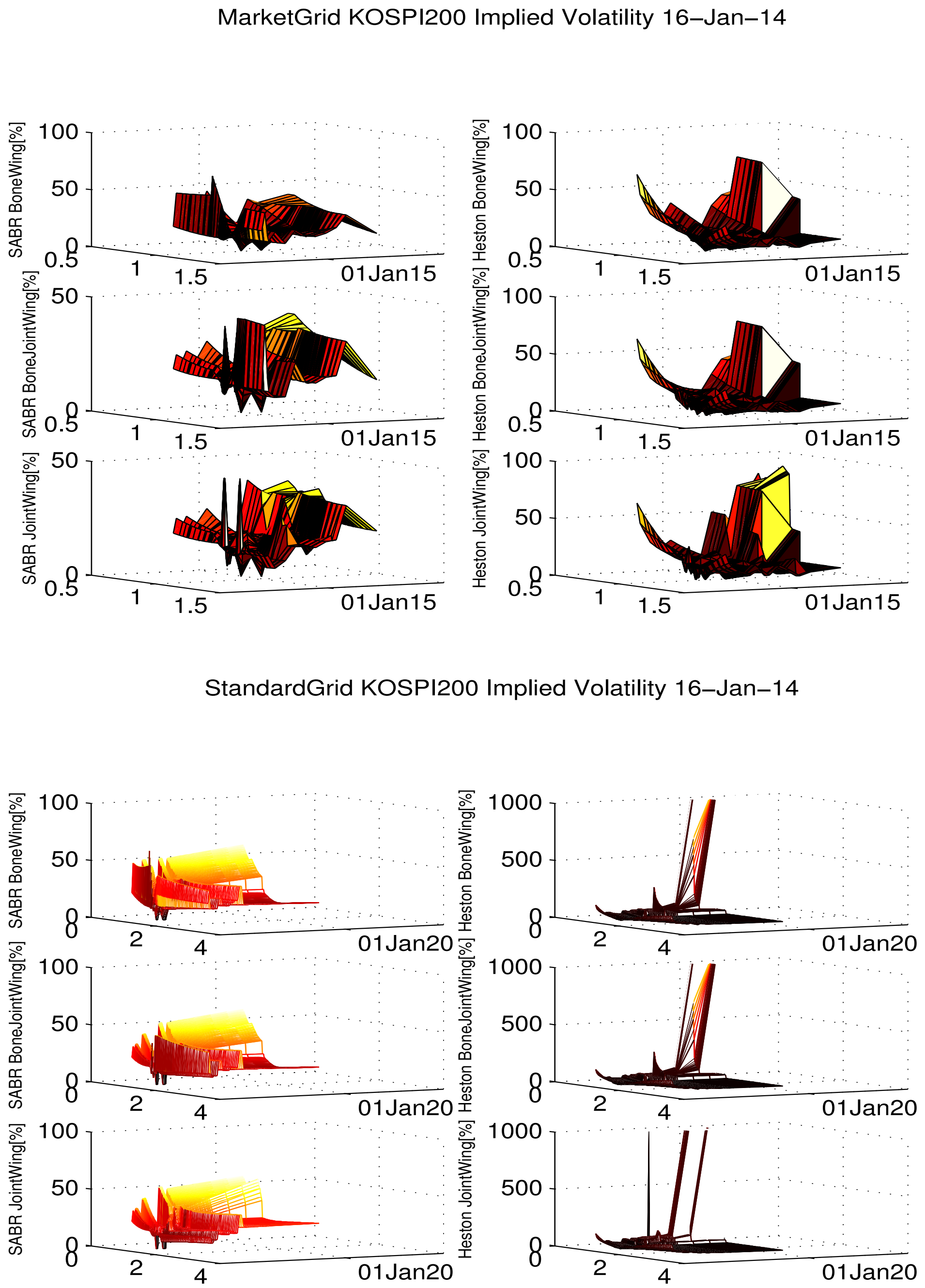


Figure 8.14: KOSPI200 Market, Standard Grid Implied Volatility (16-Jan-14)

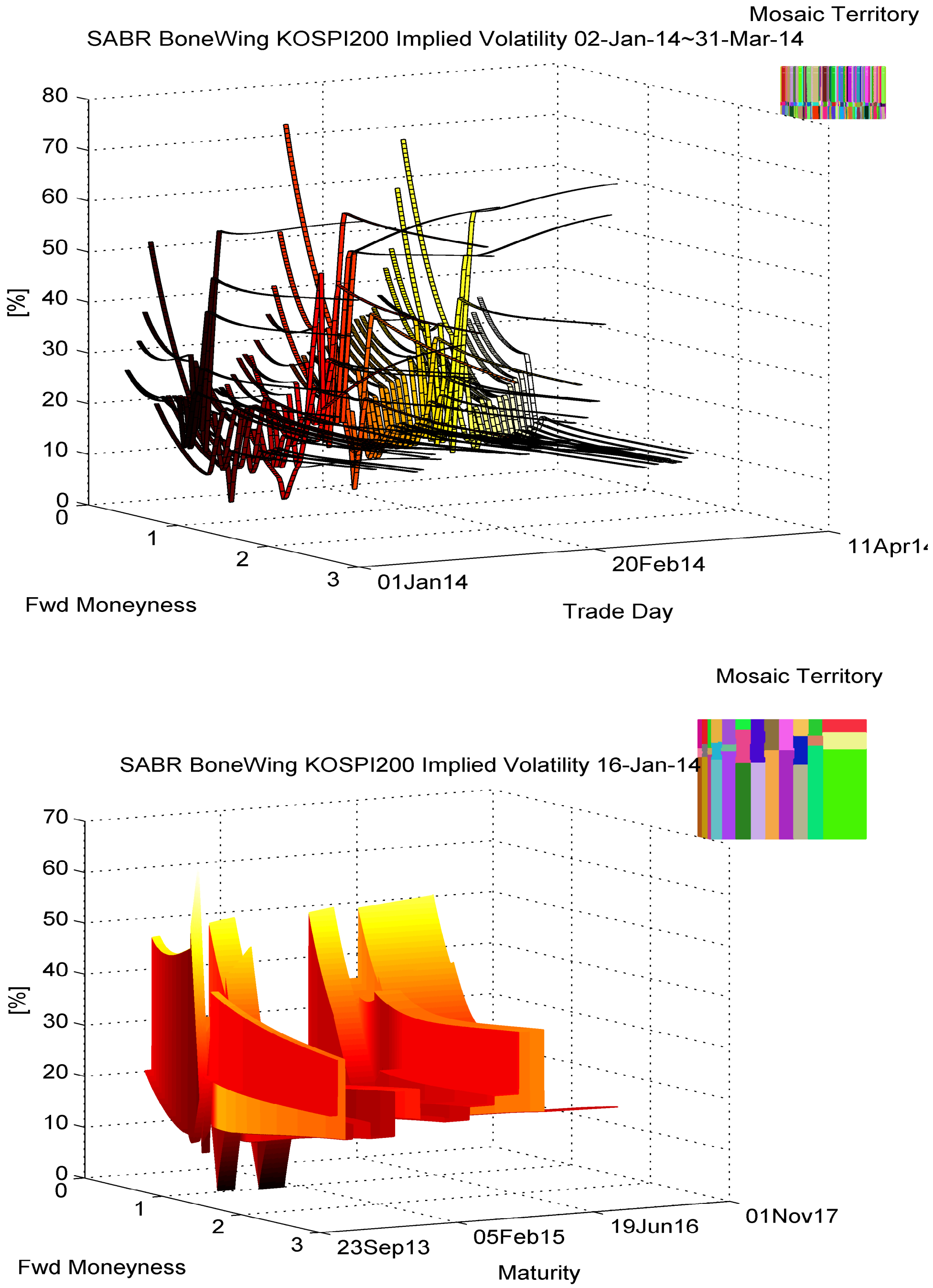
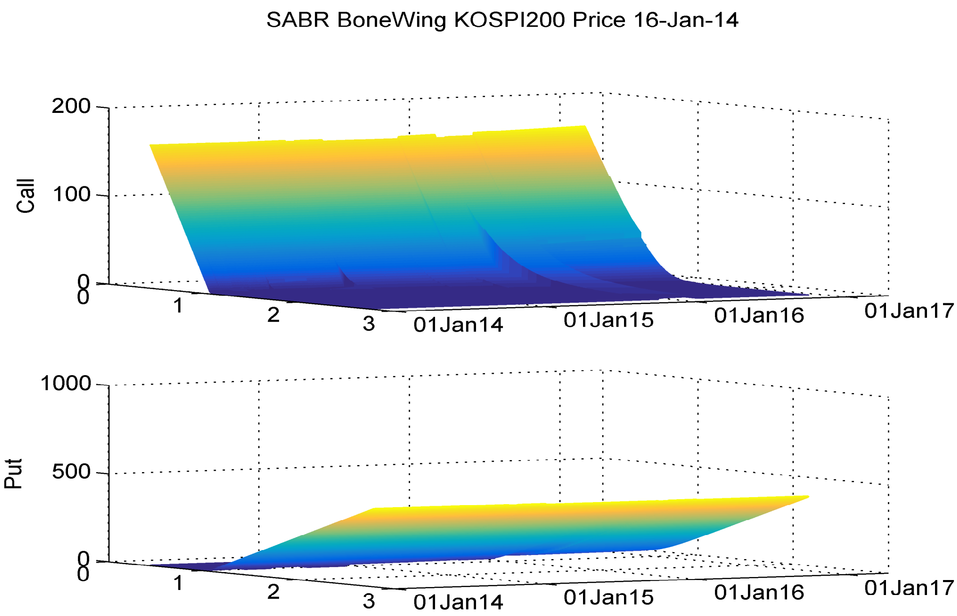


Figure 8.15: SABR BoneWing KOSPI200 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



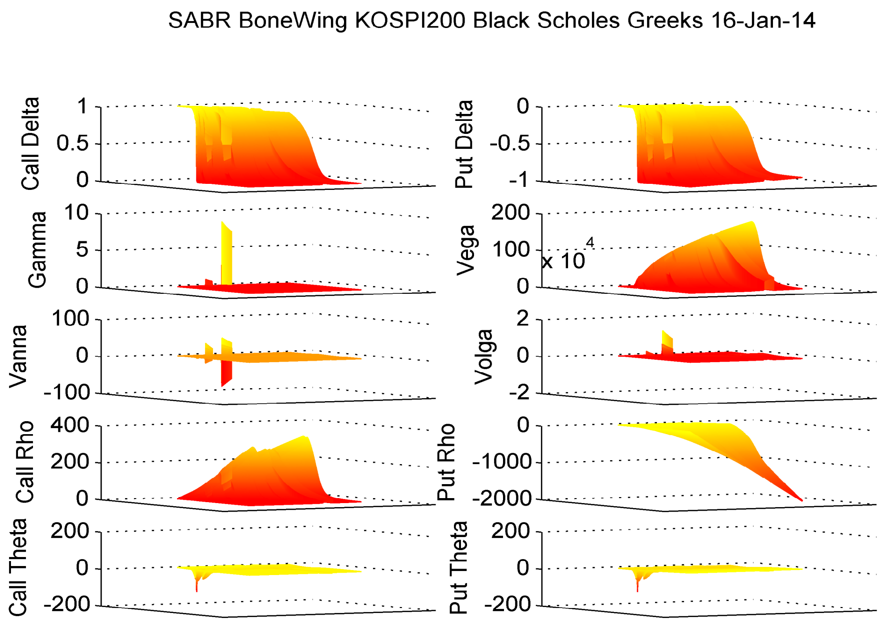
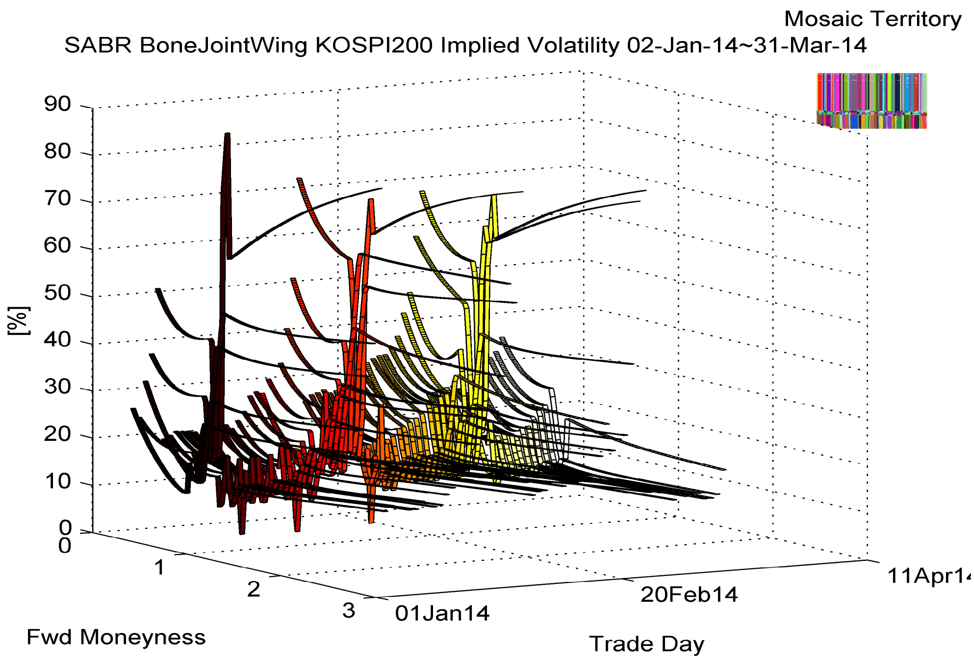


Figure 8.16: SABR BoneWing KOSPI200 Price, Black Scholes Greeks (16-Jan-14)



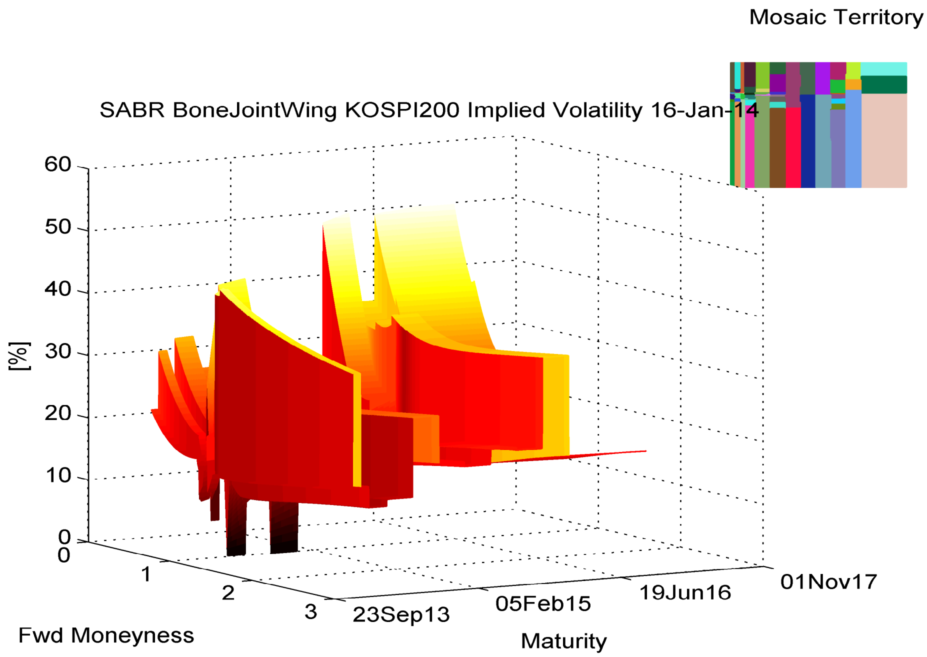
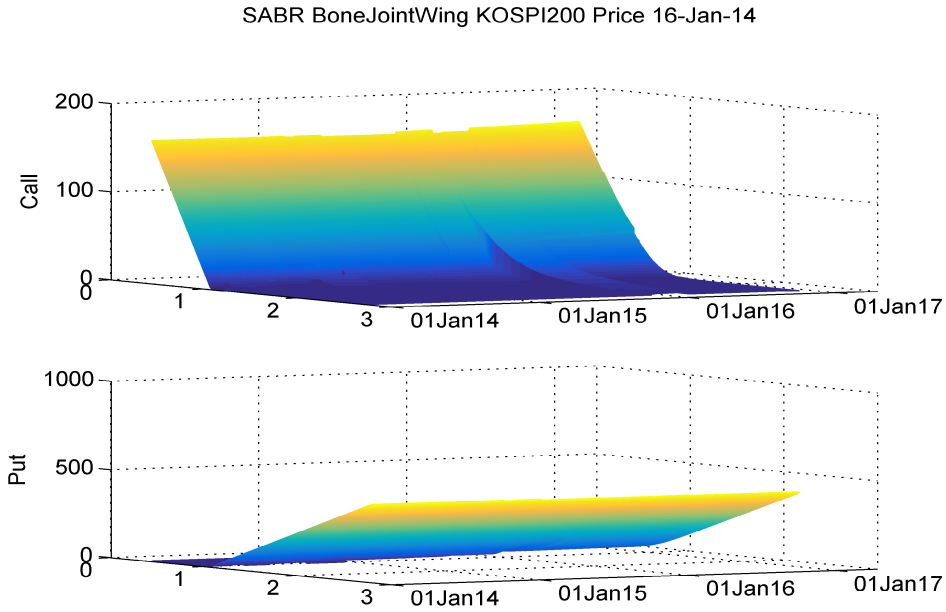


Figure 8.17: SABR BoneWing KOSPI200 Implied Volatility (Nearest) (03-Jan-14∼31-Mar-14), (16-Jan-14)



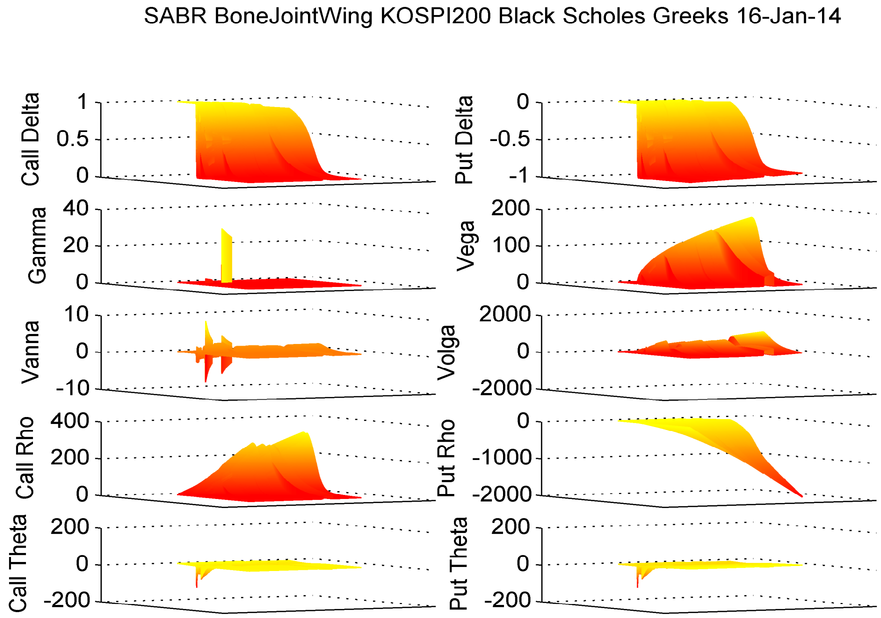
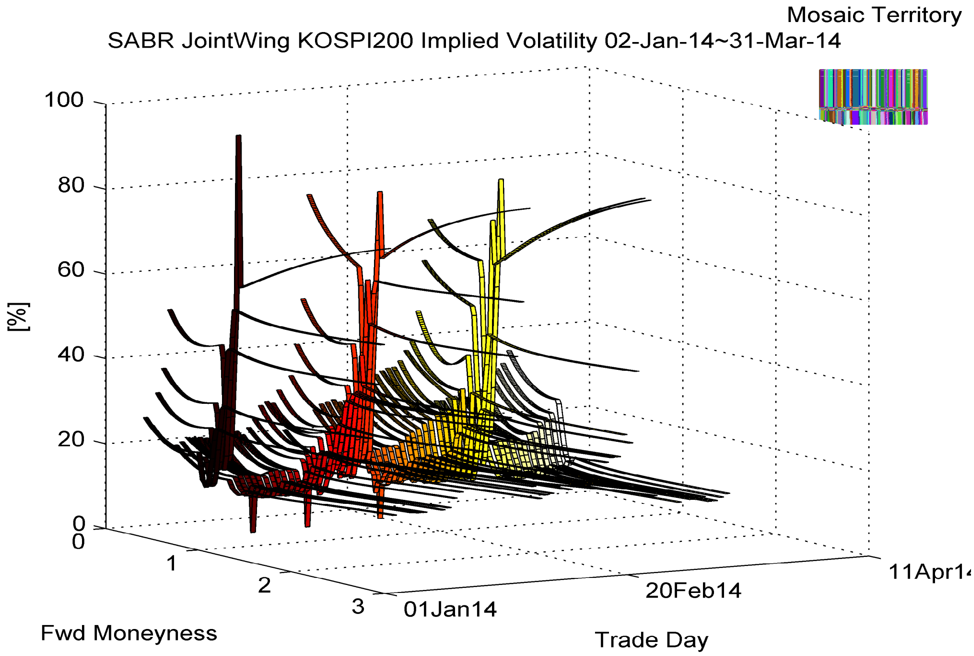


Figure 8.18: SABR BoneWing KOSPI200 Price, Black Scholes Greeks (16-Jan-14)



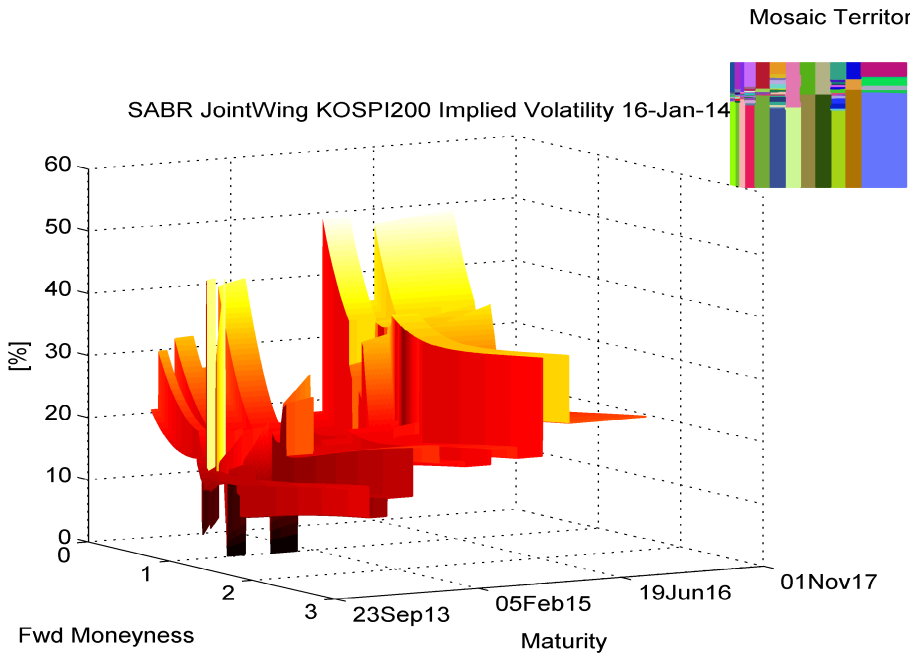


Figure 8.19: SABR BoneWing KOSPI200 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)

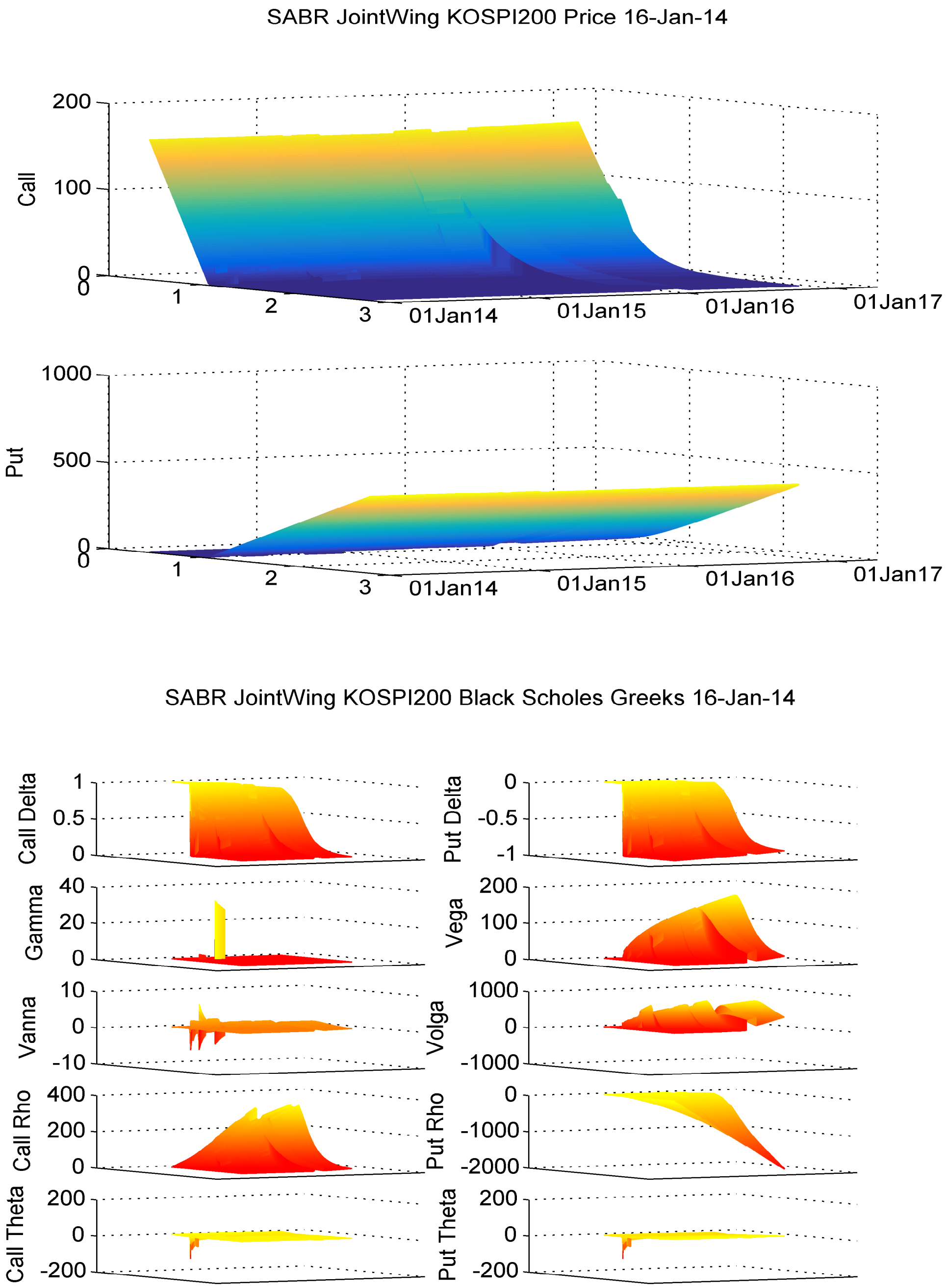
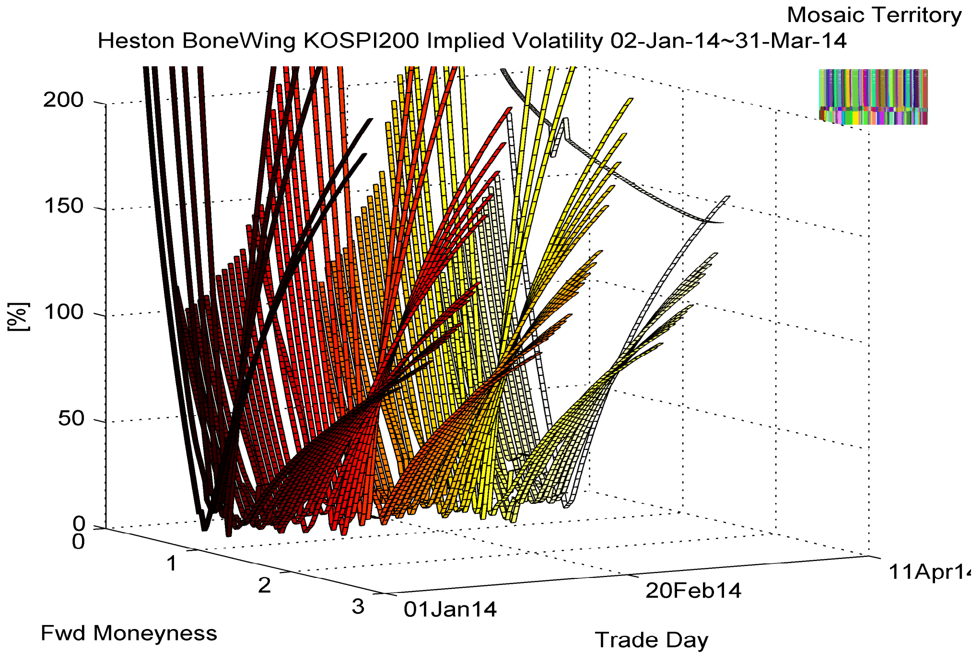


Figure 8.20: SABR BoneWing KOSPI200 Price, Black Scholes Greeks (16-Jan-14)



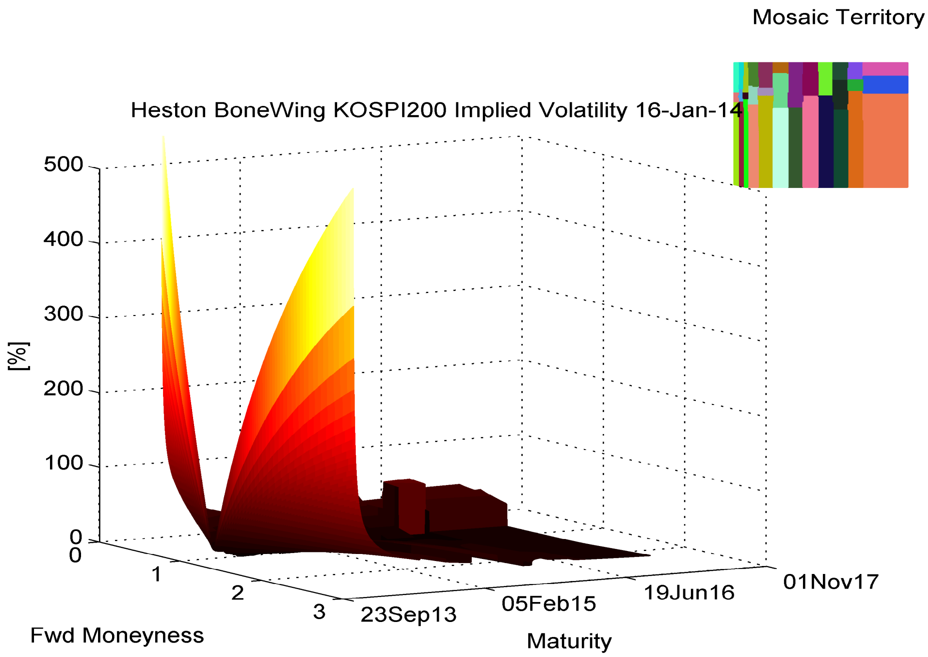


Figure 8.21: Heston BoneWing KOSPI200 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)

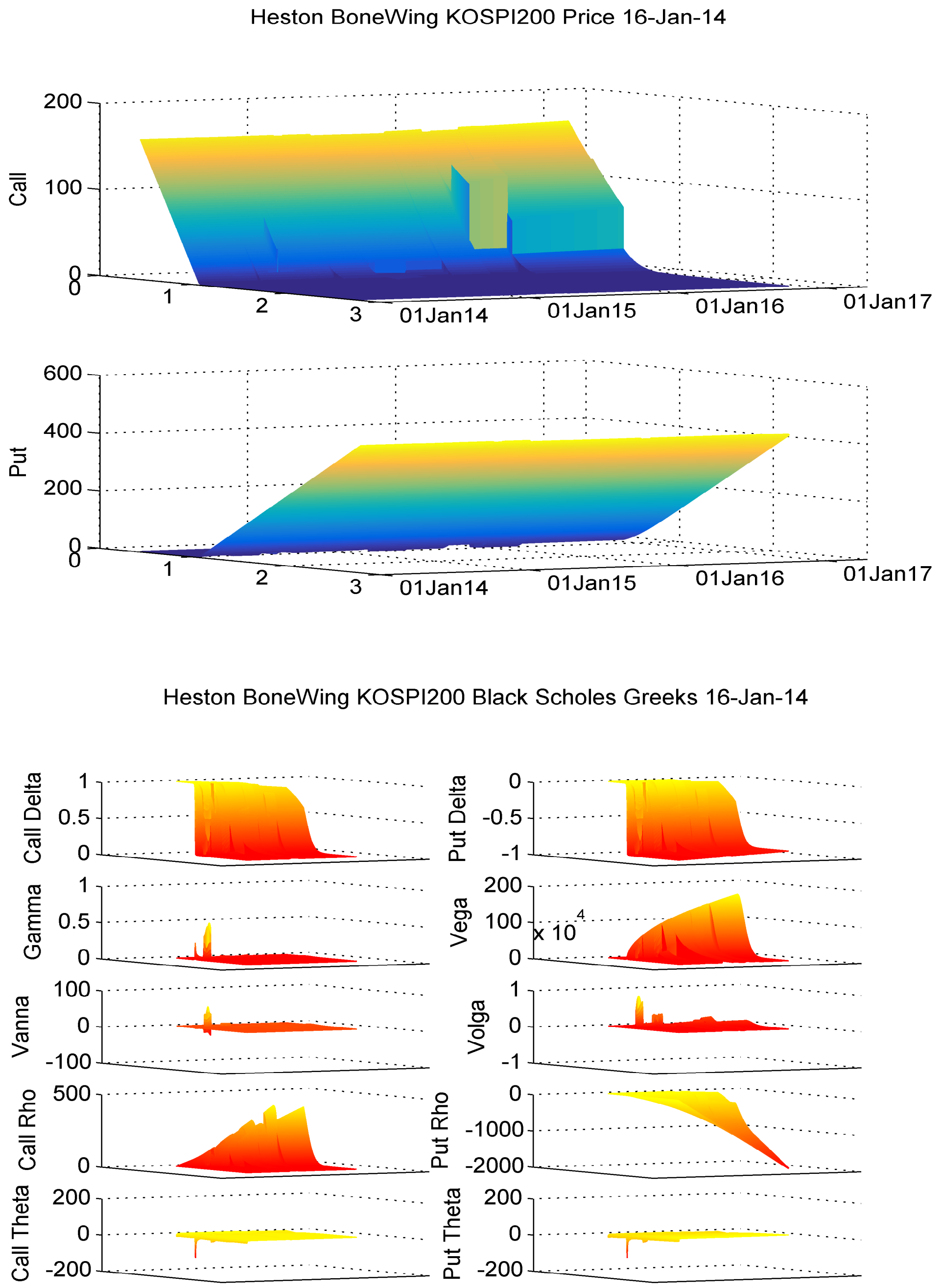
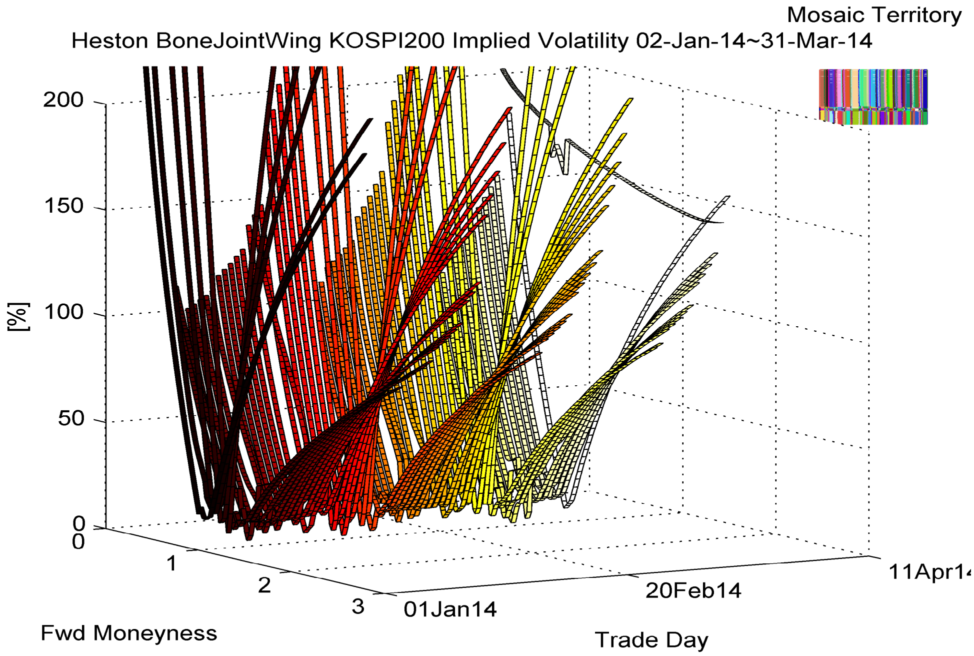


Figure 8.22: Heston BoneWing KOSPI200 Price, Black Scholes Greeks (16-Jan-14)



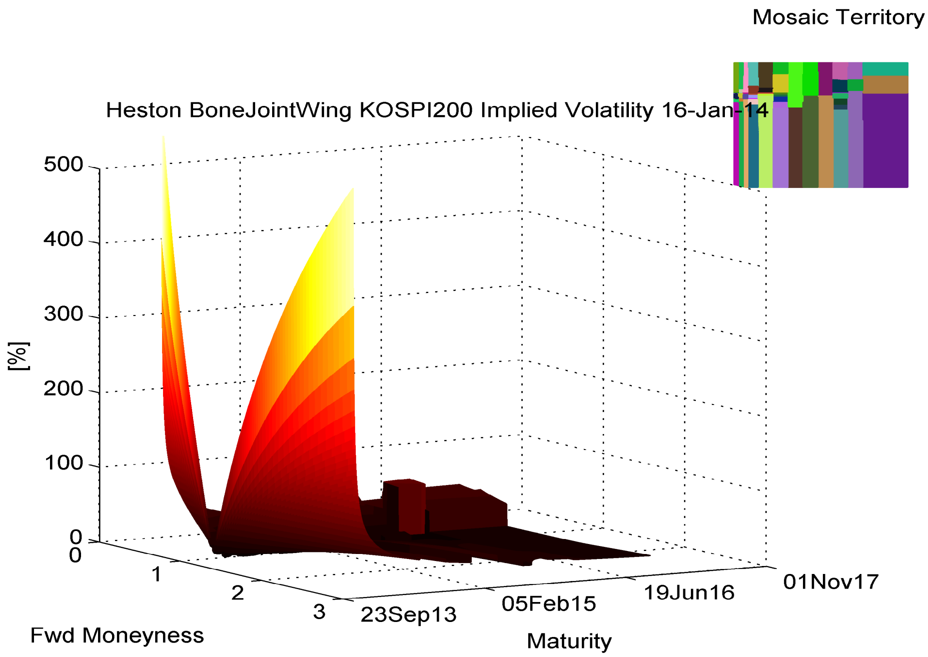


Figure 8.23: Heston BoneWing KOSPI200 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)

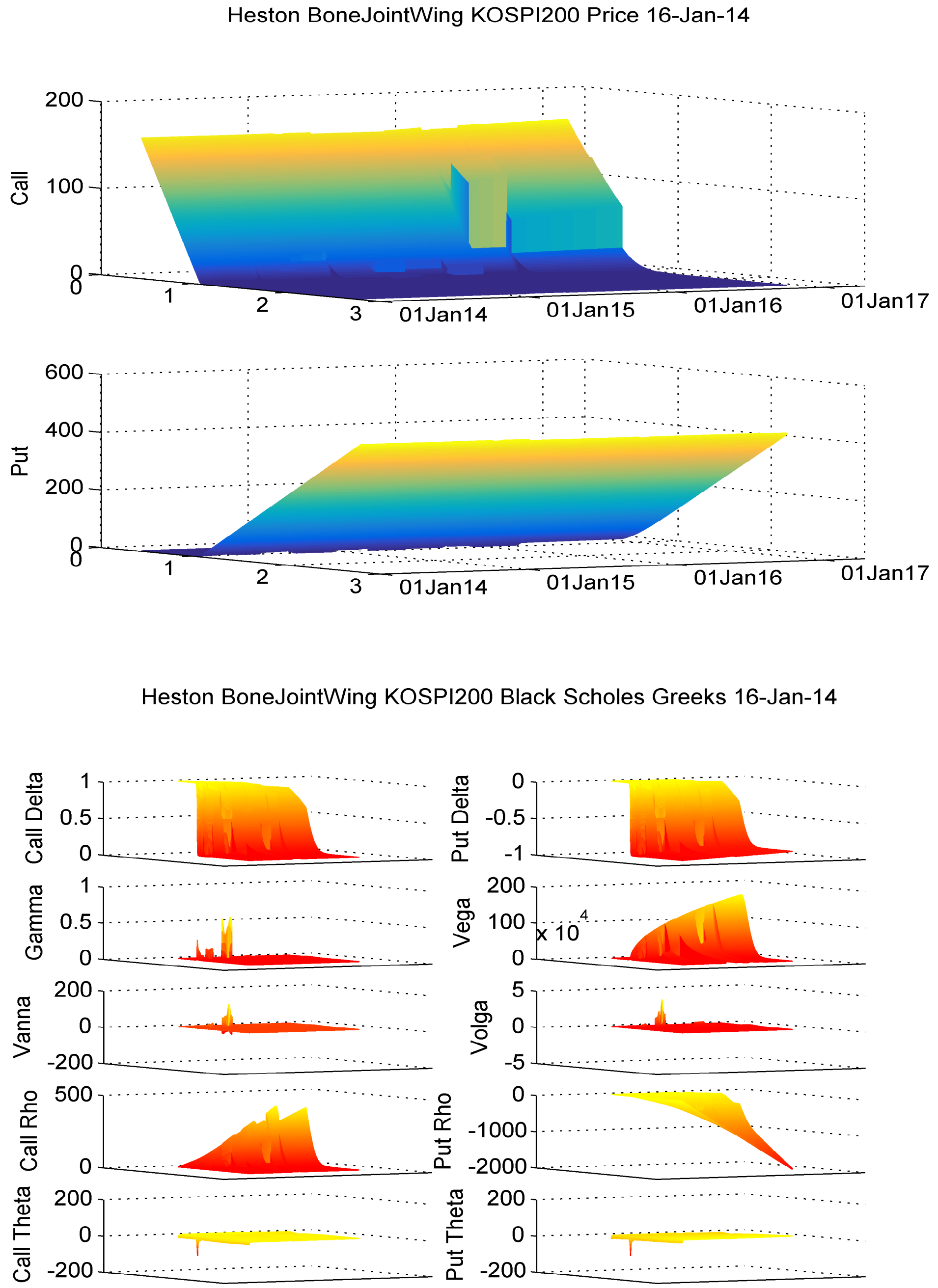
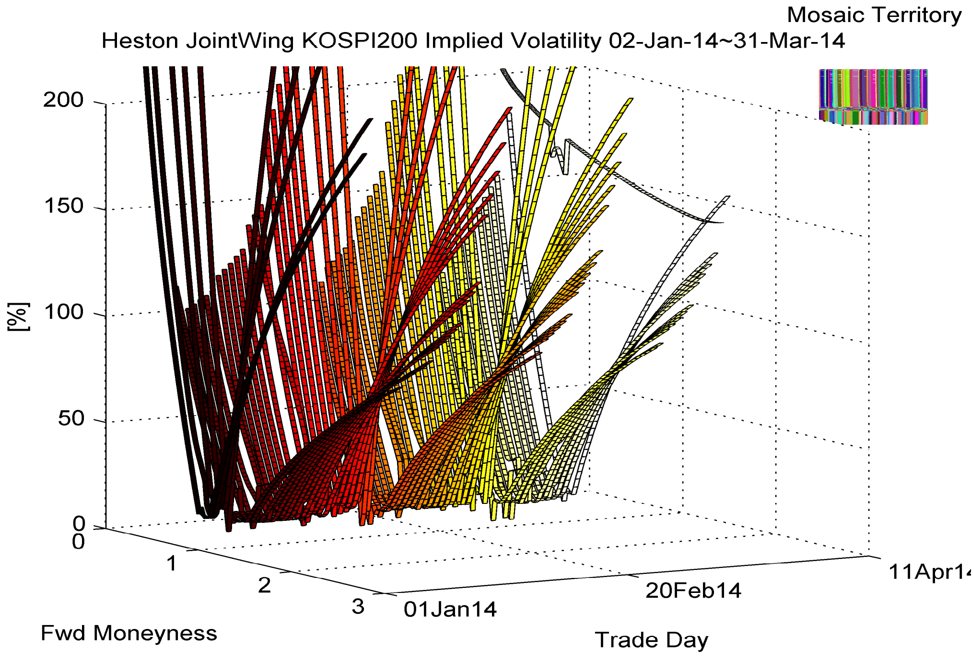


Figure 8.24: Heston BoneWing KOSPI200 Price, Black Scholes Greeks (16-Jan-14)



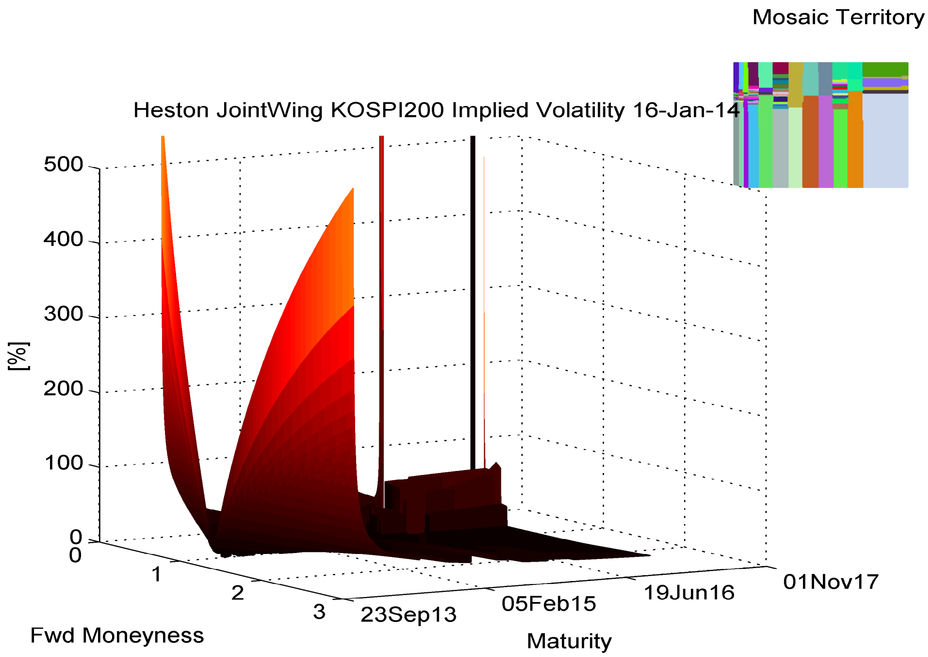
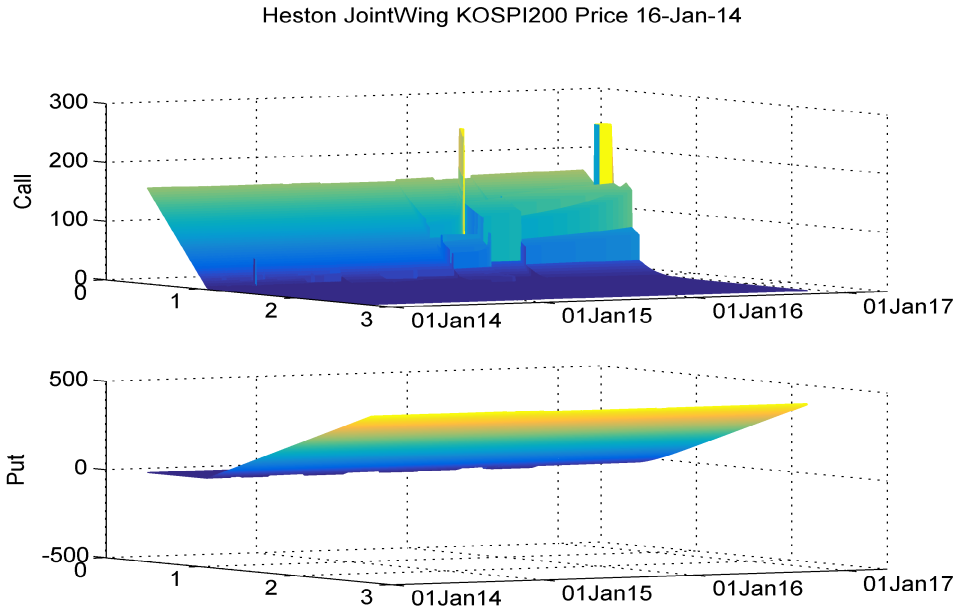


Figure 8.25: Heston BoneWing KOSPI200 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



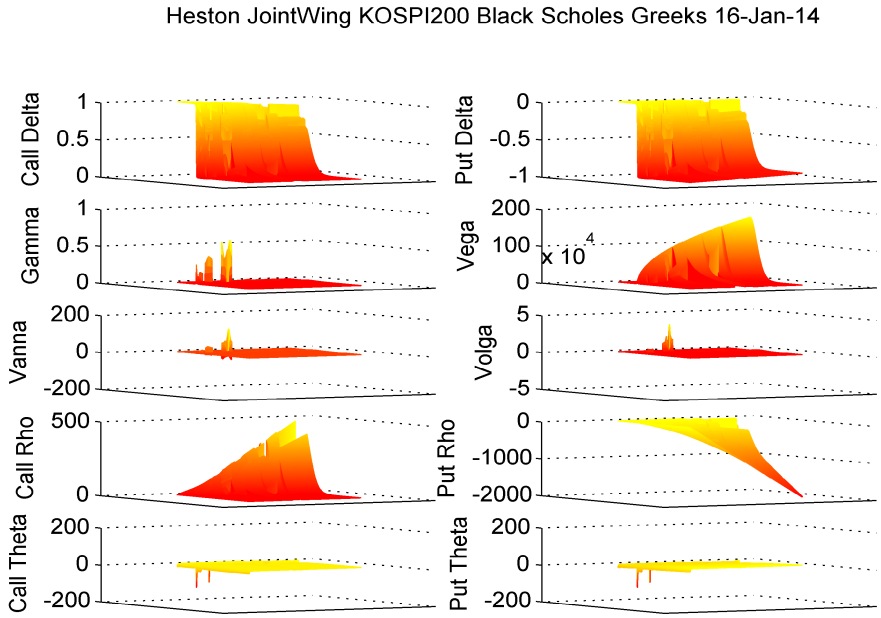


Figure 8.26: Heston BoneWing KOSPI200 Price, Black Scholes Greeks (16-Jan-14)

Graphs of nearest futures SV Mosaic implied volatilities of KOSPI200 during January 2, 2014 to March 31, 2014 are in upper half of Figure B.133 , Figure B.135 , Figure B.137 , Figure B.139 , Figure B.141 and Figure B.143. In lower half of Figure B.133 , Figure B.135 , Figure B.137 , Figure B.139 , Figure B.141 and Figure B.143, there are graphs of SV Mosaic implied volatility of KOSPI200 on January 16, 2014. Graphs of SV Mosaic price of KOSPI200 on January 16, 2014 are in upper half of Figure B.134 , Figure B.136 , Figure B.138 , Figure B.140 , Figure B.142 and Figure B.144. In lower half of Figure B.134 , Figure B.136 , Figure B.138 , Figure B.140 , Figure B.142 and Figure B.144, there are graphs of SV Mosaic Black Scholes delta, gamma, vega, vanna, volga, rho, theta of KOSPI200 on January 16, 2014. IVS domain grid for numerical pricing is applied in Figure B.133 , Figure B.134 , Figure B.135 , Figure B.136 , Figure B.137 , Figure B.138 , Figure B.139 , Figure B.140 , Figure B.141 , Figure B.142 , Figure B.143 and Figure B.144. SABR model with 'Bone Wing' Mosaic system is implemented in Figure B.119 , Figure B.120 , Figure B.133 and Figure B.134. Heston model with 'Bone Wing' Mosaic system is implemented in Figure B.125 , Figure B.126 , Figure B.139 and Figure B.140. SABR model with 'Bone Joint Wing' Mosaic system is implemented in Figure B.121 , Figure B.122 , Figure B.135 and Figure B.136. Heston model with 'Bone Joint Wing' Mosaic system is implemented in Figure B.127 , Figure B.128 , Figure B.141 and Figure B.142. SABR model with 'Joint Wing' Mosaic system is implemented in Figure B.123 , Figure B.124 , Figure B.137 and Figure B.138. Heston model with 'Joint Wing' Mosaic system is implemented in Figure B.129 , Figure B.130 , Figure B.143 and Figure B.144.

On Heston Mosaic IVS, we could observe extremely huge IV outside of the market data(in 'Standard Grid' or grid for numerical pricing). This is the typical problem caused by 'Inaccurate target implied volatility'.

Chapter 9

Constraint Local Volatility Surface

9.1 Constraint LVS

A LVS is generated by the operation to plug (3.1) formula into SV Mosaic IVS with constraint that makes local variance positive.

Algorithm 9.1 Constraint Local Volatility

1. Calculate local variance
2. Calculate non-zero constraint local variance with (5.6)
3. Calculate non-zero upper constraint local variance in order to neutralize the bias caused by non zero constraint.
4. Calculate constraint local volatility from constraint local variance

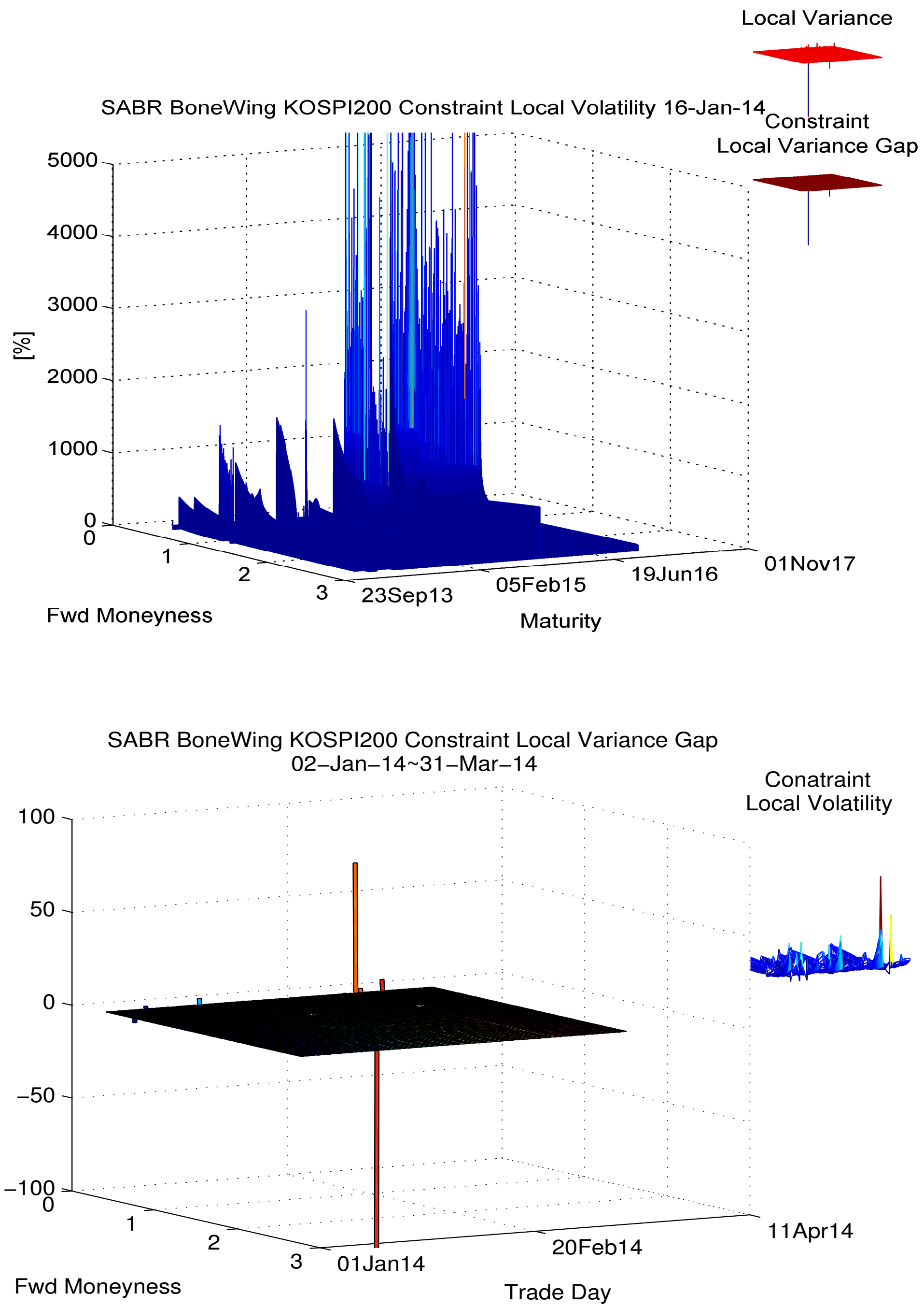


Figure 9.1: SABR BoneWing KOSPI200 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)

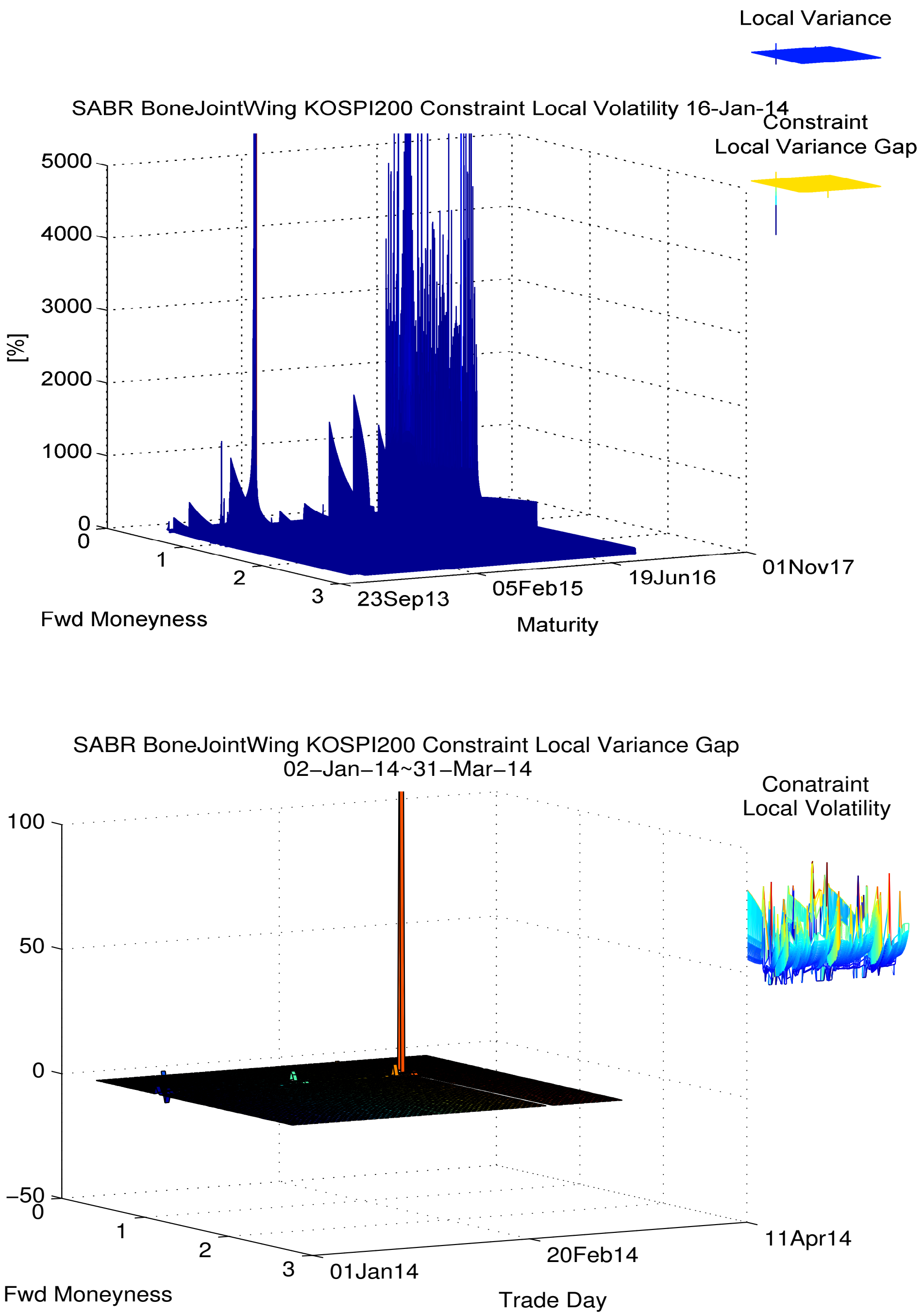
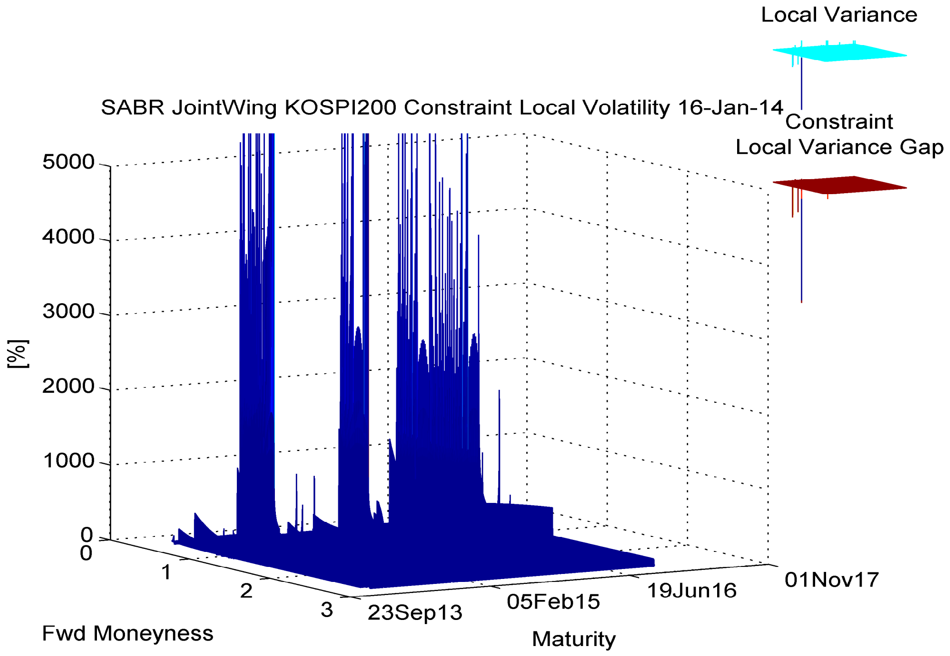


Figure 9.2: SABR BoneJointWing KOSPI200 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)



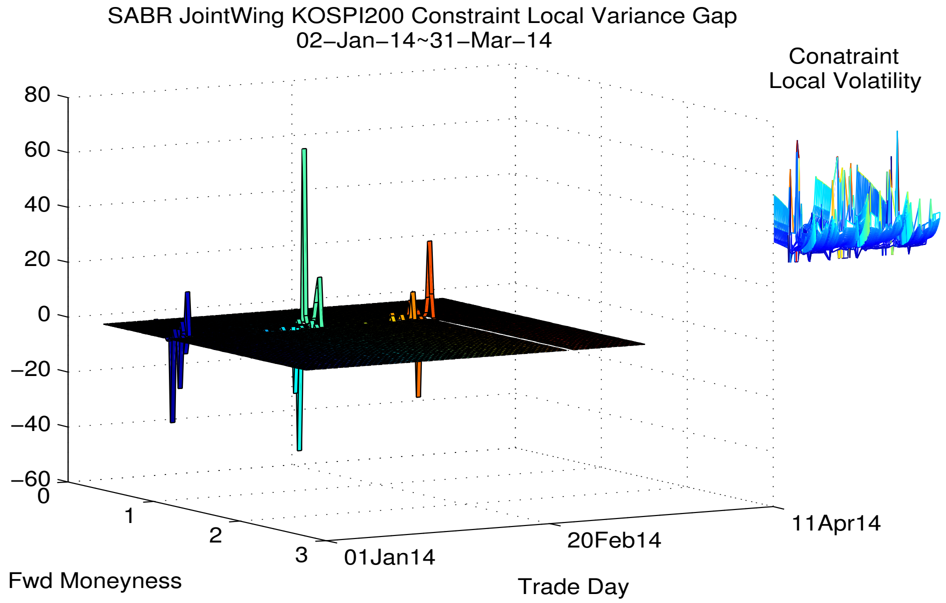
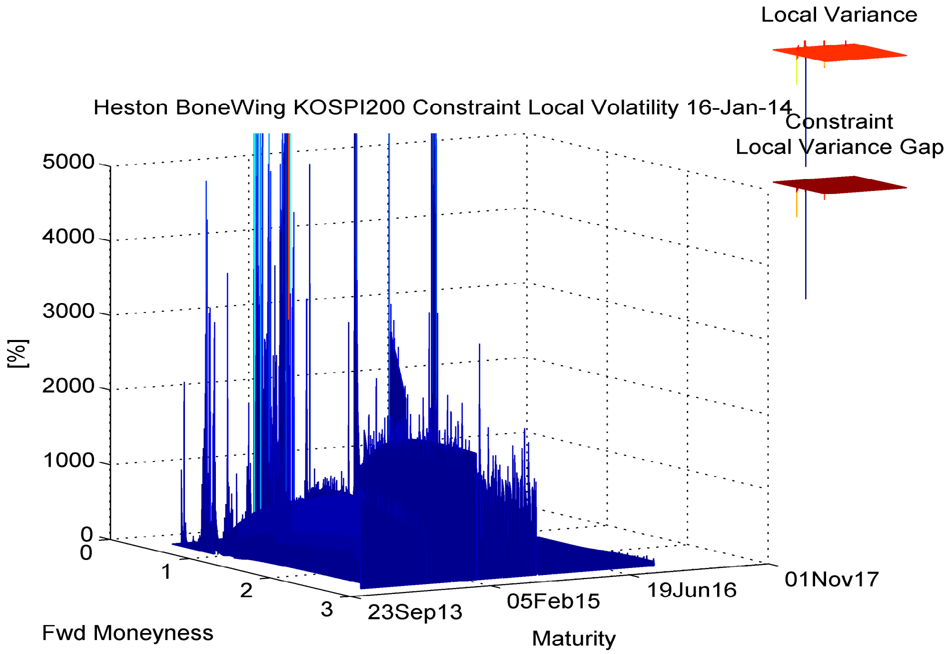


Figure 9.3: SABR JointWing KOSPI200 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



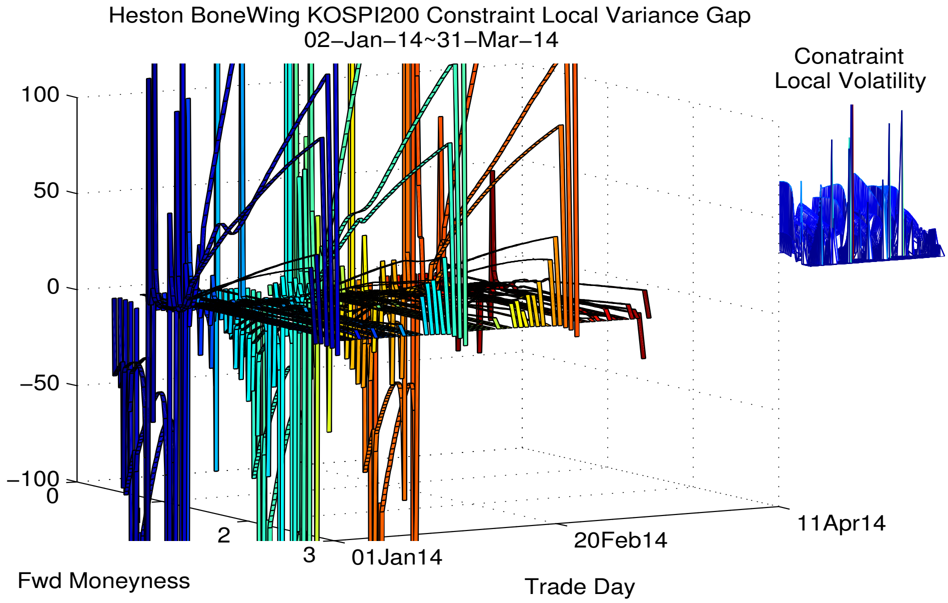
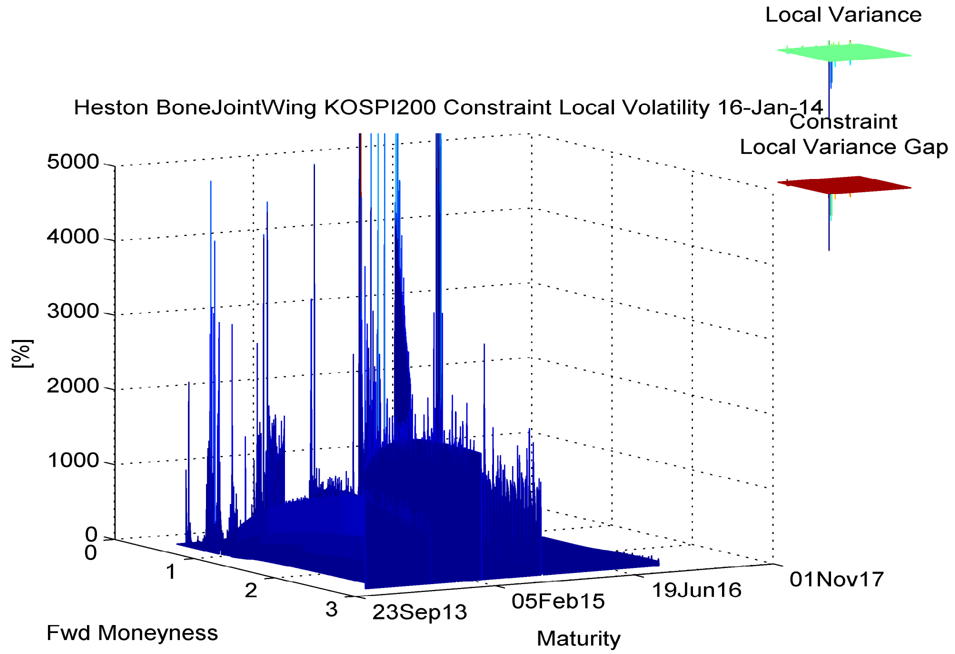


Figure 9.4: Heston BoneWing KOSPI200 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



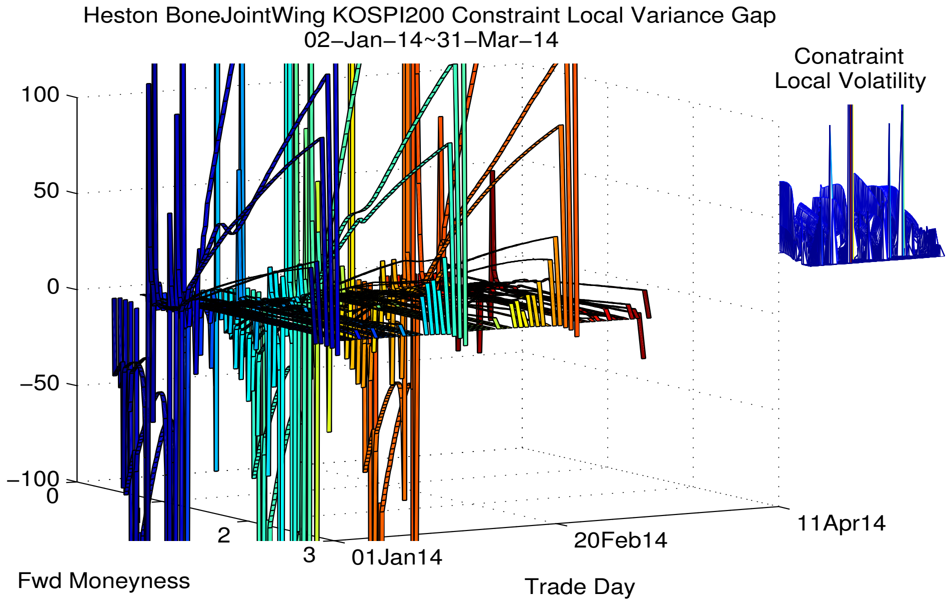


Figure 9.5: Heston BoneJointWing KOSPI200 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)

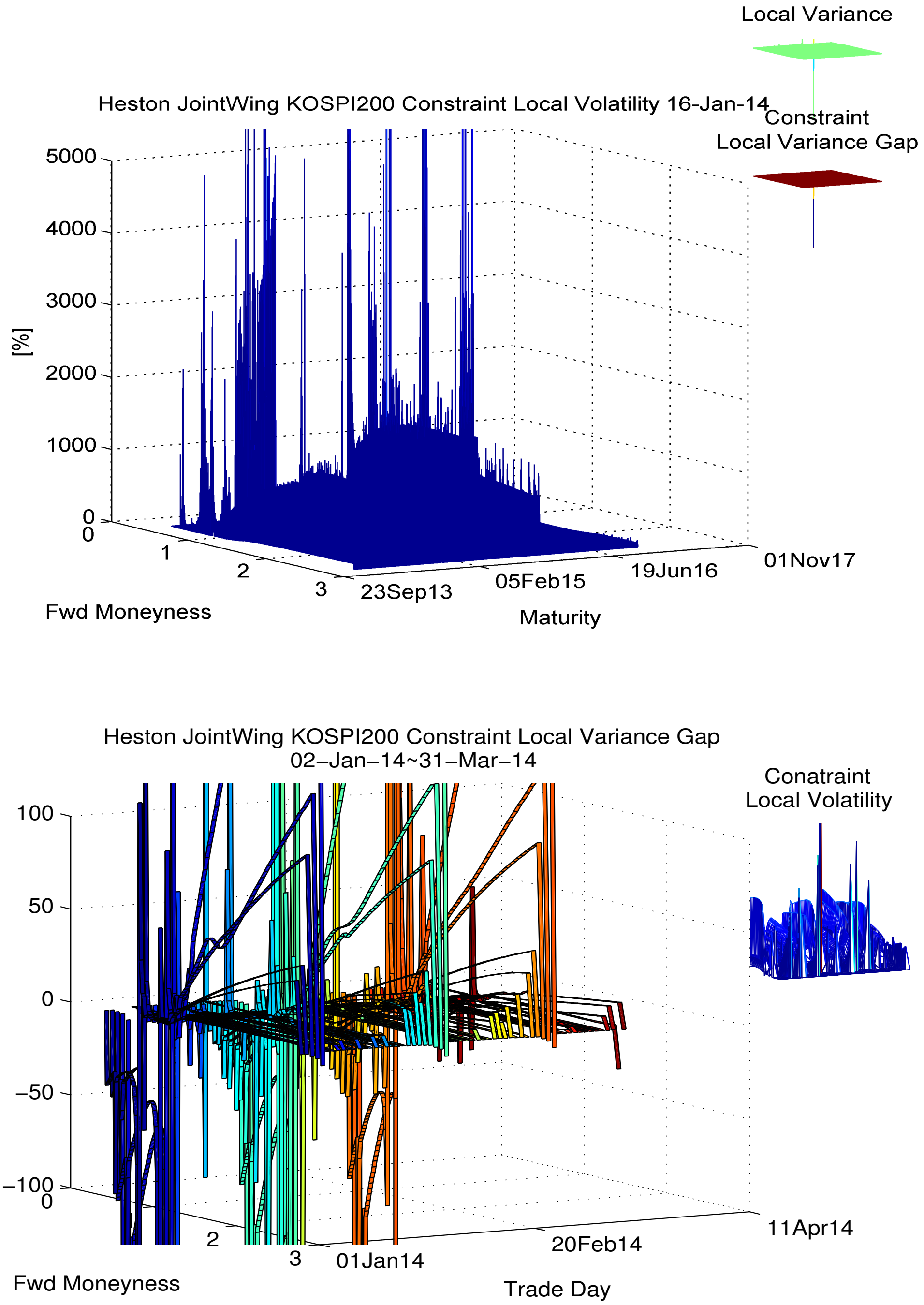


Figure 9.6: Heston JointWing KOSPI200 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)

Table 9.1: Root Mean Absolute KOSPI200 Constraint Local Variance Gap (02-Jan-14∼31-Mar-14)

SABR BoneWing	SABR BoneJoint Wing	SABR JointWing	Heston BoneWing	Heston BoneJoint Wing	Heston JointWing
7.948E-01	7.091E-01	8.249E-01	1.432E+0	1.469E+0	1.524E+0

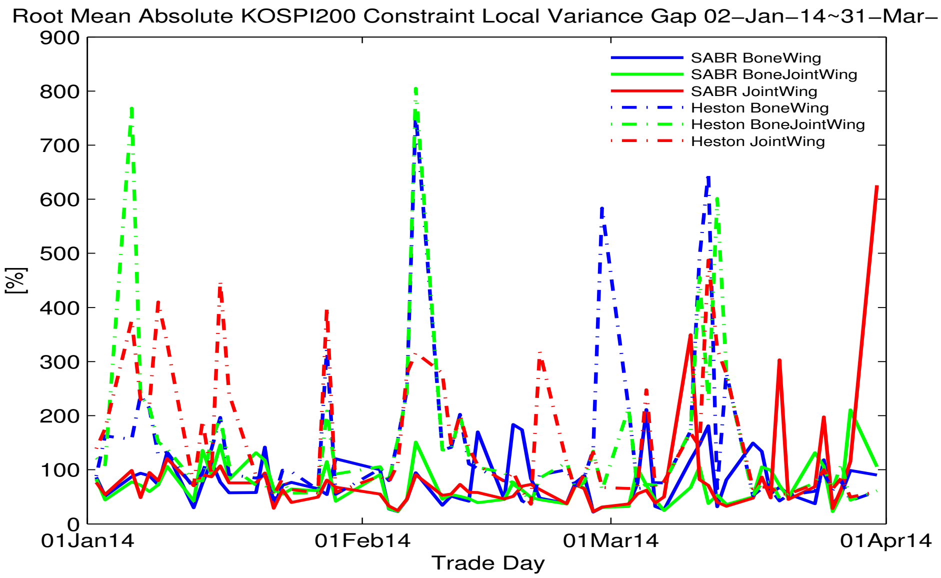


Figure 9.7: Root Mean Absolute KOSPI200 Local Variance Gap (02-Jan-14∼31-Mar-14)

Graphs of constraint local volatility of KOSPI200 on January 16, 2014 are in upper half of Figure B.166 , Figure B.167 , Figure B.168 , Figure B.169 , Figure B.170 and Figure B.171. In lower half of Figure B.166 , Figure B.167 , Figure B.168 , Figure B.169 , Figure B.170 and Figure B.171, there are graphs of the differences between nearest futures constraint local variances of KOSPI200 for local variance obtained by (3.1) during January 2, 2014 to

March 31, 2014. SABR model with 'Bone Wing' Mosaic system is implemented in Figure B.166. Heston model with 'Bone Wing' Mosaic system is implemented in Figure B.169. SABR model with 'Bone Joint Wing' Mosaic system is implemented in Figure B.167. Heston model with 'Bone Joint Wing' Mosaic system is implemented in Figure B.170. SABR model with 'Joint Wing' Mosaic system is implemented in Figure B.168. Heston model with 'Joint Wing' Mosaic system is implemented in Figure B.171. And there are root mean absolute difference and time series of differences between constraint local variances of KOSPI200 for local variance obtained by (3.1) during January 2, 2014 to March 31, 2014 in Table 9.1 and Figure B.172.

Constraint local volatility is not a whole Local-volatility. Accordingly, the result is limited to interpret. The result will imply various meanings when whole Local-volatility is implemented, it is equivalent to 'Arbitrage-free'.

Appendices

Appendix A

Source code

- VBA code of Figure 4.1, Figure 4.2, Figure 4.3 and Table 4.1:

```
Private Function PrjCGConstraint(PlainSurface As MarketGrid, G() As Double, c() As Double, a
() As Double, Subj As String, Constraint As String, xjplus1() As Double, idx As Integer, epsilon
As Double) As Variant

    Dim X() As Double, p() As Double, rs() As Double, gr() As Double, d() As Double, hs() As
    Double
    Dim alpha As Double, beta As Double, rsPnormsq As Double, dGnormsq As Double,
    rsPnormsqold As Double

    Call PrjCGInitiate(G(), X(), c(), a(), p(), rs(), gr(), d(), rsPnormsq)

    Do
        alpha = PrjCGStepsize(G(), d(), rsPnormsq, dGnormsq)
        Call PrjCGOneStep(G(), X(), p(), rs(), gr(), d(), alpha, beta, rsPnormsq)
    Loop Until Abs(rsPnormsq) < epsilon


    Select Case Subj
        Case "Price"
```

```
        Select  Case Constraint
            Case "NoArbitrage"

                Select  Case PlainSurface.SF
                    Case eStrike
                        hs() = deltaCoeff(PlainSurface.Strike(), PlainSurface.CILCntInfo(),
                        idx)
                    Case eSpotMoneyness
                        hs() = deltaCoeff(PlainSurface.SpotMoneyness(), PlainSurface.
                        CILCntInfo(), idx)
                    Case eFwdMoneyness
                        hs() = deltaCoeff(PlainSurface.FwdMoneyness(), PlainSurface.
                        CILCntInfo(), idx)
                End Select

                epsilon = epsilon * 100000

                alpha = PrjCGStepsize(G(), d(), rsPnormsq, dGnormsq)
                d() = NoArbiAdjStep(PlainSurface, alpha, X(), xjplus1(), d(), hs(), idx)
                rsPnormsqold = rsPnormsq

                Do
                    alpha = PrjCGStepsize(G(), d(), rsPnormsq, dGnormsq)
                    d() = NoArbiAdjStep(PlainSurface, alpha, X(), xjplus1(), d(), hs(), idx)
                    rsPnormsqold = rsPnormsq
                    Call PrjCGOneStep(G(), X(), p(), rs(), gr(), d(), alpha, beta,
                    rsPnormsq)
                Loop Until Abs(rsPnormsq) < epsilon Or Abs(rsPnormsqold) < Abs(
                rsPnormsq)

        End Select

    End Select

    PrjCGConstraint = X()

End Function

Public Sub FDMBSPDE(CP As CallPut, XF As XFrequencyType, BN As Barriertype, Security
As Double, Strike() As Double, Barrier() As Double _
                , IntrstRate As Double, DivdndRate As Double, TMaturity As Double,
                TExercise() As Double, Volatility() As Double _
                , SecurityLine() As Double, timeLine() As Double, dSecurityExponent As Double
                , dTline As Double, DrvResult As DerivCalcResult)

    Dim i As Long, j As Long, k As Long, n As Long, nx As Long, nt As Long, secIdx As Long,
```

```
outBrrIdx() As Long, tXidx() As Long
Dim v() As Double, vEnd() As Double, bdL() As Double, bdR() As Double, interv() As
Double, AL As Tridiag, AR As Tridiag, LAL As Tridiag, UAL As Tridiag, deltaAdjust As
Double
Dim FDMScheme As String, Theta() As Double

Dim aa As Double, bb As Double, cc As Double, nu1 As Double, nu2 As Double

FDMScheme = "Implicit"

n = UBound(Barrier(), 1) 'UBound(TExercise(), 1)
ReDim outBrrIdx(n), tXidx(n)

nx = UBound(SecurityLine, 1)
nt = UBound(timeLine(), 1)

Select Case FDMScheme
    Case "Implicit"

    ReDim Theta(nt)
    For j = nt To 1 Step -1

        Theta(j) = 1

'           ' CrankNicolson with Rannicher's time stepping
'           Select Case j
'               Case Is > nt - 10
'                   Theta(j) = 1
'               Case Else
'                   Theta(j) = 0.5
'           End Select

    Next j

End Select

For i = 1 To nx - 1

    If Security >= SecurityLine(i) And Security < SecurityLine(i + 1) Then secIdx = i: Exit
    For

Next i

For k = 1 To n

    Select Case BN
```

```
        Case eUpandOut
            For i = 1 To nx - 1

                If Barrier(k) > SecurityLine(i) And Barrier(k) <= SecurityLine(i + 1) Then
                outBrrIdx(k) = i + 1: Exit For

            Next i
            If Barrier(k) >= SecurityLine(nx) Then outBrrIdx(k) = nx
        Case Else
            outBrrIdx(k) = nx
    End Select

    Select Case XF
        Case eBermudan
            For j = 1 To nt - 1

                If TExercise(k) >= timeLine(j) And TExercise(k) < timeLine(j + 1) Then
                tXidx(k) = j: Exit For

            Next
            If TExercise(k) >= timeLine(nt) Then tXidx(k) = nt
        Case Else
            tXidx(k) = nt
    End Select

Next k

ReDim DrvResult.Value(nx, nt)
ReDim DrvResult.Delta(nx, nt)
ReDim DrvResult.Gamma(nx, nt)
ReDim DrvResult.Theta(nx, nt)

k = UBound(TExercise(), 1)

ReDim v(outBrrIdx(k) - 1)
v() = FDMBSTermiVec(CP, XF, BN, SecurityLine(), Strike(k), Barrier(k), IntrstRate,
DivdndRate, TMaturity, TExercise(k), v())
vEnd() = v()
bdR() = FDMBSBoundVec(CP, XF, BN, SecurityLine(), Strike(k), outBrrIdx(k), IntrstRate,
DivdndRate, TMaturity, TExercise(k))

For j = nt To 1 Step -1

    Call FDMBSCoeffMatL(Theta(j), AL, SecurityLine(), IntrstRate, DivdndRate, Volatility
    (), dSecurityExponent, dTline, outBrrIdx(k) - 1, j)
    Call FDMBSCoeffMatR(1 - Theta(j), AR, SecurityLine(), IntrstRate, DivdndRate,
```

```
Volatility(), dSecurityExponent, dTline, outBrrIdx(k) − 1, j)
Call TDMALU(AL, LAL, UAL, 1, outBrrIdx(k) − 1)

bdL() = FDMBSBoundVec(CP, XF, BN, SecurityLine(), Strike(k), outBrrIdx(k),
IntrstRate, DivdndRate, TMaturity, timeLine(j))

v() = FDMBSInterv(AR, AL, v(), bdR(), bdL(), SecurityLine(), IntrstRate, DivdndRate,
Volatility(), dSecurityExponent, dTline, outBrrIdx(k) − 1, j)

v() = TDMASolve(LAL, UAL, v(), 1, outBrrIdx(k) − 1)

bdR() = bdL()

If tXidx(k) = j And k <> UBound(tXidx(), 1) Then

    k = k − 1
    v() = FDMBSTermiVec(CP, XF, BN, SecurityLine(), Strike(k), Barrier(k), IntrstRate
    , DivdndRate, TMaturity, TExercise(k), v())

End If

For i = 1 To outBrrIdx(k) − 1

    DrvResult.Value(i, j) = v(i)

Next i

For i = 1 To outBrrIdx(k) − 1

    Select Case i
        Case 1

            DrvResult.Delta(i, j) = (DrvResult.Value(i + 1, j) − DrvResult.Value(i, j))
            / (SecurityLine(i + 1) − SecurityLine(i)) 'dSecurityExponent
            DrvResult.Gamma(i, j) = 0

        Case Else

            DrvResult.Delta(i, j) = (DrvResult.Value(i + 1, j) − DrvResult.Value(i − 1,
            j)) / (SecurityLine(i + 1) − SecurityLine(i − 1)) '(2 * dSecurityExponent)
            DrvResult.Gamma(i, j) = (DrvResult.Value(i + 1, j) − 2 * DrvResult.Value(i,
             j) + DrvResult.Value(i − 1, j)) / (SecurityLine(i + 1) − SecurityLine(i − 1)
            ) ^ 2

    End Select
```

```
            DrvResult.Theta(i, j) = −0.5 ∗ Volatility(i, j) ^ 2 ∗ SecurityLine(i) ∗ DrvResult.
            Gamma(i, j) − IntrstRate ∗ SecurityLine(i) ∗ DrvResult.Delta(i, j) + IntrstRate ∗
            DrvResult.Value(i, j)

        Next i

        If j = 1 Then

            DrvResult.SpotValue = DrvResult.Value(secIdx, j)
            DrvResult.SpotDelta = DrvResult.Delta(secIdx, j)
            DrvResult.SpotGamma = DrvResult.Gamma(secIdx, j)
            DrvResult.SpotTheta = DrvResult.Theta(secIdx, j)

        End If

    Next j

End Sub
```

- Matlab code of Algorithm 5.1:

```
function dividend = calcImpliedDividend(securitySpot, timetoMaturity, interestRate,
securityForward, strike, callValue, putValue)
   n = size(timetoMaturity,1);
   dividend = zeros(n,1);
   if isempty(strike)
       for idx = 1: n
           dividend(idx) = FinancialMarket.getImpliedDividendbyForwardCostIdentity(
           securitySpot, timetoMaturity(idx), interestRate(idx), securityForward(idx));
       end
   else
       for idx = 1: n
           if isempty(strike{idx})
               dividend(idx) = FinancialMarket.getImpliedDividendbyForwardCostIdentity(
               securitySpot, timetoMaturity(idx), interestRate(idx), securityForward(idx));
           else
               dividend(idx) = FinancialMarket.getImpliedDividendbyPutCallParity(securitySpot,
                timetoMaturity(idx), interestRate(idx), strike{idx}, callValue{idx}, putValue{idx
               });
           end
       end
```

```
    end
end
function dividend = getImpliedDividendbyForwardCostIdentity(securitySpot, timetoMaturity,
interestRate, securityForward)
    dividend = −log(securityForward/securitySpot)./timetoMaturity+interestRate;
    dividend(dividend < 0) = nan;
end
function dividend = getImpliedDividendbyPutCallParity(securitySpot, timetoMaturity,
interestRate, strike, callValue, putValue, cushioncell, dividendBoundcell)
    dividend = log((putValue + securitySpot − callValue − strike∗exp(−interestRate∗
    timetoMaturity))/securitySpot+1)/timetoMaturity;
    if nargin < 7
        cushionL = exp(−10);
        cushionU = 0;
        cushioncell = {cushionL; cushionU};
    end
    if nargin < 8
        dividendLB = 0;
        dividendUB = realmax;
        dividendBoundcell = {dividendLB,dividendUB};
    end
    dividend = FinancialMarket.getBoundCushionedValues(dividend, cushioncell,
    dividendBoundcell);
    dividend = median(dividend);
end
function y = getBoundCushionedValues(x, cushioncell, boundcell)
    lcushion = cushioncell{1};
    ucushion = cushioncell{2};
    lowerbound = boundcell{1};
    upperbound = boundcell{2};
    if isscalar (lcushion)
        lcushion = repmat(lcushion,size(x));
    end
    if isscalar (ucushion)
        ucushion = repmat(ucushion,size(x));
    end
    if isscalar (lowerbound)
        lowerbound = repmat(lowerbound,size(x));
    end
    if isscalar (upperbound)
        upperbound = repmat(upperbound,size(x));
    end
    y = arrayfun(@getBCValue, x, lcushion, ucushion, lowerbound, upperbound);
    function y = getBCValue(x, lcushion, ucushion, lowerbound, upperbound)
        if x < lowerbound+lcushion
            y = lowerbound+lcushion∗exp(x−lowerbound−lcushion);
```

```
        elseif  upperbound − ucushion <= x
            y =  upperbound−ucushion∗(exp(−x+upperbound−ucushion));
        else
            y  = x;
        end
    end
end
```

- Matlab code of Algorithm 5.2:

```
function market = mergeMarket(obj, secMarket)
    primMarket = obj;
    tradeday = primMarket.Spot.Tradeday;
    securitySpot = primMarket.Spot.SecuritySpot;
    exDivMonth = primMarket.Spot.ExDivMonth;
    primTimetoMaturity = primMarket.Term.TimetoMaturity;
    primDividend = primMarket.Term.Dividend;
    primSecurityForward = primMarket.Term.SecurityForward;
    if isempty(secMarket)
        secTimetoMaturity = [];
        secDividend = [];
        secSecurityForward = [];
    else
        secTimetoMaturity = secMarket.Term.TimetoMaturity;
        secDividend = secMarket.Term.Dividend;
        secSecurityForward = secMarket.Term.SecurityForward;

        secLgclIdx = FinancialMarket.getSecondLogicalIndex(primTimetoMaturity,
        secTimetoMaturity);
        dropDivPrimLgclIdx = FinancialMarket.getDroppedPrimeLogicalIndex(
        primTimetoMaturity, secTimetoMaturity, primDividend);
        selectDivSecLgclIdx = FinancialMarket.getSelectedSecondLogicalIndex(
        primTimetoMaturity, secTimetoMaturity, primDividend);
        primDividend(dropDivPrimLgclIdx) = secDividend(selectDivSecLgclIdx);
        dropSFPrimLgclIdx = FinancialMarket.getDroppedPrimeLogicalIndex(
        primTimetoMaturity, secTimetoMaturity, primSecurityForward);
        selectSFSecLgclIdx = FinancialMarket.getSelectedSecondLogicalIndex(
        primTimetoMaturity, secTimetoMaturity, primSecurityForward);
        primSecurityForward(dropSFPrimLgclIdx) = secSecurityForward(selectSFSecLgclIdx);
        secTimetoMaturity = secTimetoMaturity(secLgclIdx);
        secDividend = secMarket.Term.Dividend(secLgclIdx);
```

```
        secSecurityForward = secMarket.Term.SecurityForward(secLgclIdx);
    end

    timetoMaturity = [primTimetoMaturity; secTimetoMaturity];
    dividend = [primDividend; secDividend];
    securityForward = [primSecurityForward; secSecurityForward];

    [timetoMaturity, ia] = sort(timetoMaturity);
    maturity = FinancialMarket.getMaturity(tradeday, timetoMaturity);
    exDivTimecell = FinancialMarket.calcexDivTimeToMaturity(tradeday, maturity, exDivMonth
    );
    dividend = dividend(ia);
    nIndex = ~isnan(dividend);
    dividend = FinancialMarket.interextp1(timetoMaturity(nIndex), dividend(nIndex),
    timetoMaturity,'lumpsumDividend','', exDivTimecell);
    securityForward = securityForward(ia);
    interestRate = FinancialMarket.getImpliedInterestRatebyForwardDividendIdentity(
    securitySpot, timetoMaturity, dividend, securityForward);
    secinterpIndex = isnan(interestRate);
    interestRate = FinancialMarket.interextp1(timetoMaturity(~secinterpIndex),interestRate(~
    secinterpIndex),timetoMaturity);
    dividend = dividend-interestRate.*(interestRate<0);
    interestRate = interestRate.*(interestRate>=0);
    securityForward(secinterpIndex) = securitySpot * exp((interestRate(secinterpIndex)-dividend
    (secinterpIndex)).*timetoMaturity(secinterpIndex));
    market = FinancialMarket([],tradeday,securitySpot, exDivMonth,maturity,timetoMaturity,
    interestRate, exDivTimecell,dividend,securityForward, [], [], [], []) ;
end
function logicalIndex = getSecondLogicalIndex(primeX, secondX)
    n = size(secondX,1);
    logicalIndex = false(n,1);
    for idx = 1: n
        logicalIndex(idx) = ~any(primeX == secondX(idx));
    end
end
function logicalIndex = getDroppedPrimeLogicalIndex(primeX, secondX, primeV)
    n = size(primeX,1);
    logicalIndex = true(n,1);
    for idx = 1: n
        logicalIndex(idx) = and(any(primeX(idx) == secondX), isnan(primeV(idx)));
    end
end
function logicalIndex = getSelectedSecondLogicalIndex(primeX, secondX, primeV)
    droppedPrimeLgclIdx = FinancialMarket.getDroppedPrimeLogicalIndex(primeX, secondX,
    primeV);
    m = size(secondX,1);
```

```
    logicalIndex = false(m,1);
    droppedPrimeIndcs = find(droppedPrimeLgclIdx);
    n = size(droppedPrimeIndcs,1);
    for idx = 1: n
        logicalIndex = or(logicalIndex, primeX(droppedPrimeIndcs(idx)) == secondX);
    end
end
function vq = interextp1(x,v,xq,interpMethod,extrapMethod,exDivtimecell)
    if isempty(v)
        vq = v;
        return;
    elseif isscalar (x)
        vq = repmat(v,size(xq));
        return;
    end
    if nargin < 4
        interpMethod = 'linear';
    end
    if nargin < 5
        extrapMethod = 'nearest';
    end
    switch interpMethod
        case 'lumpsumDividend'
            if isempty(extrapMethod)
                extrapMethod = 'nearest';
            end
            n = size(xq,1);
            vq = nan(n,1);
            for idx = 1: n
                jdx = find(x==xq(idx),1);
                if isempty(jdx)
                    sdx = find(x<xq(idx),1,'last');
                    edx = find(x>xq(idx),1,'first');
                    if isempty(edx)
                        if ~isempty(sdx)
                            vq(idx) = v(sdx);
                        end
                    else
                        if isempty(sdx)
                            sdx = edx;
                        end
                        Ds = 1-exp(-v(sdx)*x(sdx));
                        De = 1-exp(-v(edx)*x(edx));
                        exX = exDivtimecell{edx};
                        m = size(exX,1);
                        if Ds == De
```

```
                D = repmat(Ds,m+1,1);
            else
                D = (De:(Ds−De)/(m+1):Ds)';
                D = D(2:end);
            end
            X = [exX; x(edx)];
            jdx = find(X>=xq(idx),1,'first') ;
            vq(idx) = −log(1−D(jdx))/xq(idx);
        end
    else
        vq(idx) = v(jdx);
    end
   end
  otherwise
   vq = interp1(x, v, xq, interpMethod);
 end
 extrapIndex = find(isnan(vq));
 vq(extrapIndex) = interp1(x, v, xq(extrapIndex), extrapMethod,'extrap');
end
function interestRate = getImpliedInterestRatebyForwardDividendIdentity(securitySpot,
timetoMaturity, dividend, securityForward)
 interestRate = log(securityForward/securitySpot)./timetoMaturity+dividend;
end
```

- Matlab code of Algorithm 5.3:

```
function market = getSecurityForwardMarket(obj)
 market = obj.EOMarket;
 marketName = obj.UnderlyingName;
 tradeday = datenum(obj.Tradedaystr);
 securitySpot = obj.EXMarket.Spot.SecuritySpot;
 exDivMonth = obj.EXMarket.Spot.ExDivMonth;
 rlTimetoMaturity = obj.Numcell{3}(:,8);
 rlInterstRate = obj.Numcell{3}(:,10);
 exDivTimecell = market.Term.ExDivTimecell;
 dividend = market.Term.Dividend;
 maturity = market.Term.Maturity;
 timetoMaturity = market.Term.TimetoMaturity;
 interestRate = FinancialMarket.interextp1(rlTimetoMaturity, rlInterstRate, timetoMaturity);
 securityForward = securitySpot ∗ exp((interestRate−dividend).∗timetoMaturity);
```

```
    market = FinancialMarket(marketName, tradeday, securitySpot, exDivMonth, maturity,
    timetoMaturity, interestRate, exDivTimecell, dividend, securityForward, [], [], [], []) ;
end
```

- Matlab code of Algorithm 6.1:

```
function obj = setBase(obj, nPData)
    soW = nPData.getOTCSyntheticOptionWeight;
    W = obj.getBaseCoefficientMatrix(soW);

    [~, soRights, soUnderlying, soStrike, soRate, soTime, ~, ~, ~, ~, ~, ~, ~, ~] = ...
    nPData.readOTCSyntheticOption;
    uLB = obj.getBaseUnknownLBVector(soUnderlying, soStrike, soRate, soTime);

    exotcop = nPData.getExchangeOTCOptionPrice;
    k = obj.getBaseKnownVector(exotcop);

    soP = obj.getBaseParityMatrix(soRights, soUnderlying, soStrike, soRate, soTime);

    s = nPData.getOTCSyntheticOptionPrice;

    field1 = 'W';
    field2 = 'uLB';
    field3 = 'k';
    field4 = 'o';
    field5 = 's';
    field6 = 'soW';
    field7 = 'soP';
    field8 = 'SOs';

    obj.Base = struct(field1, W, field2, uLB, field3, k, field4, [], field5, s, field6, soW,
    field7, soP, field8, []) ;
end
function uLBVector = getBaseUnknownLBVector(obj, refUnderlying, refStrike, refRate, refTime)
    rights = Option.rightsChange(obj.Key{Layer.Mono}.r);
    indices = uint64(obj.Key{Layer.Mono}.Iu);
    n = size(indices,1) ;
    underlying = nan(n,1);
    strike = nan(n,1);
    rate = nan(n,1);
    time = nan(n,1);
```

```
    for idx = 1: n
        if and(indices(idx,1)~=0, indices(idx,2)~=0)
            underlying(idx) = refUnderlying(indices(idx,1), indices(idx,2));
            strike(idx) = refStrike(indices(idx,1), indices(idx,2));
            rate(idx) = refRate(indices(idx,1), indices(idx,2));
            time(idx) = refTime(indices(idx,1), indices(idx,2));
        end
    end
    uLBVector = max(0, Option.calcBSParityValue(rights, underlying, strike, rate, time));
end
function kVector = getBaseKnownVector(obj, referenceVector)
    indices = obj.Key{Layer.Mono}.Ik;
    n = size(indices,1);
    kVector = zeros(n,1);
    lgclIdx = ~isnan(indices);
    kVector(lgclIdx) = referenceVector(indices(lgclIdx));
end
function pMatrix = getBaseParityMatrix(obj, refRights, refUnderlying, refStrike, refRate,
refTime)
    uRrights = obj.Key{Layer.Mono}.r;
    indices = obj.Key{Layer.Mono}.ISOs;
    [m,n] = size(indices);
    rights = nan(m,n);
    for jdx = 1: n
        idcs = find(~isnan(indices(:, jdx)));
        rights(idcs, jdx) = uRrights(indices(idcs, jdx));
    end
    pMatrix = Option.calcBSParityValue(rights, refUnderlying, refStrike, refRate, refTime,
    refRights);
end
function rights = rightsChange(rights)
    rights = (rights==ORights.Call)*double(ORights.Put) + (rights==ORights.Put)*double(
    ORights.Call);
end
function parityvalue = calcBSParityValue(rights, underlying, strike, rate, time, parityrights)
    if nargin < 6
        parityrights = Option.rightsChange(rights);
    end
    lgclrights = (rights==ORights.Call);
    lgclparityrights = (parityrights==ORights.Call);
    parityvalue = xor(lgclrights, lgclparityrights).*(2*lgclparityrights-1).*(underlying-strike)
    .*exp(-rate.*time);
end
```

- Matlab code of Algorithm 6.2:

```
function obj = setDerivative(obj)
    Wbase = obj.Base.W;
    uLBbase = obj.Base.uLB;
    kbase = obj.Base.k;
    sbase = obj.Base.s;
    soWbase = obj.Base.soW;
    soPbase = obj.Base.soP;
    [m, n] = size(Wbase);
    rowLgclIdx = true(m,1);
    delrowLgclIdx = false(m, 1);
    colLgclIdx = true(n,1);

    colLgclIdx(kbase > 0) = false;

    cnt = 0;
    maxIter = min(m,n);
    status = false;
    while cnt < maxIter && ~status
        [rowLgclIdx, colLgclIdx, status] = ...
         BootstrappingSOs.calcLeftInvertibleMatrixLogicalIndex(Wbase, rowLgclIdx, colLgclIdx);

        cnt = cnt + 1;
        if status
            W = Wbase(rowLgclIdx, colLgclIdx);
            s = sbase(rowLgclIdx);
            soW = soWbase(rowLgclIdx, :);
            soP = soPbase(rowLgclIdx, :);

            [u, resnorm, residual] = lsqlin(W,s-nansum(soW.*soP,2));
            uLB = uLBbase(colLgclIdx);
            if any(u < uLB)
                delrowLgclIdx = BootstrappingSOs.calcLowerBoundOverStepLogicalIndex(W, u,
                uLB, residual, rowLgclIdx, delrowLgclIdx);
                rowLgclIdx(delrowLgclIdx) = false;
                status = false;
            else
                if max(size(W)) > rank(W)
                    delrowLgclIdx = BootstrappingSOs.calcResidualDominantLogicalIndex(W,
                    residual, rowLgclIdx, delrowLgclIdx);
                    rowLgclIdx(delrowLgclIdx) = false;
                    status = false;
                end
```

```
            end
        end
    end

    rowIdcs = find(rowLgclIdx);
    delrowIdcs = find(delrowLgclIdx);
    colIdcs = find(colLgclIdx);

    field1 = 'rowIdcs';
    field2 = 'delrowIdcs';
    field3 = 'colIdcs';
    B2D = struct(field1, rowIdcs, field2, delrowIdcs, field3, colIdcs);

    field1 = 'W';
    field2 = 'u';
    field3 = 'uLB';
    field4 = 's';
    field5 = 'soW';
    field6 = 'soP';
    field7 = 'B2D';

    obj.Derivative = struct(field1, W, field2, u, field3, uLB, field4, s, field5, soW, field6,
    soP, field7, B2D);
end
function nnzVector = getNnzVector(A, rcIdx)
    n = size(A,rcIdx);
    nnzVector = zeros(n,1);
    for idx = 1: n
        switch rcIdx
            case 1
                nnzVector(idx) = nnz(A(idx, :));
            case 2
                nnzVector(idx) = nnz(A(:, idx));
        end
    end
end
function [rowLgclIdx, colLgclIdx] = calcLinealyIndependenceLogicalIndex(A,nnzColIndices,
rowLgclIdx, colLgclIdx)
    rowIdcs = find(rowLgclIdx);
    colIdcs = find(colLgclIdx);
    n = size(A,2);
    for jdx = 2: n
        if rank(A(:,nnzColIndices(1:jdx))) == rank(A(:,nnzColIndices(1:jdx-1)))
            depcolIdx = nnzColIndices(jdx);
            deprowIdcs = find(A(:,depcolIdx));
```

```
                colLgclIdx(colIdcs(depcolIdx)) = false;
                rowLgclIdx(rowIdcs(deprowIdcs)) = false;
            end
        end
    end
    function [rowLgclIdx, colLgclIdx, status] = calcLeftInvertibleMatrixLogicalIndex(AInBase,
    rowLgclIdx, colLgclIdx)
            A = AInBase(rowLgclIdx, colLgclIdx);

            nnzCol = BootstrappingSOs.getNnzVector(A,2);

            [~, InnzCol] = sort(nnzCol);
            [rowLgclIdx, colLgclIdx] = BootstrappingSOs.calcLinealyIndependenceLogicalIndex(A,
            InnzCol,rowLgclIdx, colLgclIdx);

            A = AInBase(rowLgclIdx, colLgclIdx);
            rowIdcs = find(rowLgclIdx);
            colIdcs = find(colLgclIdx);

            nnzCol = BootstrappingSOs.getNnzVector(A,2);
            nzColLgclIdx = (nnzCol == 0);
            colLgclIdx(colIdcs(nzColLgclIdx)) = false;

            A = AInBase(rowLgclIdx, colLgclIdx);

            nnzRow = BootstrappingSOs.getNnzVector(A,1);
            nzRowLgclIdx = (nnzRow == 0);
            rowLgclIdx(rowIdcs(nzRowLgclIdx)) = false;

            A = AInBase(rowLgclIdx, colLgclIdx);

            if min(size(A)) > rank(A)
                status = false;
            else
                status = true;
            end
    end
    function delrowLgclIdx = calcLowerBoundOverStepLogicalIndex(A, x, xLB, residual, rowLgclIdx,
    delrowLgclIdx)
        delrowJdcscell = arrayfun(@(y) {find(A(:,y) ~= 0)}, find(x < xLB));
        delrowJdcsInrowIdcs = sort(cell2mat(cellfun(@(y) {getScalarIndex(y)}, delrowJdcscell)));
        function index = getScalarIndex(indices)
            if isscalar(indices)
                index = indices;
            else
                index = [];
```

```
        end
    end
    delrowJdcsInrowIdcs = sort([delrowJdcsInrowIdcs; cell2mat(cellfun(@(y) {getArrayIndex(y)},
    delrowJdcscell))]);
    function index = getArrayIndex(indices)
        if any(arrayfun(@(y) any(y == delrowJdcsInrowIdcs), indices))
            index = [];
            return;
        end
        [~,index] = sort(abs(residual(indices)),'descend');
        index = indices(index(1));
    end
    rowIdcs = find(rowLgclIdx);
    delrowLgclIdx(rowIdcs(delrowJdcsInrowIdcs)) = true;
end
function delrowLgclIdx = calcResidualDominantLogicalIndex(A, residual, rowLgclIdx,
delrowLgclIdx)
    [~,delrowJdcsInrowIdcs] = sort(abs(residual),'descend');
    n = size(residual,1)-rank(A);
    if n > 0
        delrowJdcsInrowIdcs = delrowJdcsInrowIdcs(n);
    else
        return;
    end
    rowIdcs = find(rowLgclIdx);
    delrowLgclIdx(rowIdcs(delrowJdcsInrowIdcs)) = true;
end
```

- Matlab code of Algorithm 7.1:

```
function [baseLine, preBaseLine] = getMosaicBaseLine(obj)
    m = size(obj.MosaicBaseLine,1);
    baseLine = cell(m,1);
    sectionMoneyness = obj.OptionMarkets{Faction.Actual}.Skew.ForwardMoneyness;
    Idcs = MosaicDesginCPs.getArrayInCellFromArrayinCell('getNearestIndex', obj.
    OptionMarkets{Faction.Supposed}.Skew.ForwardMoneyness, sectionMoneyness);
    moneynessLower = cellfun(@(x, Ix) {x(Ix(MoneynessFrameWork.CallITM.Component))},
    sectionMoneyness, Idcs);
    moneynessUpper = cellfun(@(x, Ix) {x(Ix(MoneynessFrameWork.CallOTM.Component))},
    sectionMoneyness, Idcs);
    % BoneWing
```

```
    baseLine{MosaicFrameWork.BoneWing.mosaicIndex} = cellfun(@(x, y) {unique([realmin; x; y;
     realmax])}, moneynessLower, moneynessUpper);
    % BoneJointWing
    preBaseLine = obj.MosaicPreBaseLine;
    preBaseLine = cellfun(@(x, y) {preBaseLine(and(x < preBaseLine, preBaseLine < y))},
    moneynessLower, moneynessUpper);
    preBaseLine = cellfun(@(x, y, z) {preBJWbl(x,y,z)}, moneynessLower, moneynessUpper,
    preBaseLine);
    function prebl = preBJWbl(x,y,z)
        if isempty(z)
            prebl = unique([x; y]) ;
        else
            prebl = unique([x(x<z(1)); z; y(z(end)<y)]);
        end
    end
    baseLine{MosaicFrameWork.BoneJointWing.mosaicIndex} = cellfun(@(x, y) {
    MosaicDesginCPs.getBoneJointWingMapValue('setBaseLine',x,y)}, preBaseLine,
    sectionMoneyness);
    baseLine{MosaicFrameWork.BoneJointWing.mosaicIndex} = cellfun(@(x) {unique([realmin; x;
     realmax])}, baseLine{MosaicFrameWork.BoneJointWing.mosaicIndex});
    % JonitWing
    baseLine{MosaicFrameWork.JointWing.mosaicIndex} = cellfun(@(x) {unique([realmin; x;
    realmax])}, sectionMoneyness);
end
function components = getMosaicComponents(obj, mosaicIndex)
    baseLine = obj.MosaicBaseLine{mosaicIndex};
    sectionMoneyness = obj.OptionMarkets{Faction.Actual}.Skew.ForwardMoneyness;
    switch mosaicIndex
        case MosaicFrameWork.BoneWing.mosaicIndex
            % BoneWing
            components = cellfun(@(x) {[MosaicComponent.Wing; repmat(MosaicComponent.
            Bone, size(x,1)-3, 1); MosaicComponent.Wing]}, baseLine);
        case MosaicFrameWork.BoneJointWing.mosaicIndex
            % BoneJointWing
            preBaseLine = obj.MosaicPreBaseLine;
            numSectionMoneyness = cellfun(@(x, y) {MosaicDesginCPs.
            getBoneJointWingMapValue('betweenNumber',x,y)}, preBaseLine, sectionMoneyness);
            components = cellfun(@(x, y) {MosaicDesginCPs.getBoneJointWingMapValue('
            setComponent',x,y)}, preBaseLine, sectionMoneyness);
            components = cellfun(@(x) {[MosaicComponent.Wing; x; MosaicComponent.Wing]},
            components);
        case MosaicFrameWork.JointWing.mosaicIndex
            % JonitWing
            components = cellfun(@(x) {[MosaicComponent.Wing; repmat(MosaicComponent.
            Joint, size(x,1)-3, 1); MosaicComponent.Wing]}, baseLine);
    end
```

```
end
function map = getSkewSectionMap(obj, components, mosaicIndex)
    moneyness = obj.MosaicBaseLine{mosaicIndex};
    interval = cellfun(@(x, y, z) {MosaicDesginCPs.getBoneJointWingMapValue('setInterval',x
    ,[],y)}, moneyness, components);

    field1 = 'Lower';
    field2 = 'Upper';
    map.Bounds = struct(field1, {cellfun(@(x) {x(1:end-1)}, moneyness)}, field2, {cellfun(@(x) {
    x(2:end)}, moneyness)});
    map.EndPoints = struct(field1, {cellfun(@(x) {x(:,1)}, interval)}, field2 , {cellfun(@(x) {x
    (:,2)}, interval)});
end
function map = getTermSectionMap(obj)
    time = 0;
    time = [time; obj.OptionMarkets{Faction.Actual}.Term.TimetoMaturity];
    time = [time(1:end-1); realmax];
    n = size(time,1) - 1;
    interval = [Interval.Open Interval.Closed];
    interval = repmat(interval,n,1);

    field1 = 'Lower';
    field2 = 'Upper';
    map.Bounds = struct(field1, time(1:end-1), field2, time(2:end));
    map.EndPoints = struct(field1, interval (:,1) , field2 , interval (:,2) );
end
function targetPointer = getTargetPointer(obj, mosaicDesgin, mosaicName, factionIndex)
    sLB = obj.Map.Skew.Bounds.Lower;
    sUB = obj.Map.Skew.Bounds.Upper;
    targetMoneyness = mosaicDesgin.OptionMarkets{Faction.Actual}.Skew.ForwardMoneyness;
    if factionIndex == Faction.Supposed
        targetMoneyness = repmat({mosaicDesgin.OptionMarkets{factionIndex}.Skew.
        ForwardMoneyness}, size(targetMoneyness,1),1);
    end
    Idcs = cellfun(@(x,y,z) {CPSections.findModelIntervalIndices(x,y,z)}, targetMoneyness, sLB,
    sUB);

    tLB = obj.Map.Term.Bounds.Lower;
    tUB = obj.Map.Term.Bounds.Upper;
    tLE = obj.Map.Term.EndPoints.Lower;
    tUE = obj.Map.Term.EndPoints.Upper;
    targetTime = mosaicDesgin.OptionMarkets{factionIndex}.Term.TimetoMaturity;
    Jdcs = cell2mat(CPSections.findModelIntervalIndices(targetTime, tLB, tUB, tLE, tUE));

    components = obj.MosaicComponents;
    source = cellfun(@(x,y,z,w) {CPSections.getTargetSource(x, y,z,w)}, Idcs, repmat({
```

```
    mosaicName.mosaicIndex},size(Idcs)), components, repmat({factionIndex},size(Idcs)));

    field1  = 'SkewIndex';
    field2  = 'TermIndex';
    field3  = 'Source';
    targetPointer = struct( field1 , {Idcs}, field2 , {Jdcs}, field3 , {source});
end
function arrayInCell = getArrayInCellFromArrayinCell(funcName, varArr, varCell, varCell2)
    if nargin < 4
        varCell2 = repmat({[]},size(varCell));
    end
    arrayInCell = cellfun(@getArrayFromArray, varCell, varCell2);
    function array = getArrayFromArray(varArrInCell, varArrInCell2)
        switch funcName
            case 'getNearestIndex'
                array = {arrayfun(@getNearestIndex, varArr)};
            case 'getNearestValue'
                array = {arrayfun(@getNearestValue, varArr)};
            case 'getNearestValueDotImplMid'
                array = {arrayfun(@getNearestValueDotImplMid, varArr)};
        end
        function index = getNearestIndex(varScalarInArr)
            [~, index] = sort(abs(varScalarInArr-varArrInCell));
            index = index(1);
        end
        function value = getNearestValue(varScalarInArr)
            index = getNearestIndex(varScalarInArr);
            value = varArrInCell(index);
        end
        function value = getNearestValueDotImplMid(varScalarInArr)
            index = getNearestIndex(varScalarInArr);
            value = varArrInCell2(index).ImplVol.Mid;
        end
    end
end
function value = getBoneJointWingMapValue(funcName, baseLine, sectionMoneyness, component
)
    switch funcName
        case 'betweenNumber'
            if size(baseLine,1) == 1
                value = arrayfun(@betweenNumber, baseLine);
            else
                value = arrayfun(@betweenNumber, baseLine(1:end-1), baseLine(2:end));
            end
        case 'setBaseLine'
            if size(baseLine,1) == 1
```

```
            value = arrayfun(@(x) {setBaseLine(x)}, baseLine);
        else
            value = arrayfun(@(x, y) {setBaseLine(x,y)}, baseLine(1:end−1), baseLine(2:end)
            );
        end
        value = cell2mat(value(:));
    case 'setComponent'
        value = setComponent;
    case 'setInterval'
        value = setInterval;
end
function Indices = betweenIndices(blLower, blUpper)
    if nargin < 2
        Indices = find(blLower == sectionMoneyness);
        return;
    end
    if blLower < 1 && 1 <= blUpper
        Indices = find(and(blLower <= sectionMoneyness, sectionMoneyness <= blUpper));
    elseif 1 <= blLower
        if all(sectionMoneyness >= 1) && blLower == min(sectionMoneyness)
            Indices = find(and(blLower <= sectionMoneyness, sectionMoneyness <= blUpper
            ));
        else
            Indices = find(and(blLower < sectionMoneyness, sectionMoneyness <= blUpper))
            ;
        end
    elseif blUpper < 1
        if all(sectionMoneyness < 1) && blUpper == max(sectionMoneyness)
            Indices = find(and(blLower <= sectionMoneyness, sectionMoneyness <= blUpper
            ));
        else
            Indices = find(and(blLower <= sectionMoneyness, sectionMoneyness < blUpper))
            ;
        end
    end
end
function number = betweenNumber(blLower, blUpper)
    if nargin < 2
        Indices = betweenIndices(blLower);
    else
        Indices = betweenIndices(blLower, blUpper);
    end
    number = numel(Indices);
end
function baseElement = setBaseLine(blLower, blUpper)
    if nargin < 2
```

```
        Indices = betweenIndices(blLower);
    else
        Indices = betweenIndices(blLower, blUpper);
    end
    switch numel(Indices)
        case 0
            baseElement = [];
        case 1
            baseElement = sectionMoneyness(Indices);
        otherwise
            baseElement = [sectionMoneyness(Indices(1)); sectionMoneyness(Indices(end))];
    end
end
function component = setComponent
    if size(baseLine,1) == 1
        Indicescell = arrayfun(@(x) {betweenIndices(x)}, baseLine);
    else
        Indicescell = arrayfun(@(x, y) {betweenIndices(x,y)}, baseLine(1:end−1), baseLine
        (2:end));
    end
    n = size( Indicescell ,1) ;
    component = [];
    for idx = 1: n
        if idx == ceil((n+1)/2)
            switch numel(Indicescell{idx})
                case 0
                    if isscalar (component)
                        component = [];
                    else
                        component = component(1:end−1);
                    end
                case 1
                otherwise
                    component = [component; MosaicComponent.Bone];
            end
        else
            switch numel(Indicescell{idx})
                case 0
                case 1
                    component = [component; MosaicComponent.Joint];
                otherwise
                    if idx > ceil(n+1)/2
                        component = [component; MosaicComponent.Joint;
                        MosaicComponent.Bone];
                    elseif idx < ceil(n+1)/2
                        component = [component; MosaicComponent.Bone;
```

```
                        MosaicComponent.Joint];
                    end
                end
            end
        end
    end
function interval = setInterval
    n = size(component,1);
    interval = [];
    for idx = 1: n
        switch component(idx)
            case MosaicComponent.Bone
                interval = [interval; [Interval.Closed Interval.Closed]];
            case MosaicComponent.Joint
                switch component(idx-1)
                    case MosaicComponent.Bone
                        iLower = Interval.Open;
                    case MosaicComponent.Joint
                        if baseLine(idx) < 1
                            iLower = Interval.Closed;
                        elseif 1 <= baseLine(idx)
                            iLower = Interval.Open;
                        end
                    case MosaicComponent.Wing
                        iLower = Interval.Closed;
                end
                switch component(idx+1)
                    case MosaicComponent.Bone
                        iUpper = Interval.Open;
                    case MosaicComponent.Joint
                        if 1 <= baseLine(idx+1)
                            iUpper = Interval.Closed;
                        elseif baseLine(idx+1) < 1
                            iUpper = Interval.Open;
                        end
                    case MosaicComponent.Wing
                        iUpper = Interval.Closed;
                end
                interval = [interval; [iLower iUpper]];
            case MosaicComponent.Wing
                interval = [interval; [Interval.Open Interval.Open]];
                switch idx
                    case 1
                        if component(idx) == component(idx+1)
                            if 1 <= baseLine(idx+1)
                                interval(end,2) = Interval.Closed;
```

```
                    end
                end
            case n
                if component(idx) == component(idx-1);
                    if baseLine(idx) < 1
                        interval(end,1) = Interval.Closed;
                    end
                end
        end
    end
  end
 end
end
function Indices = findModelIntervalIndices(v, vLB, vUB, vLE, vUE)
    if nargin < 5
        vLE = repmat(Interval.Closed,size(vLB));
        vUE = repmat(Interval.Closed,size(vUB));
    end
    Indices = arrayfun(@(x,y,z,w) {findOptionInModelIndex(x,y,z,w)}, vLB, vUB, vLE, vUE);
    function Index = findOptionInModelIndex(vLB, vUB, vLE, vUE)
        Index = find(arrayfun(@logicalIntervalIndex, v));
        function lgclIndex = logicalIntervalIndex(v)
            lgclIndex = arrayfun(@logicalEndPointIndex, vLB, vUB, vLE, vUE);
            function lgclIndex = logicalEndPointIndex(vLB, vUB, vLE, vUE)
                switch vLE
                    case Interval.Open
                        switch vUE
                            case Interval.Open
                                lgclIndex = vLB < v && v < vUB;
                            case Interval.Closed
                                lgclIndex = vLB < v && v <= vUB;
                        end
                    case Interval.Closed
                        switch vUE
                            case Interval.Open
                                lgclIndex = vLB <= v && v < vUB;
                            case Interval.Closed
                                lgclIndex = vLB <= v && v <= vUB;
                        end
                end
            end
        end
    end
end
function source = getTargetSource(targetIndices, mosaicIndex, components, factionIndex)
    n = size(targetIndices,1);
```

```
    source = cell(n,1);
    for idx = 1: n
        m = size(targetIndices{idx},1);
        if factionIndex == Faction.Supposed
            source{idx} = repmat(PriceSource.Model,m,1);
            continue;
        end
        if n == 2
            source{idx} = repmat(PriceSource.Market,m,1);
            continue;
        end
        switch mosaicIndex
            case MosaicFrameWork.BoneWing.mosaicIndex
                switch components(idx)
                    case MosaicComponent.Bone
                        source{idx} = repmat(PriceSource.Market,m,1);
                    case MosaicComponent.Wing
                        source{idx} = repmat(PriceSource.Model,m,1);
                end
            case MosaicFrameWork.BoneJointWing.mosaicIndex
                switch components(idx)
                    case MosaicComponent.Bone
                        source{idx} = repmat(PriceSource.Market,m,1);
                    case MosaicComponent.Joint
                        switch components(idx-1)
                            case MosaicComponent.Bone
                                source{idx}(1,1) = PriceSource.Model;
                            otherwise
                                source{idx}(1,1) = PriceSource.Market;
                        end
                        switch components(idx+1)
                            case MosaicComponent.Bone
                                source{idx}(m,1) = PriceSource.Model;
                            otherwise
                                source{idx}(m,1) = PriceSource.Market;
                        end
                    case MosaicComponent.Wing
                        source{idx} = repmat(PriceSource.Model,m,1);
                end
            case MosaicFrameWork.JointWing.mosaicIndex
                source{idx} = repmat(PriceSource.Market,m,1);
        end
    end
end
```

- Matlab code of Algorithm 7.2:

```
function model = setMosaicModelParameters(obj, modelIndex, mosaicIndex)
    startSecond = tic;
    model = obj.Sections(mosaicIndex).Models(modelIndex);
    options = obj.Options;
    optionMarkets = obj.OptionMarkets;
    targetPointer = obj.Sections(mosaicIndex).TargetPointer;
    mosaicComponents = obj.Sections(mosaicIndex).MosaicComponents;

    [factionMembers, ~] = enumeration('Faction');
    n = size(factionMembers,1);
    model.Target = cell(n,1);
    model.Target{Faction.Actual} = model.setMosaicModelTarget(options{Faction.Actual},
    targetPointer{Faction.Actual}, PriceSource.Market);
    model.Target{Faction.Supposed} = model.setMosaicModelSupposedTarget(options{Faction.
    Supposed}, targetPointer{Faction.Supposed});
    model.Parameters = model.setCommonParameters;
    if model.FrameWork.Bone
        model.Parameters = model.getMosaicModelParameters(options, optionMarkets,
        targetPointer, mosaicComponents, MosaicComponent.Bone);
        obj.Sections(mosaicIndex).Models(modelIndex) = model;
        switch model.Name
            case ModelName.SABR
                model.ImplVol = obj.calcModelImplVol(modelIndex, mosaicIndex);
            case ModelName.Heston
                model.Price = obj.calcModelPrice(modelIndex, mosaicIndex);
                obj.Sections(mosaicIndex).Models(modelIndex) = model;
                model.ImplVol = obj.calcModelImplVol(modelIndex, mosaicIndex);
                obj.Sections(mosaicIndex).Models(modelIndex) = model;
                model.Vega = obj.calcModelVega(modelIndex, mosaicIndex);
        end
        model.Target{Faction.Actual} = model.setMosaicModelTarget(options{Faction.Actual},
        targetPointer{Faction.Actual}, PriceSource.Model);
    end
    if model.FrameWork.Joint
        model.Parameters = model.getMosaicModelParameters(options, optionMarkets,
        targetPointer, mosaicComponents, MosaicComponent.Joint);
    end
    if model.FrameWork.Wing
        model.Parameters = model.getMosaicModelParameters(options, optionMarkets,
        targetPointer, mosaicComponents, MosaicComponent.Wing);
    end
    model.ImplVol = [];
```

```
        model.Price = [];
        model.Vega = [];
        elapsedSecond = toc(startSecond);
        toc(startSecond)
        model.ElapsedTime = struct('Second', elapsedSecond);
    end
    function target = setMosaicModelTarget(obj, options, targetPointer, priceSource)
        switch obj.Name
            case ModelName.SABR
                field1 = 'ImplVol';
                if isempty(obj.Target{Faction.Actual}) && ~isfield(obj.Target{Faction.Actual},
                field1)
                    target = {{cellfun(@(x, y, z) {VolatilityModel.takeInMosaicModelOption(field1,
                    x, y, z, priceSource)}, options, targetPointer.SkewIndex, targetPointer.Source)
                    }};
                else
                    target = obj.Target{Faction.Actual}.(field1);
                    modelValue = obj.(field1);
                    target = {{cellfun(@(x, y, z, w, v) {VolatilityModel.takeInMosaicModelOption(
                    field1, x, y, z, priceSource, w, v)}, options, targetPointer.SkewIndex,
                    targetPointer.Source, target, modelValue)}};
                end
            case ModelName.Heston
                n = 2;
                target = cell(n,1);
                field1 = 'Price';
                if isempty(obj.Target{Faction.Actual}) && ~isfield(obj.Target{Faction.Actual},
                field1)
                    target{1} = {cellfun(@(x, y, z) {VolatilityModel.takeInMosaicModelOption(
                    field1, x, y, z, priceSource)}, options, targetPointer.SkewIndex, targetPointer.
                    Source)};
                else
                    target{1} = obj.Target{Faction.Actual}.(field1);
                    modelValue = obj.(field1);
                    target{1} = {cellfun(@(x, y, z, w, v) {VolatilityModel.takeInMosaicModelOption
                    (field1, x, y, z, priceSource, w, v)}, options, targetPointer.SkewIndex,
                    targetPointer.Source, target{1}, modelValue)};
                end
                field1 = 'Vega';
                if isempty(obj.Target{Faction.Actual}) && ~isfield(obj.Target{Faction.Actual},
                field1)
                    target{2} = {cellfun(@(x, y, z) {VolatilityModel.takeInMosaicModelOption(
                    field1, x, y, z, priceSource)}, options, targetPointer.SkewIndex, targetPointer.
                    Source)};
                else
                    target{2} = obj.Target{Faction.Actual}.(field1);
```

```
                modelValue = obj.(field1);
                target{2} = {cellfun(@(x, y, z, w, v) {VolatilityModel.takeInMosaicModelOption
                (field1, x, y, z, priceSource, w, v)}, options, targetPointer.SkewIndex,
                targetPointer.Source, target{2}, modelValue)};
            end
    end
    target = obj.setTarget(target);
end
function parameters = getMosaicModelParameters(obj, options, markets, targetPointer,
mosaicComponents, componentIndex)
    rights = cellfun(@(x, y, z) {VolatilityModel.takeInMosaicModelOption('Rights', x, y, z,
    PriceSource.Market)}, options{Faction.Actual}, targetPointer{Faction.Actual}.SkewIndex,
    targetPointer{Faction.Actual}.Source);
    rights = cellfun(@(x, y, z, w) {VolatilityModel.takeInMosaicModelOption('Rights', x, y, z,
    PriceSource.Model, w)}, options{Faction.Actual}, targetPointer{Faction.Actual}.SkewIndex,
    targetPointer{Faction.Actual}.Source, rights);
    underlying = VolatilityModel.takeInMosaicModelMarket('Underlying', markets{Faction.Actual
    }, targetPointer{Faction.Actual}.SkewIndex, mosaicComponents, componentIndex);
    strike = VolatilityModel.takeInMosaicModelMarket('Strike', markets{Faction.Actual},
    targetPointer{Faction.Actual}.SkewIndex, mosaicComponents, componentIndex);
    rate = VolatilityModel.takeInMosaicModelMarket('Rate', markets{Faction.Actual},
    targetPointer{Faction.Actual}.SkewIndex, mosaicComponents, componentIndex);
    time = VolatilityModel.takeInMosaicModelMarket('Time', markets{Faction.Actual},
    targetPointer{Faction.Actual}.SkewIndex, mosaicComponents, componentIndex);
    yield = VolatilityModel.takeInMosaicModelMarket('Yield', markets{Faction.Actual},
    targetPointer{Faction.Actual}.SkewIndex, mosaicComponents, componentIndex);

    supprights = VolatilityModel.takeInMosaicModelSupposedOption('Rights', options{Faction.
    Supposed}, targetPointer{Faction.Supposed}.SkewIndex, targetPointer{Faction.Supposed}.
    Source);
    suppunderlying = VolatilityModel.takeInMosaicModelMarket('Underlying', markets{Faction.
    Supposed}, targetPointer{Faction.Supposed}.SkewIndex, mosaicComponents, componentIndex
    );
    suppstrike = VolatilityModel.takeInMosaicModelMarket('Strike', markets{Faction.Supposed},
    targetPointer{Faction.Supposed}.SkewIndex, mosaicComponents, componentIndex);
    supprate = VolatilityModel.takeInMosaicModelMarket('Rate', markets{Faction.Supposed},
    targetPointer{Faction.Supposed}.SkewIndex, mosaicComponents, componentIndex);
    supptime = VolatilityModel.takeInMosaicModelMarket('Time', markets{Faction.Supposed},
    targetPointer{Faction.Supposed}.SkewIndex, mosaicComponents, componentIndex);
    suppyield = VolatilityModel.takeInMosaicModelMarket('Yield', markets{Faction.Supposed},
    targetPointer{Faction.Supposed}.SkewIndex, mosaicComponents, componentIndex);

    switch obj.Name
        case ModelName.SABR
            implvol = obj.Target{Faction.Actual}.ImplVol;
            suppimplvol = obj.Target{Faction.Supposed}.ImplVol;
```

```
            unitparameters = obj.getMosaicSABRParameters(underlying, strike, time, implvol, ...
                                                        suppunderlying, suppstrike, supptime,
                                                        suppimplvol, mosaicComponents,
                                                        componentIndex);
        case ModelName.Heston
            price = obj.Target{Faction.Actual}.Price;
            vega = obj.Target{Faction.Actual}.Vega;
            suppprice = obj.Target{Faction.Supposed}.Price;
            suppvega = obj.Target{Faction.Supposed}.Vega;
            unitparameters = obj.getMosaicHestonParameters(rights, underlying, strike, rate,
            time, yield, price, vega, ...
                                                        supprights, suppunderlying,
                                                        suppstrike, supprate, supptime,
                                                        suppyield, suppprice, suppvega,
                                                        mosaicComponents, componentIndex);
    end
    field1 = 'Common';
    field2 = 'Unit';
    value1 = obj.Parameters.(field1);
    if isfield (obj.Parameters, field2)
        value2 = cellfun(@(x,y) {cellfun(@(z,w) {merge(z,w)},x,y)}, obj.Parameters.(field2),
        unitparameters);
    else
        value2 = unitparameters;
    end
    function oldnew = merge(old, new)
        oldnew = old;
        if isempty(oldnew)
            oldnew = new;
        end
    end
    parameters = struct(field1, value1, field2, {value2});
end
function parameters = getMosaicSABRParameters(obj, underlying, strike, time, implvol, ...
                                                suppunderlying, suppstrike, supptime,
                                                suppimplvol, mosaicComponents,
                                                componentIndex)
      parameters = cellfun(@(a,b,c,d,e,f,g,h,i) {getBandSABRParameters(a,b,c,d,e,f,g,h,i)}, ...
                            underlying, strike, time, implvol, ...
                            suppunderlying, suppstrike, supptime, suppimplvol,
                            mosaicComponents);
      function parameters = getBandSABRParameters(underlying, strike, time, implvol, ...
                                                suppunderlying, suppstrike, supptime,
                                                suppimplvol, mosaicComponents)
          parameters = cellfun(@(a,b,c,d,e,f,g,h,i) {getUnitSABRParameters(a,b,c,d,e,f,g,h,i)},
          ...
```

```
                        underlying,  strike ,  time,  implvol,  ...
                        suppunderlying, suppstrike,  supptime, suppimplvol, num2cell(int8(
                        mosaicComponents)));
        function  parameters = getUnitSABRParameters(underlying, strike, time, implvol, ...
                                                     suppunderlying, suppstrike,  supptime,
                                                     suppimplvol, mosaicComponents)
            if  mosaicComponents ~= componentIndex
                parameters = [];
                return;
            end
            Idcs  = cell2mat(arrayfun(@(x) {find(x == suppstrike./suppunderlying)}, strike./
            underlying));
            if  ~isempty(Idcs)
                suppunderlying(Idcs) = [];
                suppstrike(Idcs)  =  [];
                supptime(Idcs) = [];
                suppimplvol(Idcs) =  [];
            end
            parameters = obj.calibUnitSABRParameters([underlying; suppunderlying], [strike;
             suppstrike], [time; supptime], [implvol; suppimplvol]);
        end
    end
end
function  parameters = calibUnitSABRParameters(obj, underlying, strike, time, implvol)
    alpha = obj. ParasInitial . Alpha;
    if  isempty(obj.Parameters)
        beta = obj. ParasInitial . Beta;
    else
        beta = obj.Parameters.Common.Beta;
    end
    rho = obj. ParasInitial . Rho;
    nu = obj. ParasInitial . Nu;
    alphaLB = obj.ParasLBound.Alpha;
    alphaUB = obj.ParasUBound.Alpha;
    rhoLB = obj.ParasLBound.Rho;
    nuLB = obj.ParasLBound.Nu;
    rhoUB = obj.ParasUBound.Rho;
    nuUB = obj.ParasUBound.Nu;
    tolFun = obj.IVTol;
    maxFunEvals = obj.MaxFunEvals;

    monneyness = strike./underlying;
    if  any(monneyness == 1)
        atmimplvol = unique(implvol(monneyness == 1));
        [alpha,  rho,  nu] = VolatilityModel.calibSABRImplAlphaRhoNu(underlying, strike, time,
        implvol, ...
```

```
                                                                atmimplvol, beta, rho, nu,
                                                                 ...
                                                                rhoLB, nuLB, rhoUB, nuUB,
                                                                tolFun, maxFunEvals);
    else
        [alpha, rho, nu] = VolatilityModel.calibSABRAlphaRhoNu(underlying, strike, time,
        implvol, ...
                                                             [], [], [], [], [], ...
                                                             alpha, beta, rho, nu, ...
                                                             alphaLB, rhoLB, nuLB, alphaUB,
                                                              rhoUB, nuUB, tolFun,
                                                             maxFunEvals);
    end

    field1 = 'Alpha';
    field2 = 'Rho';
    field3 = 'Nu';
    parameters = struct(field1, alpha, field2, rho, field3, nu);
end

function target = takeInMosaicModelOption(kind, options, targetIndices, targetSource,
priceSource, target, model)
    if nargin < 6
        target = cellfun(@(x,y) {getUnitModelOption(x,y)}, targetIndices, targetSource);
    else
        target = cellfun(@(x,y,z) {getUnitModelOption(x,y,z)}, targetIndices, targetSource,
        target);
    end
    function target = getUnitModelOption(targetIndices, targetSource, target)
        if nargin <3
            target = nan(size(targetSource));
        end
        lgclIdx = targetSource == priceSource;
        if any(lgclIdx)
            switch kind
                case 'Rights'
                    targetCandidate = arrayfun(@(v) int8(v.Rights), options);
                otherwise
                    switch priceSource
                        case PriceSource.Model
                            targetCandidate = model;
                        case PriceSource.Market
                            switch kind
                                case 'ImplVol'
                                    targetCandidate = arrayfun(@(v) v.ImplVol.Mid, options);
                                case 'Price'
```

```
                    targetCandidate = arrayfun(@(v) v.Price.Mid, options);
                case 'Vega'
                    targetCandidate = arrayfun(@(v) v.Vega, options);
            end
        end
    end
    target(lgclIdx) = targetCandidate(targetIndices(lgclIdx));
  end
 end
end
function [alpha, rho, nu] = calibSABRAlphaRhoNu(underlying, strike, time, implvol, ...
                                    weight, subunderlying, substrike, subtime,
                                    synimplvol, ...
                                    alphaInit, beta, rhoInit, nuInit, ...
                                    alphaLB, rhoLB, nuLB, alphaUB, rhoUB, nuUB,
                                    tolFun, maxFunEvals)
 function fval = sabrErrorOnAlphabyRhobyNu(x)
  m1 = size(underlying, 1);
  [m2, n2] = size(subunderlying);
  residual = zeros(m1+m2, 1);
  residual(1:m1) = implvol - VolatilityModel.blackvolbysabr(x(1), beta, x(2), x(3), time,
  underlying, strike);
  if logical(m2)
    for jdx = 1: n2
      synIndex = find(~isnan(weight(:,jdx)));
      if ~isempty(synIndex)
        residual(m1+synIndex) = residual(m1+synIndex) + (synimplvol(synIndex) -
         VolatilityModel.blackvolbysabr(x(1), beta, x(2), x(3), subtime(synIndex, jdx),
         subunderlying(synIndex, jdx), substrike(synIndex, jdx)));
      end
    end
  end
  fval = max(abs(residual));
  if ~any(isnan(residual))
    xback = x;
    fvalback = fval;
  end
 end
 options = optimoptions('fmincon','Algorithm','interior-point', 'TolFun', tolFun, '
 MaxFunEvals', maxFunEvals, 'Display', 'off');
 alpharhonuInit = [alphaInit; rhoInit; nuInit];
 alpharhonuLB = [alphaLB; rhoLB; nuLB];
 alpharhonuUB = [alphaUB; rhoUB; nuUB];
 try
  xback = alpharhonuInit;
  fvalback = 0;
```

```
        [alpharhonu, fval] = fmincon(@sabrErrorOnAlphabyRhobyNu, alpharhonuInit, [], [], [], [],
         alpharhonuLB, alpharhonuUB, [], options);
    catch
        alpharhonu = xback;
        fval = fvalback;
    end
    alpha = alpharhonu(1);
    rho = alpharhonu(2);
    nu = alpharhonu(3);
end
function [alpha, rho, nu] = calibSABRImplAlphaRhoNu(underlying, strike, time, implvol, ...
                                                    atmimplvol, beta, rhoInit, nuInit, ...
                                                    rhoLB, nuLB, rhoUB, nuUB, tolFun,
                                                    maxFunEvals)
    function fval = sabrErrorOnImplAlphabyRhobyNu(x)
        residual = implvol - VolatilityModel.blackvolbysabr(atmVol2SabrAlpha(x(1), x(2)), beta,
         x(1), x(2), time, underlying, strike);
        fval = max(abs(residual));
        if ~any(isnan(residual))
            xback = x;
            fvalback = fval;
        end
    end
    options = optimoptions('fmincon','Algorithm','interior-point', 'TolFun', tolFun, '
    MaxFunEvals', maxFunEvals, 'Display', 'off');
    rhonuInit = [rhoInit; nuInit];
    rhonuLB = [rhoLB; nuLB];
    rhonuUB = [rhoUB; nuUB];
    alpharoots = @(Rho,Nu) roots([...
        (1 - beta)^2*unique(time)/24/unique(underlying)^(2 - 2*beta) ...
        Rho*beta*Nu*unique(time)/4/unique(underlying)^(1 - beta) ...
        (1 + (2 - 3*Rho^2)*Nu^2*unique(time)/24) ...
        -atmimplvol*unique(underlying)^(1 - beta)]);
    atmVol2SabrAlpha = @(Rho,Nu) min(real(arrayfun(@(x) ...
        x*(x>0) + realmax*(x<0 || abs(imag(x))>1e-6), alpharoots(Rho,Nu))));

    try
        xback = rhonuInit;
        fvalback = 0;
        [rhonu, fval] = fmincon(@sabrErrorOnImplAlphabyRhobyNu, rhonuInit, [], [], [], [],
        rhonuLB, rhonuUB, [], options);
    catch
        rhonu = xback;
        fval = fvalback;
    end
    rho = rhonu(1);
```

```
    nu = rhonu(2);
    alpha = atmVol2SabrAlpha(rho, nu);
end
```

- Matlab code of Algorithm 9.1:

```
function lclvar = getModelLocalVar(underlying, strike, rate, time, yield, implvol)
    dimplvoldTime = (implvol(:,[2:end end])−implvol)./(time(:,[2:end end])−time(:,[1:end−1 end
    −1]));
    dimplvoldStrike = (implvol([2:end end],:) −implvol([1 1:end−1],:)) ./( strike ([2: end end],:) −
    strike([1 1:end−1],:));
    d2implvoldStrike2 = (implvol([2:end end],:) +implvol([1 1:end−1],:)−2∗implvol)./(strike ([2:
    end end],:) −strike([1 1:end−1],:)) .ˆ2;

    y = log(strike ./underlying)−(rate−yield).∗time;
    timeFwd = implvol.∗(implvol+2.∗time.∗dimplvoldTime);
    skew1 = 2∗(rate−yield).∗strike.∗time.∗implvol.∗dimplvoldStrike;
    skew2 = (1−strike.∗y./implvol.∗dimplvoldStrike).ˆ2;
    skew3 = strike.∗time.∗implvol.∗(dimplvoldStrike−0.25∗strike.∗time.∗implvol.∗dimplvoldStrike
    .ˆ2+strike.∗d2implvoldStrike2);
    lclvar = (timeFwd+skew1)./(skew2+skew3);
end
function obj = setConstraintModelBased(obj, marketGrid)
    time = repmat(marketGrid.TimetoMaturity',size(marketGrid.Strike,1),1);
    lclvar = obj.ModelBased.Variance.Local;

    [ lclvolconst , lclvarconst ] = MosaicModelSurface.getConstraintModelLocalVol(time, lclvar);
    field1 = 'Local';
    volatility = struct( field1 , { lclvolconst });
    field1 = 'Local';
    variance = struct( field1 , { lclvarconst });

    field1 = ' Volatility ';
    field2 = 'Variance';
    obj.ConstraintModelBased = struct(field1, volatility , field2 , variance);
end
function [ lclvolconst , lclvarconst ] = getConstraintModelLocalVol(time, lclvar, cushioncell,
lclvarBoundcell)
    if nargin < 3
        cushionL = exp(−20);
        cushionU = exp(−20);
```

```
        cushioncell = {cushionL; cushionU};
    else
        cushionL = cushioncell{1};
        cushionU = cushioncell{2};
    end
    if nargin < 4
        lclvarLB = realmin;
        [lclvarUB, cushionU] = MosaicModelSurface.getConstraintlclvarUB(lclvarLB, cushionL,
        time, lclvar, cushionU);
        cushioncell = arrayfun(@(x) {{cushionL; x}}, cushionU);
        lclvarBoundcell = arrayfun(@(x) {{lclvarLB; x}}, lclvarUB);
        lclvarcell = mat2cell(lclvar, ones(1,size(lclvar,1)),size(lclvar,2));
        lclvarconst = cell2mat(cellfun(@(x,y,z) {FinancialMarket.getBoundCushionedValues(x,y,
        z)}, lclvarcell, cushioncell, lclvarBoundcell));
    else
        lclvarconst = FinancialMarket.getBoundCushionedValues(lclvar, cushioncell,
        lclvarBoundcell);
    end
    lclvolconst = sqrt(lclvarconst);
end
function [lclvarUB, cushionU] = getConstraintlclvarUB(lclvarLB, cushionL, time, lclvar,
cushionU)
    lcltotvarGapTimeAvgUnderLB = mean(min(0,lclvar.*time-(lclvarLB+cushionL)),2);
    lcltotvarGapTimeAvgOverUB = @(x, Idx) mean(max(0,lclvar(Idx,:).*time(Idx,:)-x*(1-
    cushionU)));
    lcltotvarGapTimeAvg = @(x, Idx) lcltotvarGapTimeAvgUnderLB(Idx) +
    lcltotvarGapTimeAvgOverUB(x, Idx);
    tol = 1e-12;
    options = optimset('fminbnd');
    options = optimset(options, 'TolX', tol, 'Display', 'off');
    lclvarUBLB = (lclvarLB+cushionL)/(1-cushionU);
    lclvarUBUB = max(lclvarUBLB, max((lclvar.*time)/(1-cushionU),[],2));
    n = size(lclvarUBUB,1);
    lclvarUB = arrayfun(@(x,Idx) fminbnd(lcltotvarGapTimeAvg, lclvarUBLB, x, options, Idx),
    lclvarUBUB, (1:n)');
    cushionU = cushionU*lclvarUB;
end
```

Appendix B

Figures and Tables

Table B.1: HSCEI Stock index futures, Dividend yield, Riskless rate (16-Jan-14)

Time-to-maturity (Days)	Stock index futures (spot: 10241.52)	Time-to-maturity (A/365F)	Dividend yield (%)	Riskless rate (%)
13	10160.37	0.04	22.71	0.38
42	10165.13	0.12	6.88	0.38
71	10173.33	0.19	3.81	0.38
162	9861.22	0.44	9.04	0.51
256	9825.79	0.70	6.42	0.51
348	9846.05	0.95	4.60	0.47
529	9568.42	1.45	5.25	0.56
713	9573.09	1.95	4.13	0.67
895	9650.61	2.45	3.29	0.86
1078	9393.30	2.95	3.98	1.06

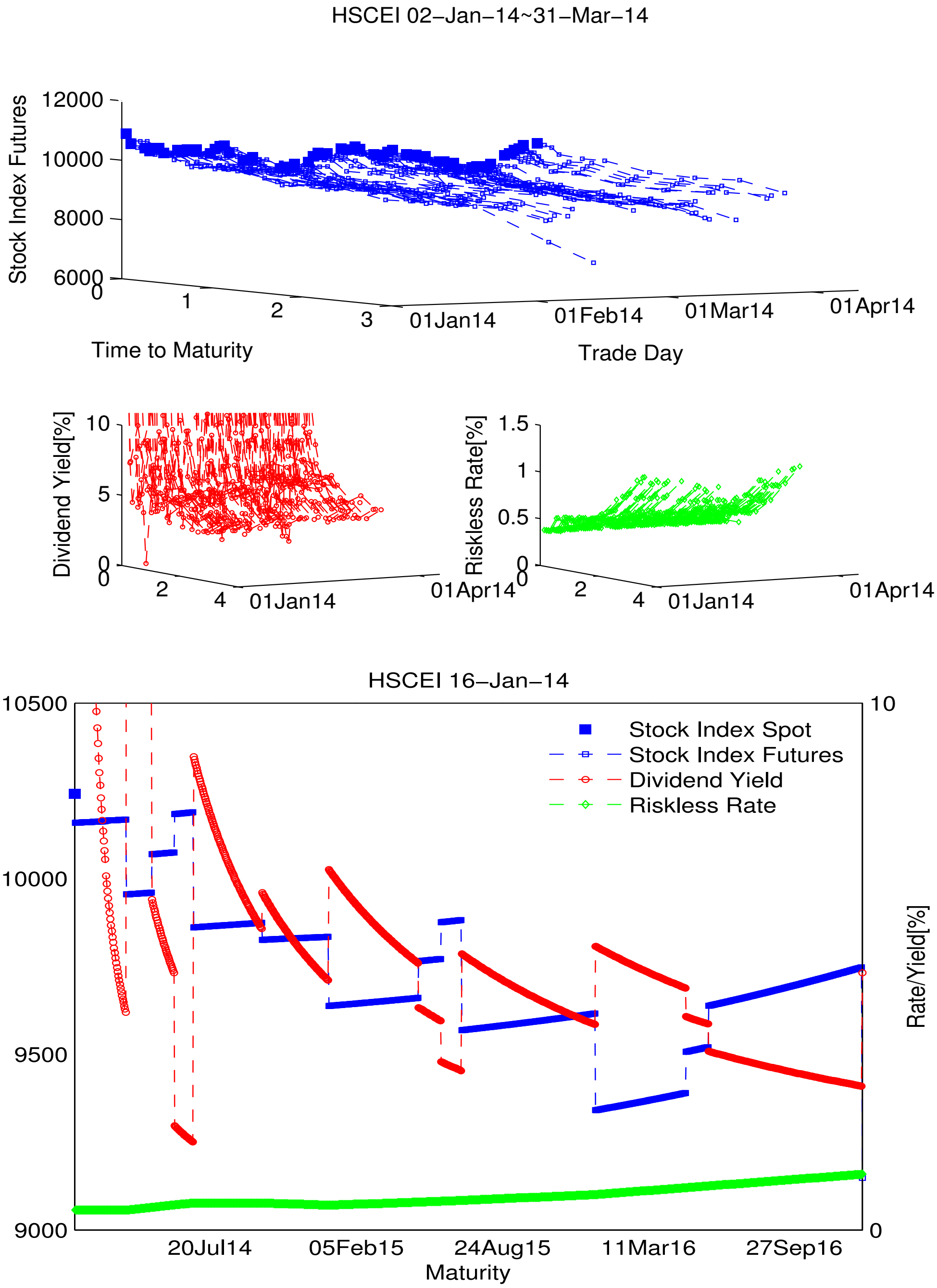


Figure B.1: HSCEI Stock index futures, Dividend yield, Riskless rate (03-Jan-14∼31-Mar-14), (16-Jan-14)

Table B.2: SnP500 Stock index futures, Dividend yield, Riskless rate (16-Jan-14)

Time-to-maturity (Days)	Stock index futures (spot: 1850.84)	Time-to-maturity (A/365F)	Dividend yield (%)	Riskless rate (%)
2	1847.15	0.01	36.68	0.24
37	1843.60	0.10	4.10	0.24
65	1841.66	0.18	3.03	0.24
93	1840.21	0.25	2.50	0.24
156	1834.88	0.43	2.33	0.31
247	1828.98	0.68	2.08	0.32
338	1822.48	0.93	1.97	0.30
366	1821.26	1.00	1.90	0.30
520	1811.31	1.42	1.90	0.38
702	1803.18	1.92	1.85	0.49
1066	1799.43	2.92	1.82	0.86

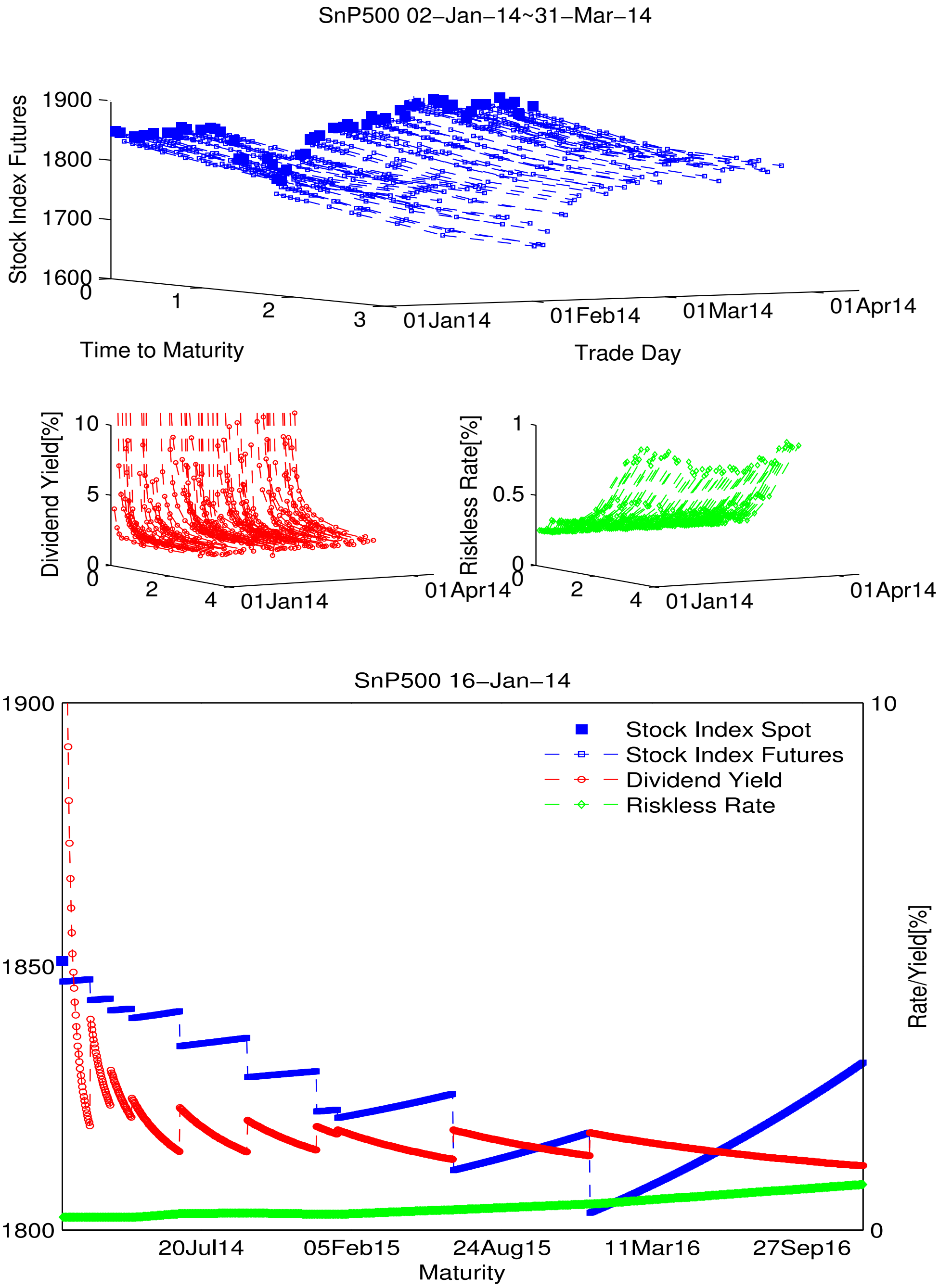


Figure B.2: SnP500 Stock index futures, Dividend yield, Riskless rate (03-Jan-14~31-Mar-14), (16-Jan-14)

Table B.3: Nikkei225 Stock index futures, Dividend yield, Riskless rate (16-Jan-14)

Time-to-maturity (Days)	Stock index futures (spot: 15941.08)	Time-to-maturity (A/365F)	Dividend yield (%)	Riskless rate (%)
29	15723.45	0.08	17.45	0.15
57	15723.60	0.16	8.94	0.15
85	15650.32	0.23	8.05	0.15
113	15652.93	0.31	6.06	0.16
148	15659.92	0.41	4.57	0.19
239	15650.06	0.65	3.02	0.21
330	15588.32	0.90	2.68	0.21
512	15499.99	1.40	2.21	0.21
694	15430.98	1.90	1.92	0.21
1058	15255.82	2.90	1.75	0.23
1422	15089.82	3.90	1.69	0.28

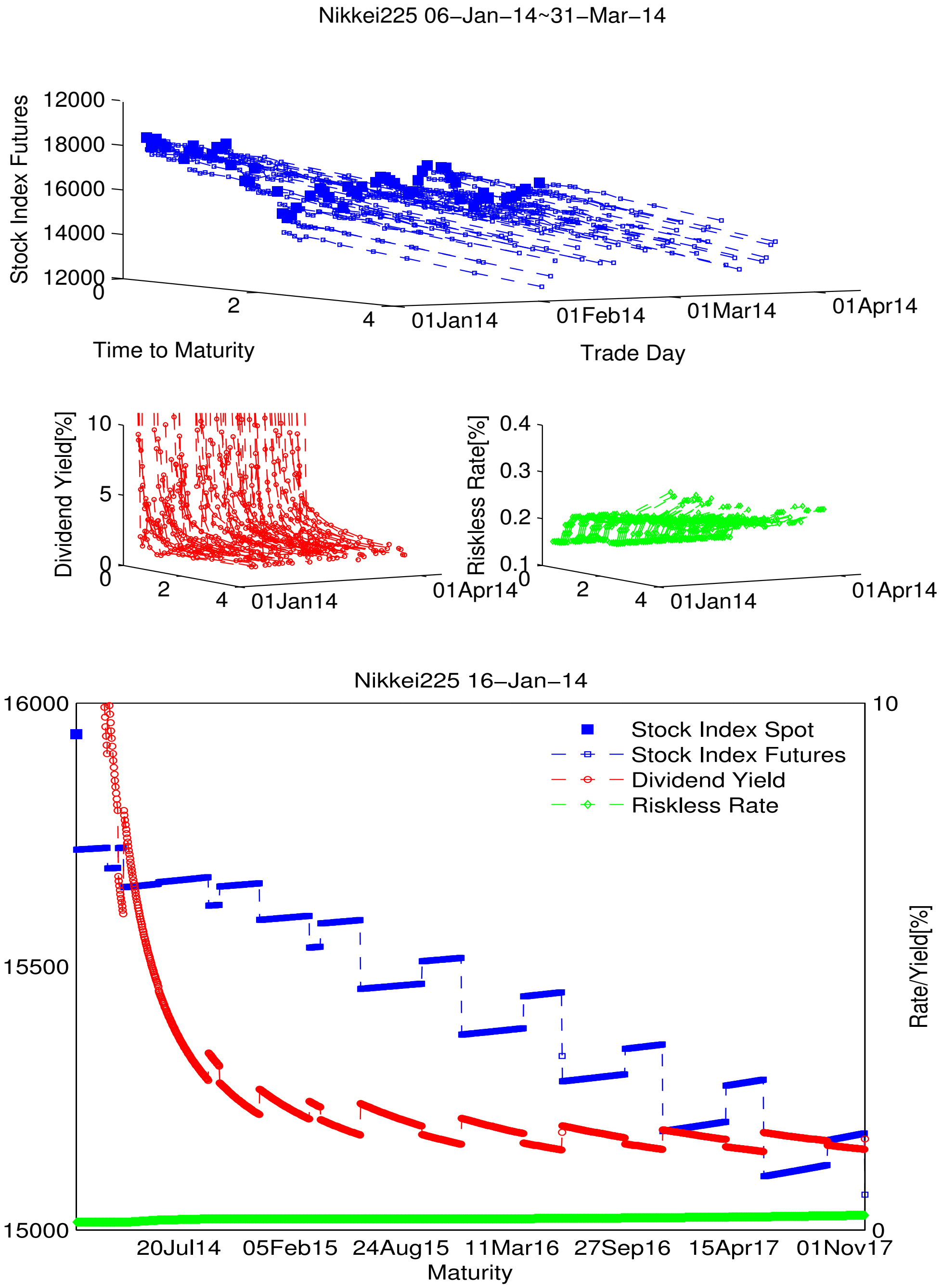


Figure B.3: Nikkei225 Stock index futures, Dividend yield, Riskless rate (03-Jan-14∼31-Mar-14), (16-Jan-14)

Table B.4: EuroStoxx50 Stock index futures, Dividend yield, Riskless rate (16-Jan-14)

Time-to-maturity (Days)	Stock index futures (spot: 3168.76)	Time-to-maturity (A/365F)	Dividend yield (%)	Riskless rate (%)
2	2983.06	0.01	1102.28	0.27
37	2980.15	0.10	60.80	0.27
65	2980.55	0.18	34.65	0.27
156	2923.44	0.43	19.19	0.33
247	2920.66	0.68	12.40	0.35
338	2910.97	0.93	9.49	0.33
520	2855.99	1.42	7.65	0.36
702	2850.52	1.92	5.90	0.40
1066	2813.03	2.92	4.65	0.57

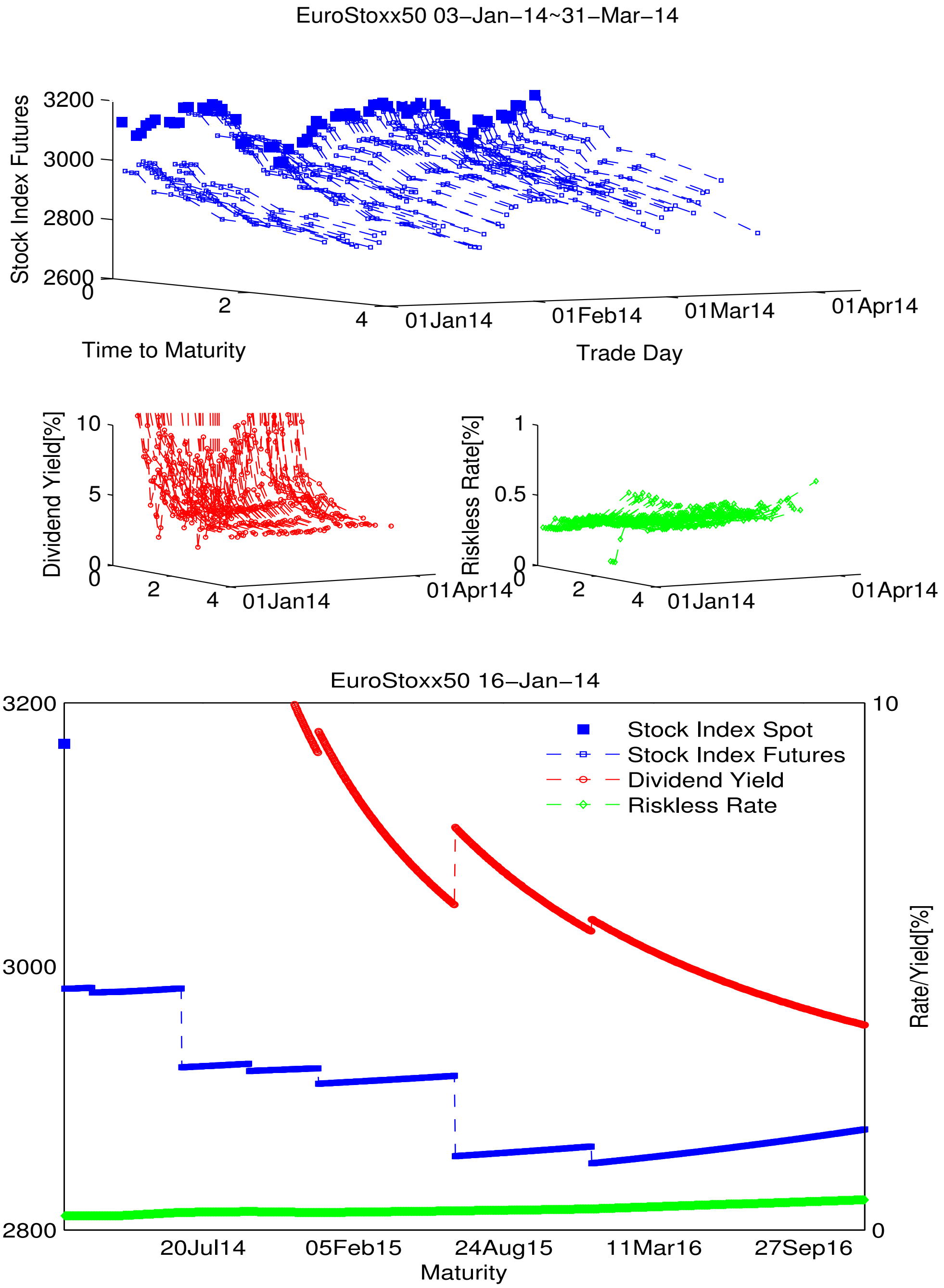


Figure B.4: EuroStoxx50 Stock index futures, Dividend yield, Riskless rate (03-Jan-14∼31-Mar-14), (16-Jan-14)

Table B.5: HSCEI OverTheCounter Synthetic Options (16-Jan-14)

Strategy	weight × rights	
Price	Bootstrapped price	
Implied volatility (%)	Forward moneyness Time-to-maturity	
Bootstrapped Type	Bootstrapped implied volatility (%)	
CallRatio	+1.0 Call	-2.0 Call
6.12E-66	6.12E-66	3.70E+01
22.85	1.00 0.04	1.04 0.04
Spot	0.13	22.62
CallRatio	+1.0 Call	-2.0 Call
0.00E+00	0.00E+00	0.00E+00
22.75	1.02 0.12	1.08 0.12
Flat	22.75	22.75
CallRatio	+1.0 Call	-2.0 Call
0.00E+00	0.00E+00	0.00E+00
23.13	1.08 0.95	1.22 0.95
Flat	23.13	23.13
CallRatio	+1.0 Call	-2.0 Call
0.00E+00	0.00E+00	0.00E+00
27.01	1.12 0.95	1.34 0.95
Flat	27.01	27.01

CallRatio	+1.0 Call	-1.5 Call	
0.00E+00	0.00E+00	0.00E+00	
26.38	1.16	1.28	
	0.95	0.95	
Flat	26.38	26.38	
CallSpread	+1.0 Call	-1.0 Call	
0.00E+00	0.00E+00	0.00E+00	
198.59	1.00	1.04	
	0.12	0.12	
Flat	198.59	198.59	
CallSpread	+1.0 Call	-1.0 Call	
0.00E+00	0.00E+00	0.00E+00	
20.99	1.04	1.10	
	0.12	0.12	
Flat	20.99	20.99	
CallButterFly	+1.0 Call	-2.0 Call	+1.0 Call
6.47E-83	6.47E-83	0.00E+00	0.00E+00
18.10	1.00	1.04	1.08
	0.12	0.12	0.12
Flat	18.10	18.10	18.10
PutCalendarSpread	-1.0 Put	+1.0 Put	
9.76E+02	4.34E+02	9.76E+02	
22.80	0.97	0.98	
	0.44	0.95	
Spot	21.35	28.88	
PutCalendarSpread	-1.0 Put	+1.0 Put	

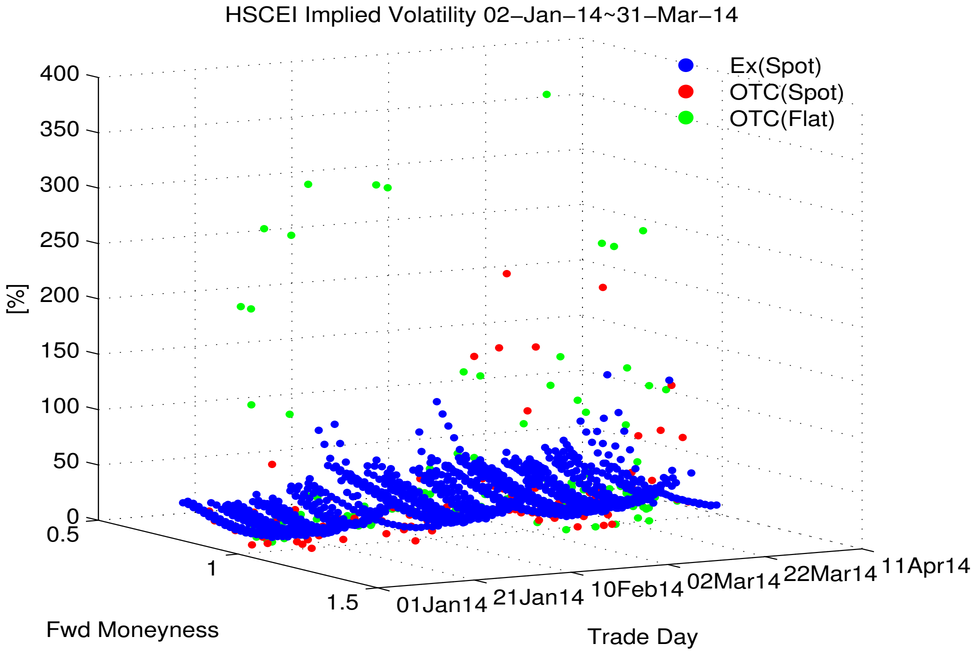
1.75E+03	2.24E+03	3.98E+03
23.61	1.02	1.05
	0.95	1.95
Spot	56.53	72.65
PutCalendarSpread	-2.0 Put	+1.0 Put
2.14E+03	2.32E+02	2.60E+03
41.81	0.79	0.84
	0.95	2.45
Spot	26.64	63.44
PutCalendarSpread	-1.0 Put	+1.0 Put
2.37E+03	2.32E+02	2.60E+03
22.42	0.79	0.84
	0.95	2.45
Spot	26.64	63.44
PutCalendarSpread	-1.0 Put	+1.0 Put
8.21E+02	2.40E+03	3.22E+03
20.71	0.88	0.91
	1.95	2.45
Spot	61.12	68.06
PutCalendarSpread	-1.0 Put	+1.0 Put
1.18E+03	2.40E+03	3.58E+03
22.67	0.88	0.90
	1.95	2.95
Spot	61.12	72.80
PutRatio	+1.0 Put	-2.0 Put
7.76E+02	7.76E+02	1.15E+02
28.76	1.02	0.92

	0.19	0.19
Spot	36.23	22.22
PutRatio	+1.0 Put	-1.5 Put
1.43E+03	1.43E+03	1.85E+02
34.72	1.06	0.89
	0.44	0.44
Spot	42.39	22.66
PutRatio	+1.0 Put	-2.0 Put
-8.16E+02	3.98E+03	2.40E+03
24.68	1.05	0.88
	1.95	1.95
Spot	72.65	61.12
PutSpread	+1.0 Put	-1.0 Put
2.33E-32	2.33E-32	0.00E+00
18.31	0.96	0.91
	0.12	0.12
Flat	18.31	18.31
PutSpread	+1.0 Put	-1.0 Put
6.73E+02	6.73E+02	4.22E-26
17.75	0.99	0.89
	0.44	0.44
Flat	17.75	17.75
PutSpread	+1.0 Put	-1.0 Put
1.58E+03	3.98E+03	2.40E+03
17.64	1.05	0.88
	1.95	1.95
Spot	72.65	61.12

PutButterFly	+1.0 Put	-2.0 Put	+1.0 Put
1.43E+02	1.43E+02	3.91E-290	0.00E+00
28.26	1.00	0.96	0.93
	0.04	0.04	0.04
Flat	28.26	28.26	28.26
PutButterFly	+1.0 Put	-2.0 Put	+1.0 Put
5.80E+02	7.85E+02	2.52E+01	8.25E+01
20.46	1.06	0.98	0.90
	0.19	0.19	0.19
Spot	21.28	4.74	22.90
RiskReversal	+1.0 Put	-1.0 Call	
1.07E+01	1.07E+01	0.00E+00	
47.22	0.98	1.02	
	0.04	0.04	
Flat	47.22	47.22	
RiskReversal	-1.0 Put	+1.0 Call	
-5.60E+02	5.60E+02	0.00E+00	
82.58	0.96	1.04	
	0.12	0.12	
Flat	82.58	82.58	
RiskReversal	-1.0 Put	+1.0 Call	
-1.15E-38	1.15E-38	0.00E+00	
18.90	0.94	1.04	
	0.19	0.19	
Flat	18.90	18.90	
RiskReversal	+1.0 Put	-1.0 Call	

6.34E-106	6.34E-106	0.00E+00
11.32	0.94	1.06
	0.19	0.19
Flat	11.32	11.32
RiskReversal	+1.0 Put	-1.0 Call
3.64E+02	3.64E+02	1.55E+02
25.03	0.93	1.06
	0.44	0.19
Spot	25.22	21.28
RiskReversal	+1.0 Put	-1.0 Call
1.18E+03	1.18E+03	2.99E+02
32.91	0.92	1.12
	0.70	0.70
Spot	49.88	21.58
RiskReversal	+1.0 Put	-1.0 Call
6.19E+03	6.19E+03	0.00E+00
72.35	1.01	1.26
	1.45	1.45
Flat	72.35	72.35
Straddle	+1.0 Put	+1.0 Call
4.35E+02	2.33E+02	2.03E+02
21.28	1.00	1.00
	0.19	0.19
Spot	12.14	12.14
Straddle	+1.0 Put	+1.0 Call
1.74E+03	1.06E+03	6.76E+02
21.70	1.04	1.04

	0.70	0.70	
Spot	25.72	25.72	
Straddle	+1.0 Put	+1.0 Call	
4.65E+03	2.69E+03	1.96E+03	
22.71	1.08	1.08	
	2.45	2.45	
Spot	38.20	38.20	



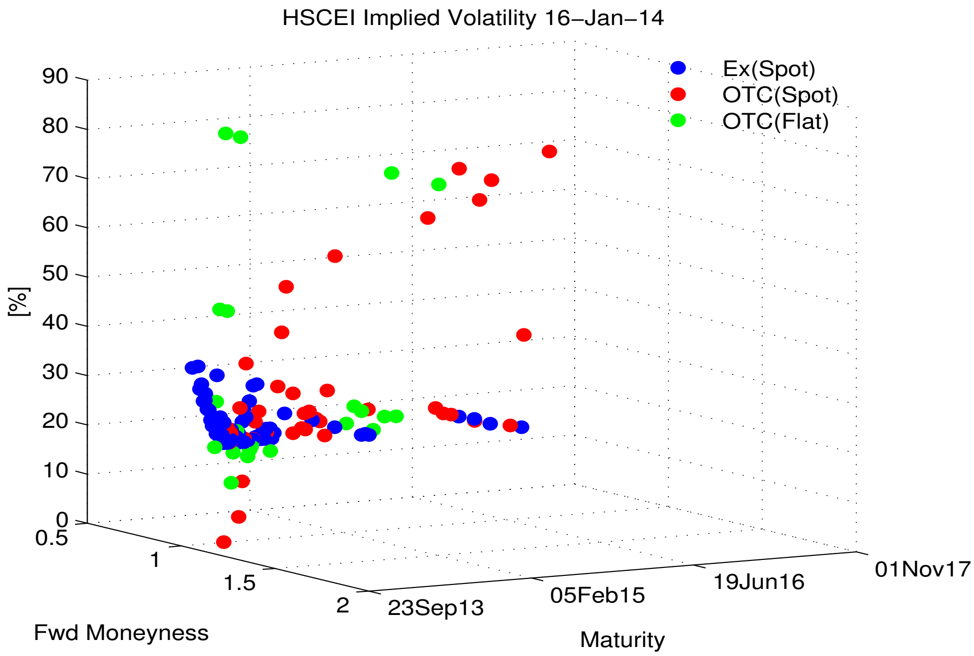


Figure B.5: HSCEI Implied Volatility (Nearest) (03-Jan-14∼31-Mar-14), (16-Jan-14)

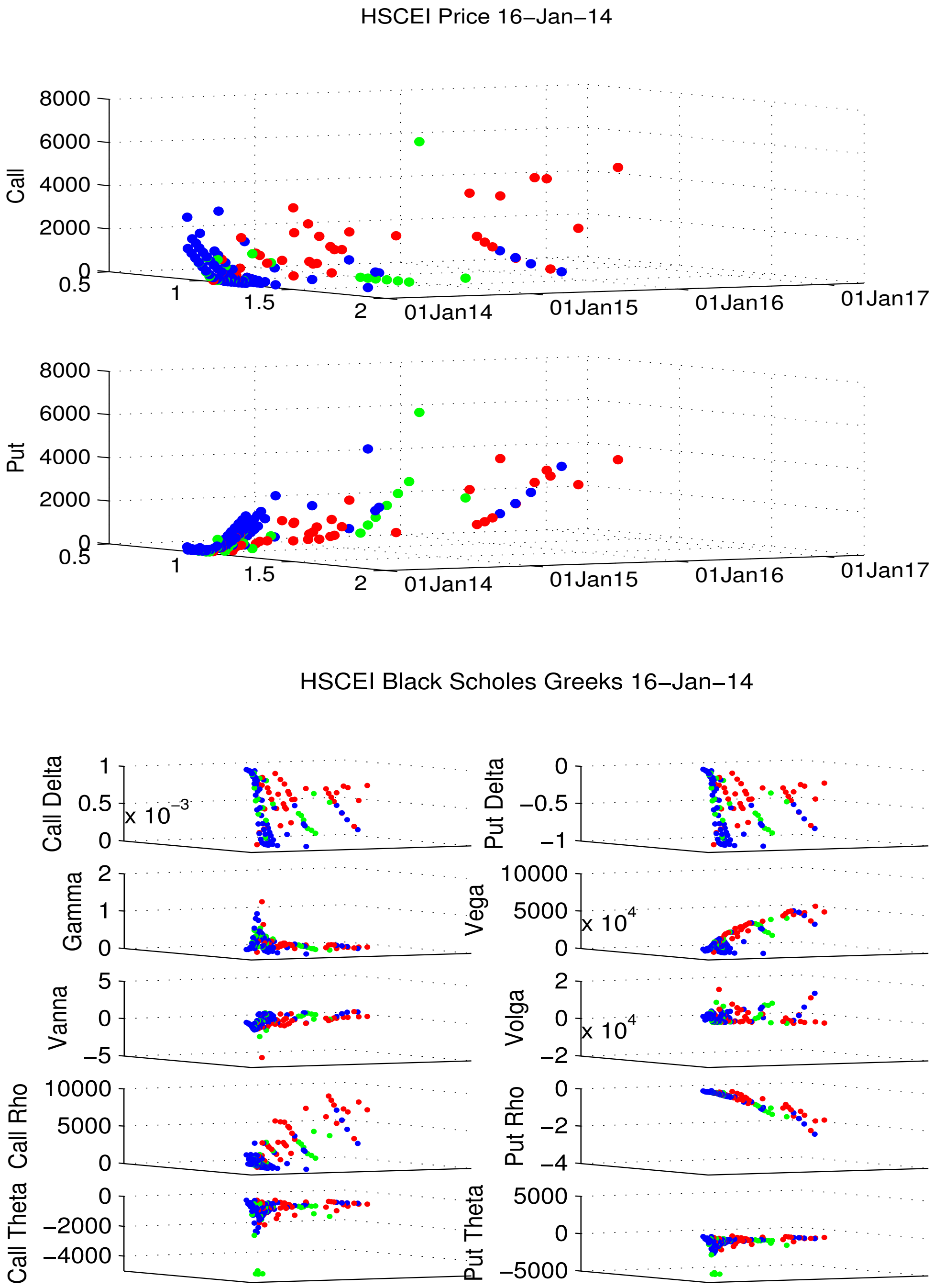
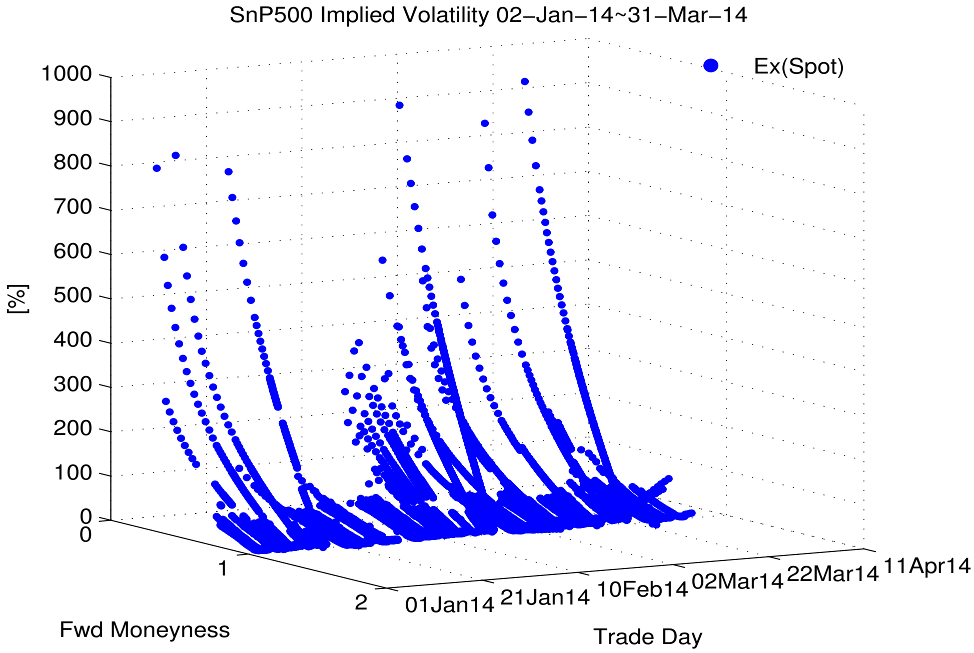


Figure B.6: HSCEI Price, Black Scholes Greeks (16-Jan-14)



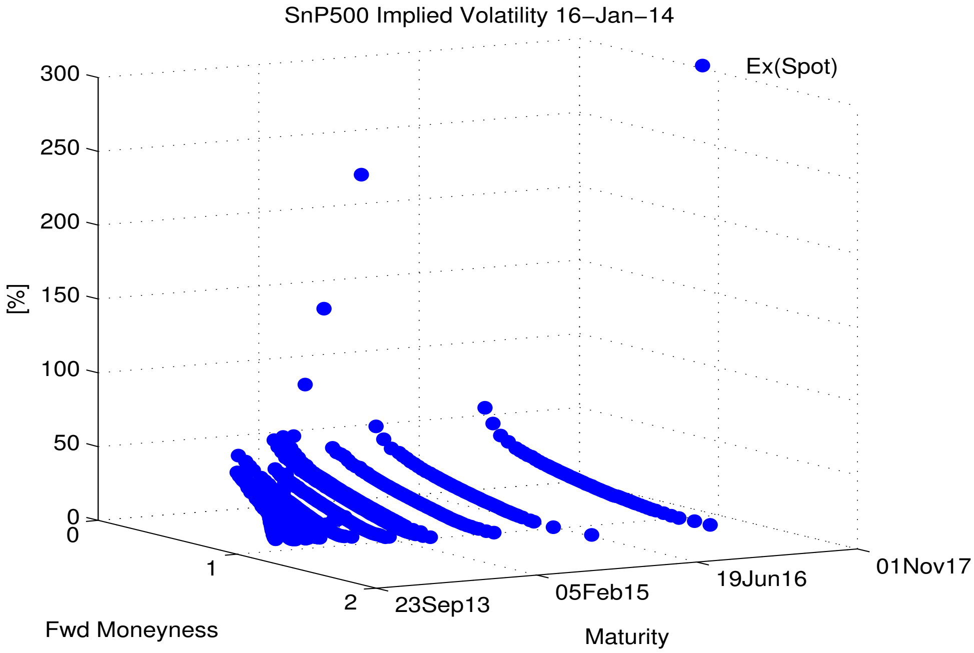


Figure B.7: SnP500 Implied Volatility (Nearest) (03-Jan-14∼31-Mar-14), (16-Jan-14)

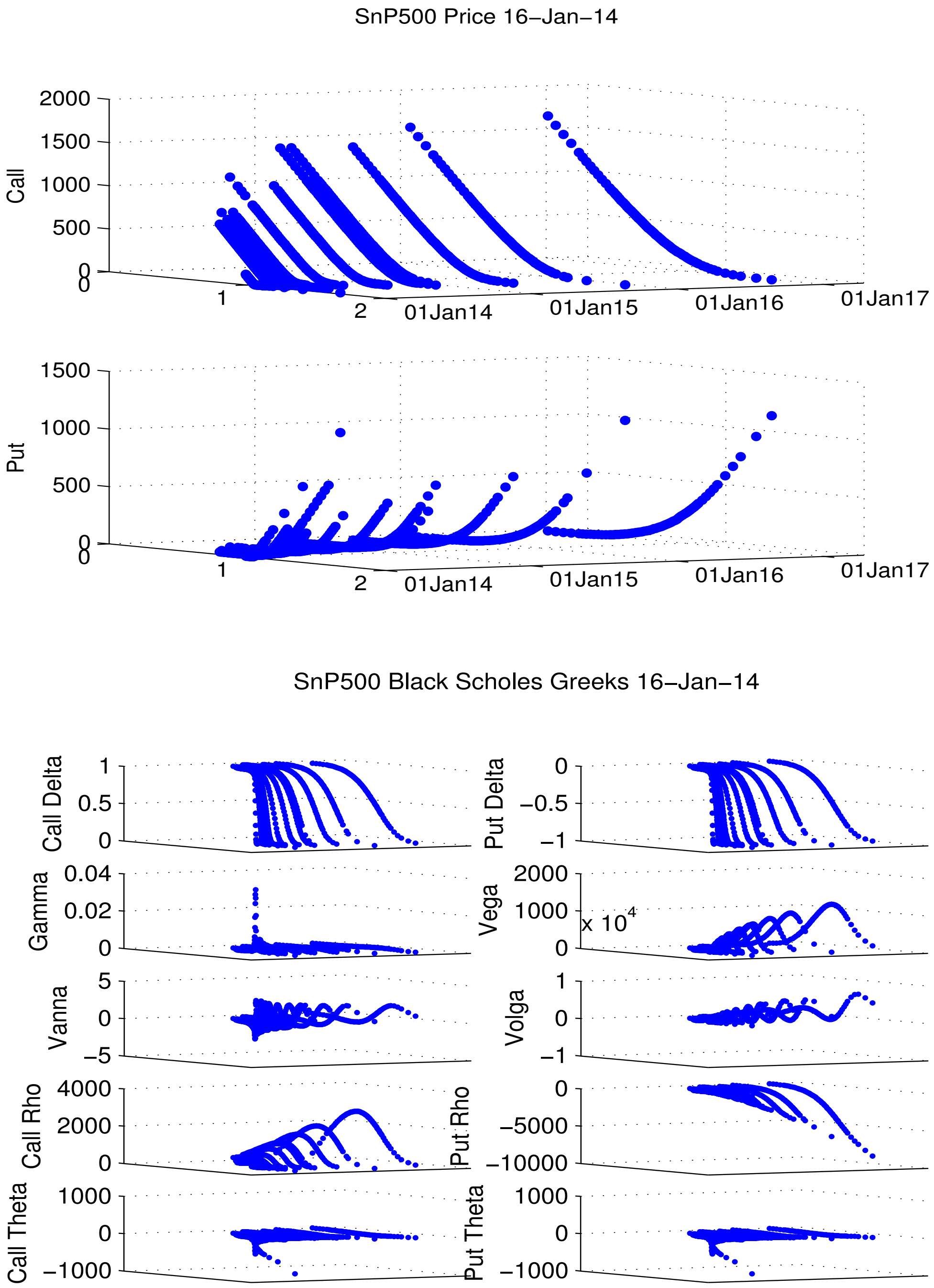


Figure B.8: SnP500 Price, Black Scholes Greeks (16-Jan-14)

Table B.6: Nikkei225 OverTheCounter Synthetic Options (16-Jan-14)

Strategy Price Implied volatility (%) Bootstrapped Type	weight × rights Bootstrapped price Forward moneyness Time-to-maturity Bootstrapped implied volatility (%)	
CallCalendarSpread	-1.0 Call	+1.0 Call
0.00E+00	0.00E+00	0.00E+00
27.35	1.03	1.03
	0.08	0.16
Flat	27.35	27.35
CallCalendarSpread	-1.0 Call	+1.0 Call
0.00E+00	0.00E+00	0.00E+00
27.26	1.08	1.08
	0.08	0.16
Flat	27.26	27.26
CallCalendarSpread	-1.0 Call	+1.0 Call
0.00E+00	0.00E+00	0.00E+00
21.85	1.02	1.02
	0.16	0.41
Flat	21.85	21.85
CallCalendarSpread	-1.0 Call	+1.0 Call
0.00E+00	0.00E+00	0.00E+00
22.39	1.05	1.05
	0.23	0.41
Flat	22.39	22.39

CallCalendarSpread	-1.0 Call	+1.0 Call
0.00E+00	0.00E+00	0.00E+00
19.00	1.13	1.13
	1.40	1.90
Flat	19.00	19.00
CallDiagonal	-1.0 Call	+1.0 Call
0.00E+00	0.00E+00	0.00E+00
24.84	1.05	1.08
	0.08	0.16
Flat	24.84	24.84
CallDiagonal	-2.0 Call	+1.0 Call
0.00E+00	0.00E+00	0.00E+00
24.63	1.05	1.02
	0.08	0.41
Flat	24.63	24.63
CallRatio	+1.0 Call	-2.0 Call
0.00E+00	0.00E+00	0.00E+00
18.06	1.02	1.07
	0.08	0.08
Flat	18.06	18.06
CallRatio	+1.0 Call	-1.5 Call
0.00E+00	0.00E+00	0.00E+00
27.18	1.03	1.08
	0.08	0.08
Flat	27.18	27.18
CallRatio	+1.0 Call	-1.5 Call

0.00E+00	0.00E+00	0.00E+00
28.25	1.02	1.11
	0.16	0.16
Flat	28.25	28.25
CallRatio	+1.0 Call	-1.5 Call
0.00E+00	0.00E+00	0.00E+00
21.67	1.10	1.13
	0.23	0.23
Flat	21.67	21.67
CallRatio	+1.0 Call	-2.0 Call
0.00E+00	0.00E+00	0.00E+00
21.76	1.12	1.17
	0.23	0.23
Flat	21.76	21.76
CallRatio	+1.0 Call	-2.0 Call
0.00E+00	0.00E+00	0.00E+00
21.56	1.05	1.15
	0.41	0.41
Flat	21.56	21.56
CallSpread	+1.0 Call	-1.0 Call
0.00E+00	0.00E+00	0.00E+00
155.93	1.02	1.05
	0.08	0.08
Flat	155.93	155.93
CallSpread	+1.0 Call	-1.0 Call
0.00E+00	0.00E+00	0.00E+00
38.61	1.03	1.05

	0.08	0.08
Flat	38.61	38.61
CallSpread	+1.0 Call	-1.0 Call
0.00E+00	0.00E+00	0.00E+00
29.12	1.03	1.08
	0.08	0.08
Flat	29.12	29.12
CallSpread	+1.0 Call	-1.0 Call
0.00E+00	0.00E+00	0.00E+00
24.03	1.05	1.08
	0.08	0.08
Flat	24.03	24.03
CallSpread	+1.0 Call	-1.0 Call
0.00E+00	0.00E+00	0.00E+00
27.42	1.07	1.08
	0.08	0.08
Flat	27.42	27.42
CallSpread	+1.0 Call	-1.0 Call
0.00E+00	0.00E+00	0.00E+00
47.63	1.00	1.03
	0.16	0.16
Flat	47.63	47.63
CallSpread	+1.0 Call	-1.0 Call
0.00E+00	0.00E+00	0.00E+00
57.25	1.02	1.03
	0.16	0.16
Flat	57.25	57.25

CallSpread	+1.0 Call	-1.0 Call	
0.00E+00	0.00E+00	0.00E+00	
149.48	1.02	1.08	
	0.16	0.16	
Flat	149.48	149.48	
CallSpread	+1.0 Call	-1.0 Call	
0.00E+00	0.00E+00	0.00E+00	
261.54	1.03	1.08	
	0.16	0.16	
Flat	261.54	261.54	
CallSpread	+1.0 Call	-1.0 Call	
0.00E+00	0.00E+00	0.00E+00	
104.68	1.04	1.07	
	0.23	0.23	
Flat	104.68	104.68	
CallSpread	+1.0 Call	-1.0 Call	
0.00E+00	0.00E+00	0.00E+00	
272.11	1.09	1.15	
	0.31	0.31	
Flat	272.11	272.11	
CallButterFly	+1.0 Call	-2.0 Call	+1.0 Call
0.00E+00	0.00E+00	0.00E+00	0.00E+00
19.25	1.02	1.03	1.05
	0.08	0.08	0.08
Flat	19.25	19.25	19.25
CallButterFly	+1.0 Call	-2.0 Call	+1.0 Call

0.00E+00	0.00E+00	0.00E+00	0.00E+00
21.80	1.00	1.02	1.03
	0.16	0.16	0.16
Flat	21.80	21.80	21.80
PutCalendarSpread	-1.0 Put	+1.0 Put	
0.00E+00	0.00E+00	0.00E+00	
25.09	0.92	0.92	
	0.08	0.16	
Flat	25.09	25.09	
PutCalendarSpread	-1.0 Put	+1.0 Put	
6.44E+02	1.31E-183	6.44E+02	
21.64	0.95	0.96	
	0.16	0.41	
Flat	21.64	21.64	
PutCalendarSpread	-1.0 Put	+1.0 Put	
1.19E+03	1.72E+03	2.91E+03	
17.96	0.96	0.97	
	0.90	1.40	
Flat	17.96	17.96	
PutRatio	+1.0 Put	-1.5 Put	
0.00E+00	0.00E+00	0.00E+00	
46.46	0.92	0.89	
	0.08	0.08	
Flat	46.46	46.46	
PutRatio	+1.0 Put	-1.5 Put	
1.17E+03	3.65E+03	1.65E+03	
63.36	0.89	0.77	

	0.65	0.65
Flat	63.36	63.36
PutRatio	+1.0 Put	-1.5 Put
3.79E+02	9.32E+02	3.68E+02
25.52	0.99	0.93
	0.65	0.65
Spot	20.04	16.84
PutRatio	+1.0 Put	-2.0 Put
1.53E+03	4.45E+03	1.46E+03
66.79	0.83	0.64
	0.90	0.90
Flat	66.79	66.79
PutRatio	+1.0 Put	-3.0 Put
-3.62E+03	1.07E+04	4.78E+03
44.76	0.89	0.50
	3.90	3.90
Flat	44.76	44.76
PutSpread	+1.0 Put	-1.0 Put
1.77E+02	1.77E+02	2.55E+02
1.90	1.01	0.99
	0.08	0.08
Spot	4.29	20.19
PutSpread	+1.0 Put	-1.0 Put
0.00E+00	0.00E+00	0.00E+00
23.70	0.92	0.86
	0.16	0.16
Flat	23.70	23.70

PutSpread	+1.0 Put	-1.0 Put	
4.36E+02	4.36E+02	2.30E+02	
19.75	0.87	0.77	
	0.90	0.90	
Spot	21.89	25.30	
PutSpread	+1.0 Put	-1.0 Put	
2.96E+01	2.96E+01	3.40E+01	
30.41	0.42	0.36	
	1.90	1.90	
Spot	30.41	36.59	
PutButterFly	+1.0 Put	-2.0 Put	+1.0 Put
1.82E-12	8.03E+03	7.03E+03	6.03E+03
411.00	0.70	0.64	0.57
	0.41	0.41	0.41
Flat	411.00	411.00	411.00
PutButterFly	+1.0 Put	-2.0 Put	+1.0 Put
2.00E+02	5.84E+02	4.35E+02	2.00E+02
24.31	0.96	0.93	0.89
	0.41	0.41	0.41
Spot	22.49	23.48	20.17
RiskReversal	-1.0 Put	+1.0 Call	
-1.70E+03	1.70E+03	0.00E+00	
186.74	0.97	1.03	
	0.08	0.08	
Flat	186.74	186.74	
RiskReversal	+1.0 Put	-1.0 Call	

1.37E+03	1.37E+03	0.00E+00
156.58	0.97	1.05
	0.08	0.08
Flat	156.58	156.58
RiskReversal	-1.0 Put	+1.0 Call
-3.77E+03	3.77E+03	0.00E+00
368.73	0.99	1.02
	0.08	0.08
Flat	368.73	368.73
RiskReversal	-1.0 Put	+1.0 Call
-3.92E+03	3.92E+03	0.00E+00
223.75	0.95	1.08
	0.16	0.16
Flat	223.75	223.75
RiskReversal	+1.0 Put	-1.0 Call
1.47E+03	1.47E+03	0.00E+00
95.87	0.95	1.08
	0.16	0.16
Flat	95.87	95.87
RiskReversal	-1.0 Put	+1.0 Call
-5.61E+03	5.61E+03	3.77E+02
296.37	0.99	1.03
	0.16	0.16
Spot	240.93	24.06
RiskReversal	+1.0 Put	-1.0 Call
1.53E+02	1.53E+02	3.82E+02
22.48	0.96	1.05

	0.23	0.23
Spot	13.32	23.72
RiskReversal	+1.0 Put	-1.0 Call
2.81E-269	2.81E-269	0.00E+00
14.58	0.93	1.09
	0.31	0.31
Flat	14.58	14.58
RiskReversal	+1.0 Put	-1.0 Call
2.72E+03	2.72E+03	0.00E+00
70.36	0.93	1.15
	0.41	0.41
Flat	70.36	70.36
RiskReversal	+1.0 Put	-1.0 Call
4.52E+03	4.52E+03	0.00E+00
49.66	0.93	1.12
	0.90	0.90
Flat	49.66	49.66
Straddle	+1.0 Put	+1.0 Call
8.53E+02	8.53E+02	0.00E+00
23.78	1.02	1.02
	0.16	0.16
Flat	23.78	23.78
Straddle	+1.0 Put	+1.0 Call
1.73E+03	1.73E+03	0.00E+00
23.17	1.02	1.02
	0.41	0.41
Flat	23.17	23.17

Straddle	+1.0 Put	+1.0 Call
2.01E+03	9.32E+02	1.08E+03
22.97	0.99	0.99
	0.65	0.65
Spot	20.04	20.04
Straddle	+1.0 Put	+1.0 Call
2.49E+03	1.42E+03	1.07E+03
22.67	1.02	1.02
	0.65	0.65
Spot	24.26	24.26
Straddle	+1.0 Put	+1.0 Call
4.49E+03	2.49E+03	1.99E+03
21.49	1.03	1.03
	1.40	1.40
Spot	30.28	30.28
Straddle	+1.0 Put	+1.0 Call
7.52E+03	4.13E+03	3.39E+03
20.69	1.05	1.05
	2.90	2.90
Spot	36.11	36.11
Straddle	+1.0 Put	+1.0 Call
9.10E+03	5.00E+03	4.10E+03
20.75	1.06	1.06
	3.90	3.90
Spot	38.38	38.38
Strangle	+1.0 Put	+1.0 Call

2.87E+02	NaN	NaN
22.09	0.99	1.07
	0.08	0.08
Flat	22.09	22.09

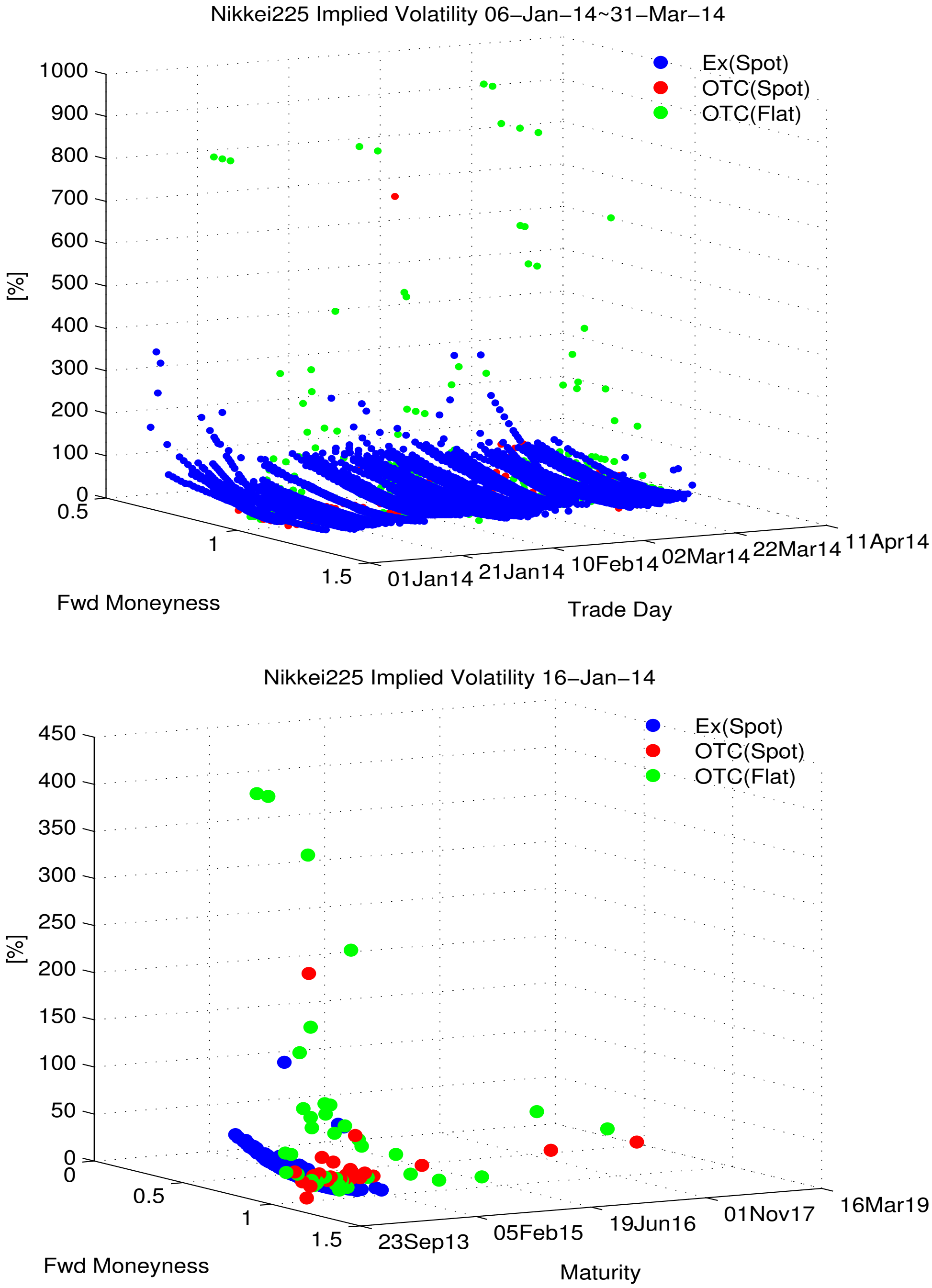
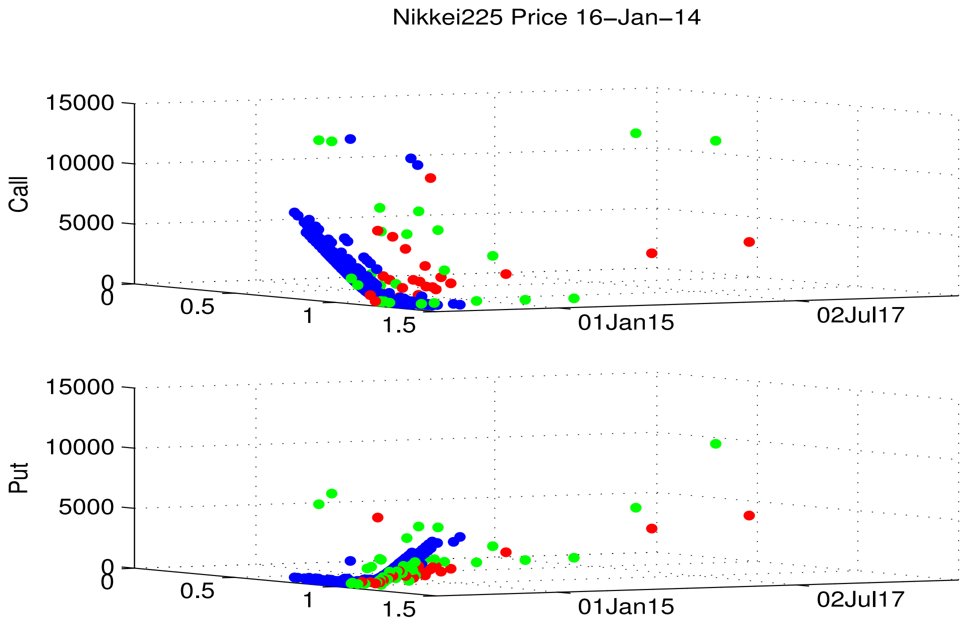


Figure B.9: Nikkei225 Implied Volatility (Nearest) (03-Jan-14∼31-Mar-14), (16-Jan-14)



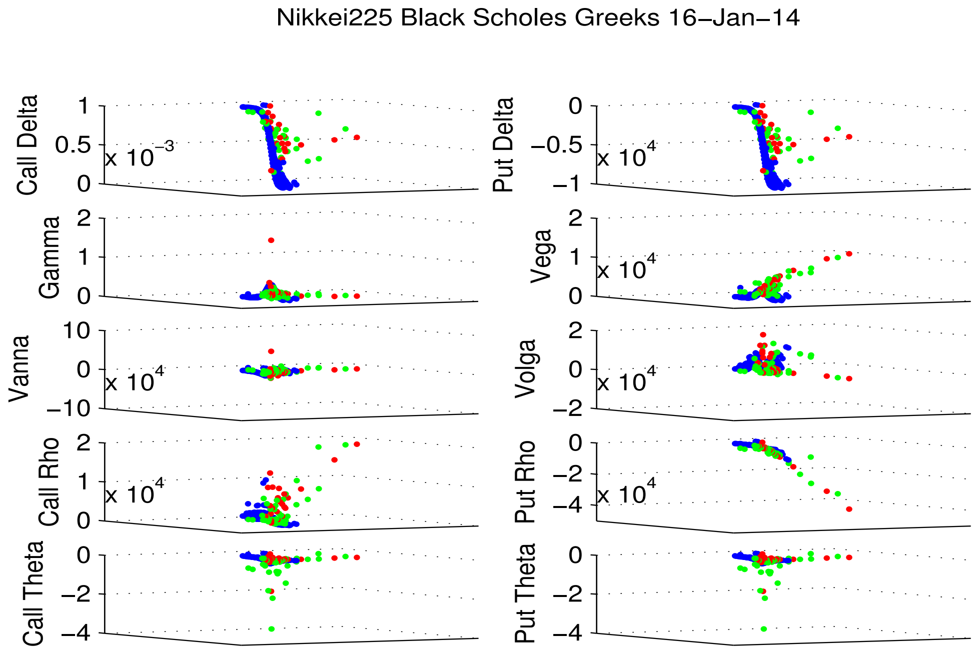
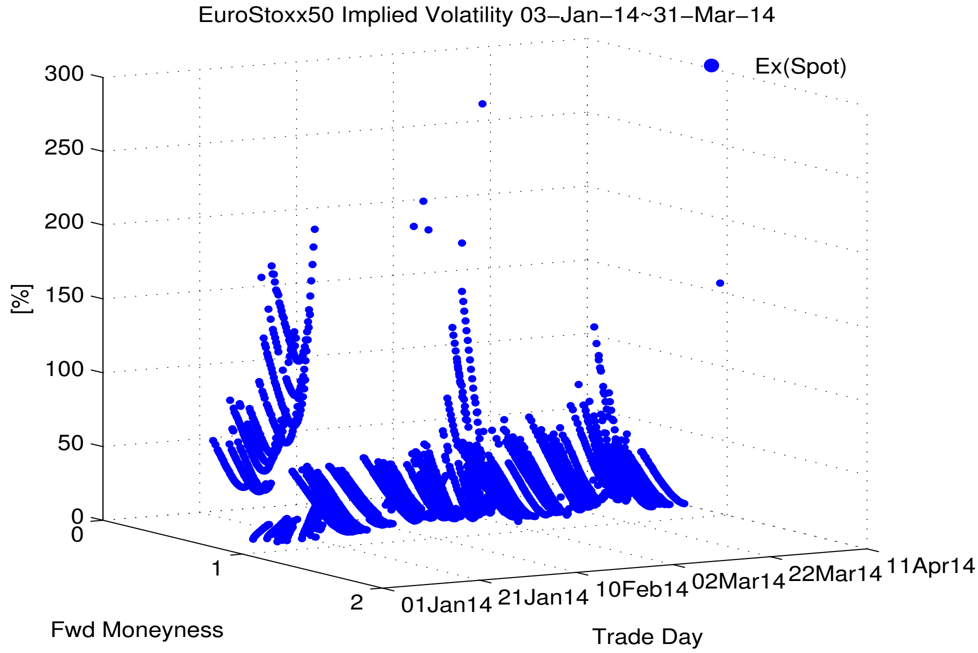


Figure B.10: Nikkei225 Price, Black Scholes Greeks (16-Jan-14)



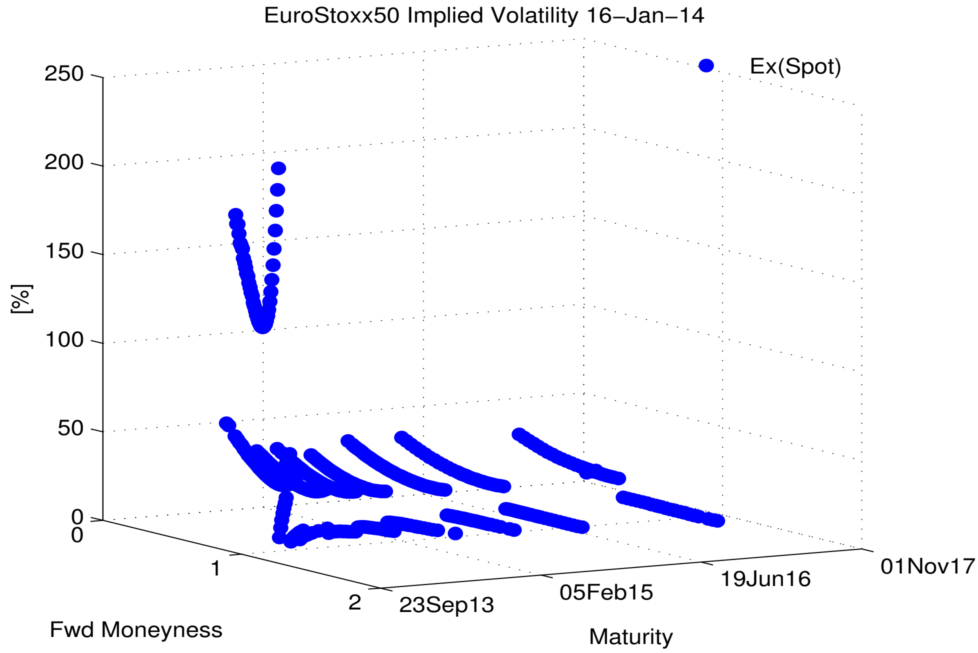


Figure B.11: EuroStoxx50 Implied Volatility (Nearest) (03-Jan-14∼31-Mar-14), (16-Jan-14)

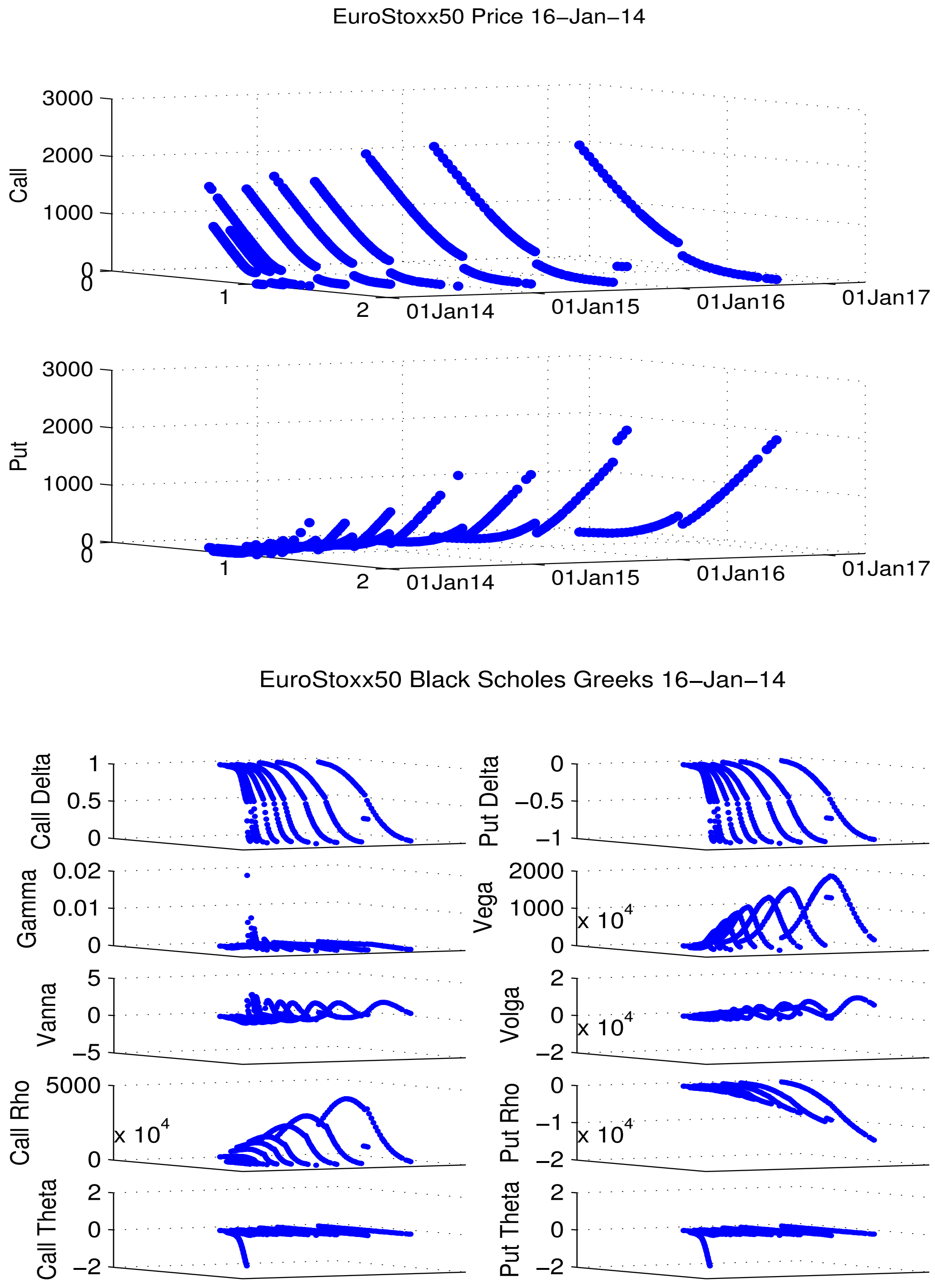


Figure B.12: EuroStoxx50 Price, Black Scholes Greeks (16-Jan-14)

Table B.7: SABR BoneWing HSCEI Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	($-\infty$, 0.87)		7.13E+00	-2.45E-01	2.75E+00
Bone	[0.87, 1.18]	(0.00, 0.04]	1.81E+01	6.34E-01	1.00E+01
Wing	(1.18,$+\infty$)		2.19E+00	4.48E-01	2.25E+00
Wing	($-\infty$, 0.83)		6.96E+00	-2.25E-01	2.69E+00
Bone	[0.83, 1.28]	(0.04, 0.12]	1.59E+01	4.46E-01	6.81E+00
Wing	(1.28,$+\infty$)		7.81E+00	3.74E-01	1.84E+00
Wing	($-\infty$, 0.74)		4.94E+00	1.33E-02	2.52E+00
Bone	[0.74, 1.16]	(0.12, 0.19]	1.17E+01	7.96E-01	6.06E+00
Wing	(1.16,$+\infty$)		5.07E-01	4.61E-03	1.68E+00
Wing	($-\infty$, 0.71)		9.59E+00	1.86E-01	1.79E+00
Bone	[0.71, 1.52]	(0.19, 0.44]	1.51E+01	3.41E-01	2.32E+00
Wing	(1.52,$+\infty$)		5.48E+00	5.86E-02	1.64E+00
Wing	($-\infty$, 0.92)		2.76E+00	8.14E-01	6.59E+00
Bone	[0.92, 1.12]	(0.44, 0.70]	1.46E+01	-6.30E-01	4.72E+00
Wing	(1.12,$+\infty$)		9.88E+00	7.47E-01	6.46E-01
Wing	($-\infty$, 0.71)		6.11E+00	7.48E-01	3.13E+00
Bone	[0.71, 1.34]	(0.70, 0.95]	1.71E+01	7.47E-01	3.97E+00
Wing	(1.34,$+\infty$)		5.61E+00	7.50E-02	1.64E+00
Wing	($-\infty$, 0.88)		2.33E+00	8.15E-01	5.01E+00
Bone	[0.88, 1.26]	(0.95, 1.45]	5.03E+01	7.60E-01	3.26E+00
Wing	(1.26,$+\infty$)		3.41E+00	3.92E-02	2.14E+00
Wing	($-\infty$, 0.88)		5.50E+00	-4.64E-02	1.99E+00
Bone	[0.88, 1.38]	(1.45, 1.95]	1.18E+01	4.72E-01	2.74E+00

Wing	(1.38,+∞)		5.65E+00	8.20E-02	1.38E+00
Wing	(-∞, 0.84)		5.68E+00	-7.64E-02	2.34E+00
Bone	[0.84, 1.08]	(1.95, 2.45]	2.79E+01	-7.36E-01	3.93E+00
Wing	(1.08,+∞)		8.24E+00	2.36E-01	8.93E-01
Wing	(-∞, 0.90)	(2.45,+∞]	5.90E+00	-7.88E-02	2.20E+00
Wing	[0.90,+∞)		6.67E+01	-3.98E-01	8.18E-01

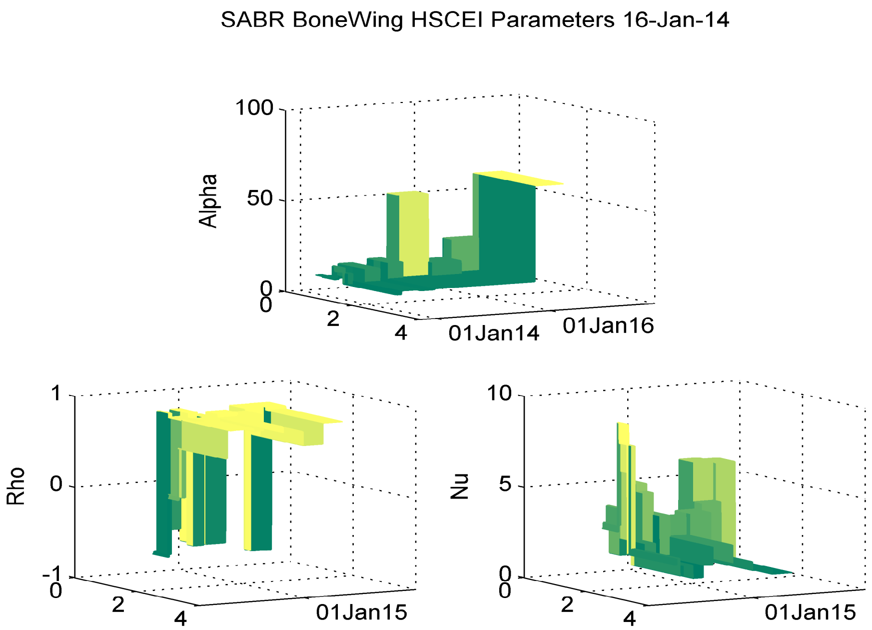


Figure B.13: SABR BoneWing HSCEI Parameters

Table B.8: Heston BoneWing HSCEI Parameters (κ = 2.00E+00, v0 = 1.00E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.87)		1.00E-01	3.50E-01	-2.50E-01
Bone	[0.87, 1.18]	(0.00, 0.04]	1.00E-01	3.50E-01	-2.50E-01
Wing	(1.18,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.83)		3.01E-01	1.93E+00	9.99E-01
Bone	[0.83, 1.28]	(0.04, 0.12]	1.00E-01	3.50E-01	-2.50E-01
Wing	(1.28,+∞)		1.08E-01	3.90E-01	-6.23E-01
Wing	(-∞, 0.74)		5.86E-01	8.29E-04	-5.05E-02
Bone	[0.74, 1.16]	(0.12, 0.19]	1.00E-01	3.50E-01	-2.50E-01
Wing	(1.16,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.71)		2.35E-02	3.65E-01	3.38E-01
Bone	[0.71, 1.52]	(0.19, 0.44]	3.96E-04	1.58E-01	9.67E-01
Wing	(1.52,+∞)		8.67E-02	1.76E-03	-3.40E-01
Wing	(-∞, 0.92)		3.58E-01	1.72E+00	-1.74E-01
Bone	[0.92, 1.12]	(0.44, 0.70]	7.15E-01	6.40E+00	-6.48E-01
Wing	(1.12,+∞)		1.39E+00	3.38E-01	-9.99E-01
Wing	(-∞, 0.71)		7.33E-02	1.79E+00	-2.07E-01
Bone	[0.71, 1.34]	(0.70, 0.95]	1.00E-01	3.50E-01	-2.50E-01
Wing	(1.34,+∞)		1.89E-04	1.57E+00	9.89E-01
Wing	(-∞, 0.88)		1.43E-02	2.86E-01	7.24E-01
Bone	[0.88, 1.26]	(0.95, 1.45]	1.00E-01	3.50E-01	-2.50E-01
Wing	(1.26,+∞)		4.18E-01	3.79E+00	8.75E-01
Wing	(-∞, 0.88)		5.18E-01	4.12E+00	-6.14E-01
Bone	[0.88, 1.38]	(1.45, 1.95]	7.50E-01	6.44E+00	-7.57E-01

Wing	(1.38,+∞)		2.29E-01	5.48E-05	-1.94E-01
Wing	(-∞, 0.84)		4.77E-01	1.25E+00	-3.17E-01
Bone	[0.84, 1.08]	(1.95, 2.45]	3.40E-01	2.43E+00	-4.12E-01
Wing	(1.08,+∞)		5.32E-01	2.17E+00	9.90E-01
Wing	(-∞, 0.90)	(2.45,+∞]	6.51E-01	7.09E-01	-6.25E-01
Wing	[0.90,+∞)		4.84E-01	4.70E-01	-9.97E-01

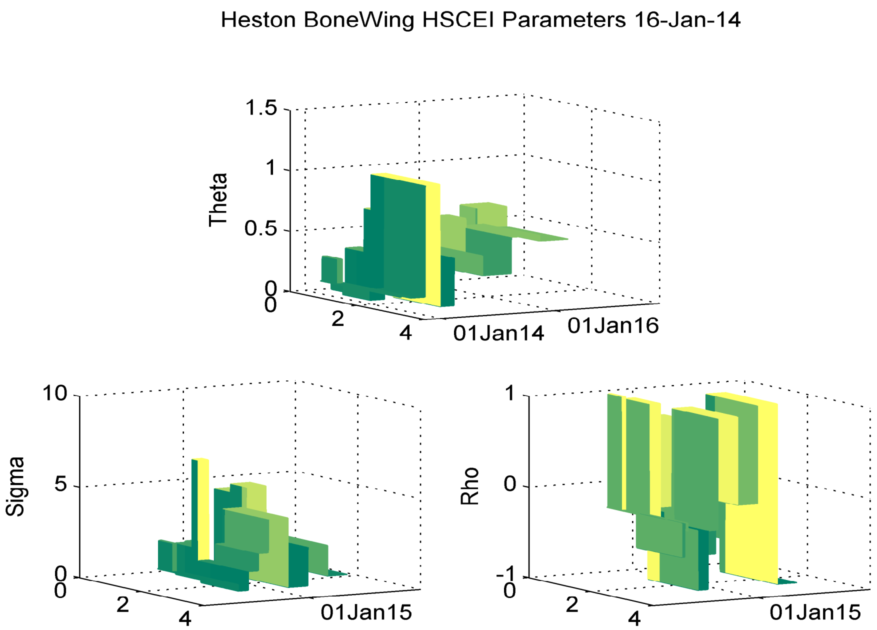


Figure B.14: Heston BoneWing HSCEI Parameters

Table B.9: SABR BoneJointWing HSCEI Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.87)		7.10E+00	-2.33E-01	2.48E+00
Bone	[0.87, 0.94]		9.54E+00	-6.03E-01	6.11E+00
Joint	(0.94, 0.96)		9.57E+00	-4.32E-01	9.33E+00
Bone	[0.96, 1.04]	(0.00, 0.04]	1.59E+01	-1.32E-01	9.88E+00
Joint	(1.04, 1.06)		1.00E+01	3.81E-01	1.00E+01
Bone	[1.06, 1.18]		1.00E+01	3.64E-01	1.00E+01
Wing	(1.18,+∞)		7.82E+00	3.56E-01	1.59E+00
Wing	(-∞, 0.83)		6.82E+00	-1.93E-01	2.04E+00
Bone	[0.83, 0.94]		9.51E+00	-4.62E-01	3.66E+00
Joint	(0.94, 0.96)		9.43E+00	-3.66E-01	6.41E+00
Bone	[0.96, 1.04]	(0.04, 0.12]	1.17E+01	3.52E-01	1.00E+01
Joint	(1.04, 1.06)		9.35E+00	2.76E-01	7.38E+00
Bone	[1.06, 1.28]		6.88E+00	1.96E-01	4.06E+00
Wing	(1.28,+∞)		7.67E+00	3.35E-01	1.15E+00
Wing	(-∞, 0.74)		6.43E+00	-1.63E-01	2.27E+00
Bone	[0.74, 0.94]		7.18E+00	-1.82E-01	3.52E+00
Joint	(0.94, 0.98)		9.78E+00	-4.97E-02	3.17E+00
Bone	[0.98, 1.04]	(0.12, 0.19]	7.08E+00	5.54E-01	9.08E+00
Joint	(1.04, 1.06)		9.32E+00	3.75E-01	5.52E+00
Bone	[1.06, 1.16]		6.44E+00	1.18E-01	4.21E+00
Wing	(1.16,+∞)		9.51E+00	6.35E-01	6.77E-01
Wing	(-∞, 0.71)		9.57E+00	2.86E-01	1.99E+00
Bone	[0.71, 0.93]	(0.19, 0.44]	7.20E+00	-1.70E-01	2.81E+00

Joint	(0.93, 0.97)		6.74E+00	-5.70E-02	3.52E+00
Bone	[0.97, 1.01]		1.13E+01	7.19E-01	7.89E+00
Joint	(1.01, 1.06)		9.27E+00	2.37E-02	4.26E+00
Bone	[1.06, 1.52]		7.67E+00	2.17E-01	3.18E+00
Wing	(1.52,+∞)		5.50E+00	6.18E-02	1.70E+00
Wing	(-∞, 0.92)		2.02E+00	7.55E-01	3.62E+00
Joint	[0.92, 1.00)		8.22E+00	4.08E-02	5.30E+00
Bone	[1.00, 1.04]	(0.44, 0.70]	2.05E+01	7.17E-01	1.75E+00
Joint	(1.04, 1.12]		7.07E+00	1.34E-01	3.49E+00
Wing	(1.12,+∞)		1.33E+00	1.43E-01	6.64E-01
Wing	(-∞, 0.71)		2.03E+00	8.09E-01	4.10E+00
Bone	[0.71, 0.94]		7.31E+00	-1.38E-01	2.14E+00
Joint	(0.94, 0.98)		7.43E+00	-2.99E-02	3.79E+00
Bone	[0.98, 1.02]	(0.70, 0.95]	4.81E+00	4.29E-01	7.77E+00
Joint	(1.02, 1.08)		8.08E+00	3.73E-02	4.13E+00
Bone	[1.08, 1.34]		5.36E+00	5.38E-02	2.92E+00
Wing	(1.34,+∞)		5.49E+00	5.58E-02	1.37E+00
Wing	(-∞, 0.88)		1.69E+00	5.43E-01	1.17E+00
Joint	[0.88, 1.01]	(0.95, 1.45]	5.10E+01	8.16E-01	4.91E+00
Joint	(1.01, 1.26]		9.20E+00	-7.51E-02	4.55E+00
Wing	(1.26,+∞)		4.72E+00	-4.09E-02	1.28E+00
Wing	(-∞, 0.88)		5.62E+00	-7.95E-02	2.20E+00
Bone	[0.88, 0.92]		4.43E+00	6.46E-02	3.64E+00
Joint	(0.92, 0.96)		5.58E+00	-2.06E-02	3.93E+00
Bone	[0.96, 1.05]	(1.45, 1.95]	4.47E+00	1.42E-02	4.88E+00
Joint	(1.05, 1.13)		7.44E+00	4.63E-02	2.91E+00
Bone	[1.13, 1.38]		1.05E+00	-7.87E-01	5.50E+00

Wing	(1.38,+∞)		6.52E+00	1.34E-01	1.02E+00
Wing	(-∞, 0.84)	(1.95, 2.45]	5.71E+00	-7.95E-02	2.39E+00
Bone	[0.84, 0.91]		6.68E+00	4.51E-01	4.36E+00
Joint	(0.91, 1.08]		3.19E+01	-6.57E-01	2.07E+00
Wing	(1.08,+∞)		4.63E+00	-5.40E-02	7.74E-01
Wing	(-∞, 0.90)	(2.45,+∞]	5.90E+00	-7.88E-02	2.20E+00
Wing	[0.90,+∞)		6.67E+01	-3.98E-01	8.18E-01

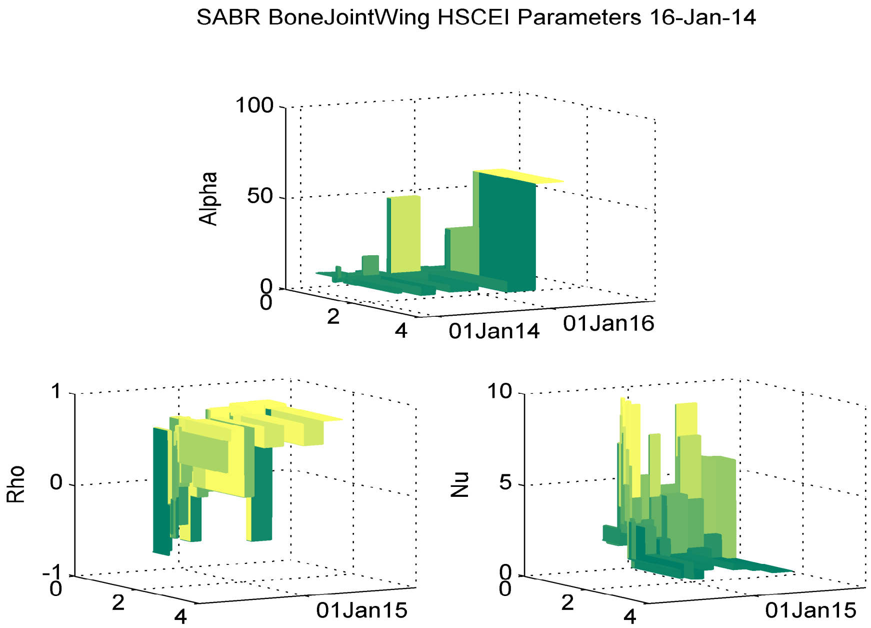


Figure B.15: SABR BoneJointWing HSCEI Parameters

Table B.10: Heston BoneJointWing HSCEI Parameters (κ = 2.00E+00, v0 = 1.00E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.87)		1.00E-01	3.50E-01	-2.50E-01
Bone	[0.87, 0.94]		1.00E-01	3.50E-01	-2.50E-01
Joint	(0.94, 0.96)		1.00E-01	3.50E-01	-2.50E-01
Bone	[0.96, 1.04]	(0.00, 0.04]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.04, 1.06)		1.00E-01	3.50E-01	-2.50E-01
Bone	[1.06, 1.18]		1.00E-01	3.50E-01	-2.50E-01
Wing	(1.18,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.83)		3.01E-01	1.93E+00	9.99E-01
Bone	[0.83, 0.94]		1.00E-01	3.50E-01	-2.50E-01
Joint	(0.94, 0.96)		1.00E-01	3.50E-01	-2.50E-01
Bone	[0.96, 1.04]	(0.04, 0.12]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.04, 1.06)		1.00E-01	3.50E-01	-2.50E-01
Bone	[1.06, 1.28]		1.00E-01	3.50E-01	-2.50E-01
Wing	(1.28,+∞)		1.08E-01	3.90E-01	-6.23E-01
Wing	(-∞, 0.74)		5.86E-01	8.29E-04	-5.05E-02
Bone	[0.74, 0.94]		3.94E-04	2.15E-01	9.87E-01
Joint	(0.94, 0.98)		6.29E-04	2.46E+00	9.32E-01
Bone	[0.98, 1.04]	(0.12, 0.19]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.04, 1.06)		3.05E-01	6.47E+00	1.94E-01
Bone	[1.06, 1.16]		2.46E-01	5.83E-01	-1.45E-01
Wing	(1.16,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.71)		2.35E-02	3.65E-01	3.38E-01
Bone	[0.71, 0.93]	(0.19, 0.44]	3.96E-04	1.58E-01	9.67E-01

Joint	(0.93, 0.97)		3.81E-02	7.70E-01	9.73E-01
Bone	[0.97, 1.01]		3.35E-01	4.23E+00	3.70E-02
Joint	(1.01, 1.06)		4.73E+00	6.28E+00	5.36E-02
Bone	[1.06, 1.52]		1.15E+00	5.51E+00	-8.76E-01
Wing	(1.52,+∞)		8.67E-02	1.76E-03	-3.40E-01
Wing	(-∞, 0.92)		4.83E-01	8.28E-01	-3.89E-01
Joint	[0.92, 1.00)		6.01E-01	5.30E+00	-4.82E-01
Bone	[1.00, 1.04]	(0.44, 0.70]	1.80E-01	2.09E+00	3.01E-02
Joint	(1.04, 1.12]		1.94E-01	1.38E+00	-7.28E-01
Wing	(1.12,+∞)		1.39E+00	3.38E-01	-9.99E-01
Wing	(-∞, 0.71)		1.12E-01	1.51E+00	-1.33E-01
Bone	[0.71, 0.94]		1.27E-01	1.57E+00	4.15E-02
Joint	(0.94, 0.98)		1.85E-01	1.06E+00	-9.27E-02
Bone	[0.98, 1.02]	(0.70, 0.95]	5.92E-01	4.39E+00	-5.43E-01
Joint	(1.02, 1.08)		5.79E-01	6.40E+00	-5.54E-01
Bone	[1.08, 1.34]		1.00E-01	3.50E-01	-2.50E-01
Wing	(1.34,+∞)		1.89E-04	1.57E+00	9.89E-01
Wing	(-∞, 0.88)		5.91E-02	3.30E-01	7.46E-01
Joint	[0.88, 1.01]	(0.95, 1.45]	6.88E-01	1.96E+00	-3.76E-01
Joint	(1.01, 1.26]		1.00E-01	3.50E-01	-2.50E-01
Wing	(1.26,+∞)		3.06E+00	6.99E+00	8.68E-01
Wing	(-∞, 0.88)		4.99E-01	4.17E+00	-6.12E-01
Bone	[0.88, 0.92]		4.11E-01	4.11E+00	-5.65E-01
Joint	(0.92, 0.96)		2.83E-01	7.46E-01	-1.63E-01
Bone	[0.96, 1.05]	(1.45, 1.95]	6.81E-01	4.75E+00	-6.47E-01
Joint	(1.05, 1.13)		5.34E-01	6.05E+00	-4.65E-01
Bone	[1.13, 1.38]		7.13E-02	6.09E-01	-7.07E-02

Wing	(1.38,+∞)		2.29E-01	5.48E-05	-1.94E-01
Wing	(-∞, 0.84)	(1.95, 2.45]	6.02E-01	1.25E+00	-4.04E-01
Bone	[0.84, 0.91]		5.46E-01	8.14E-01	-1.90E-01
Joint	(0.91, 1.08]		3.46E-01	2.83E+00	-4.68E-01
Wing	(1.08,+∞)		5.29E-01	2.16E+00	9.90E-01
Wing	(-∞, 0.90)	(2.45,+∞]	6.51E-01	7.09E-01	-6.25E-01
Wing	[0.90,+∞)		4.84E-01	4.70E-01	-9.97E-01

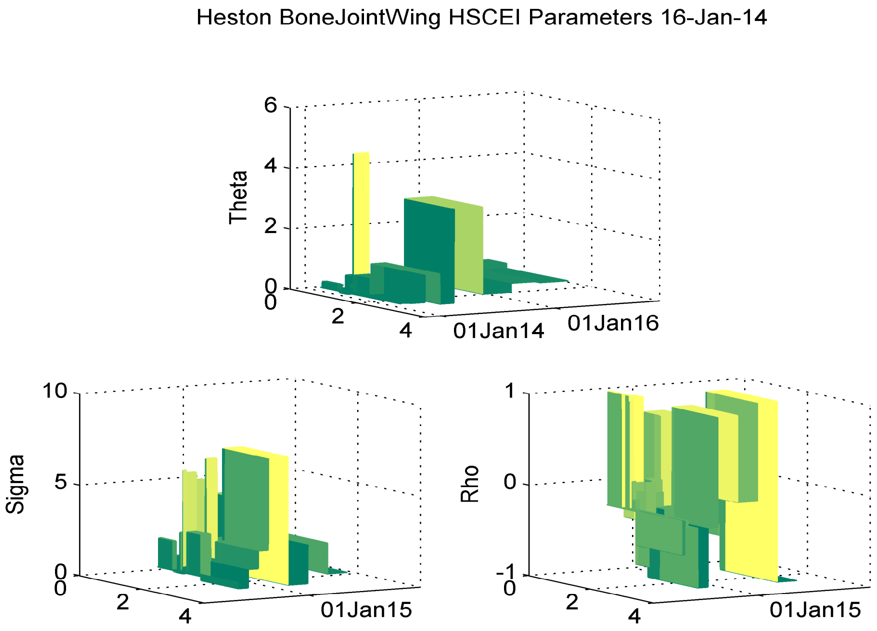


Figure B.16: Heston BoneJointWing HSCEI Parameters

Table B.11: SABR JointWing HSCEI Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.87)		7.09E+00	-2.29E-01	2.38E+00
Joint	[0.87, 0.89)		8.47E+00	-4.74E-01	5.95E+00
Joint	[0.89, 0.91)		8.67E+00	-4.96E-01	5.95E+00
Joint	[0.91, 0.93)		8.95E+00	-4.51E-01	6.26E+00
Joint	[0.93, 0.93)		6.93E+00	-1.83E-01	8.12E+00
Joint	[0.93, 0.94)		7.00E+00	-2.95E-01	8.29E+00
Joint	[0.94, 0.96)		9.45E+00	-4.70E-01	7.66E+00
Joint	[0.96, 0.96)		9.97E+00	-3.79E-01	9.98E+00
Joint	[0.96, 0.98)		1.00E+01	-3.81E-01	1.00E+01
Joint	[0.98, 0.98)		1.00E+01	-3.80E-01	1.00E+01
Joint	[0.98, 1.00]		2.04E+01	-8.11E-01	1.00E+01
Joint	(1.00, 1.00]		1.00E+01	2.45E-01	1.00E+01
Joint	(1.00, 1.02]	(0.00, 0.04]	1.00E+01	2.45E-01	1.00E+01
Joint	(1.02, 1.02]		1.00E+01	3.96E-01	1.00E+01
Joint	(1.02, 1.04]		1.00E+01	3.96E-01	1.00E+01
Joint	(1.04, 1.04]		8.15E+00	2.43E-01	8.86E+00
Joint	(1.04, 1.06]		9.30E+00	4.79E-01	7.18E+00
Joint	(1.06, 1.06]		1.00E+01	3.64E-01	1.00E+01
Joint	(1.06, 1.08]		1.00E+01	3.64E-01	1.00E+01
Joint	(1.08, 1.10]		8.72E+00	4.58E-01	5.54E+00
Joint	(1.10, 1.12]		8.47E+00	4.29E-01	5.40E+00
Joint	(1.12, 1.14]		2.63E+00	8.15E-02	8.22E+00
Joint	(1.14, 1.16]		2.81E+00	7.81E-02	8.21E+00

Joint	(1.16, 1.18]		6.52E+00	3.92E-01	6.13E+00
Wing	(1.18,+∞)		7.79E+00	3.23E-01	1.10E+00
Wing	(-∞, 0.83)		6.79E+00	-1.89E-01	1.94E+00
Joint	[0.83, 0.85)		6.35E+00	5.71E-01	5.78E+00
Joint	[0.85, 0.87)		5.65E+00	5.50E-01	6.04E+00
Joint	[0.87, 0.89)		7.15E+00	3.87E-01	5.24E+00
Joint	[0.89, 0.91)		8.54E+00	2.01E-01	4.69E+00
Joint	[0.91, 0.91)		4.36E+00	4.84E-01	6.54E+00
Joint	[0.91, 0.93)		8.87E+00	1.38E-01	4.44E+00
Joint	[0.93, 0.94)		9.19E+00	-2.86E-01	4.83E+00
Joint	[0.94, 0.96)		9.42E+00	-3.47E-01	5.55E+00
Joint	[0.96, 0.96)		6.48E+00	-3.32E-01	7.51E+00
Joint	[0.96, 0.98)		9.68E+00	-3.88E-01	7.07E+00
Joint	[0.98, 0.98)		8.22E+00	-2.61E-01	9.05E+00
Joint	[0.98, 1.00]		2.08E+01	6.91E-01	1.38E+00
Joint	(1.00, 1.00]		9.81E+00	1.06E-01	9.82E+00
Joint	(1.00, 1.02]		9.86E+00	-1.70E-01	9.04E+00
Joint	(1.02, 1.02]	(0.04, 0.12]	7.69E+00	2.50E-01	8.89E+00
Joint	(1.02, 1.04]		9.59E+00	2.36E-01	6.88E+00
Joint	(1.04, 1.04]		1.00E+01	2.06E-01	1.00E+01
Joint	(1.04, 1.06]		1.00E+01	2.06E-01	1.00E+01
Joint	(1.06, 1.06]		4.35E+00	3.47E-01	6.87E+00
Joint	(1.06, 1.08]		8.89E+00	4.88E-01	4.74E+00
Joint	(1.08, 1.08]		3.24E+00	6.18E-01	6.43E+00
Joint	(1.08, 1.10]		1.13E+00	4.51E-01	7.28E+00
Joint	(1.10, 1.10]		3.28E+00	5.48E-01	5.85E+00
Joint	(1.10, 1.12]		5.90E+00	6.29E-01	4.55E+00

Joint	(1.12, 1.14]		6.11E+00	5.91E-01	4.13E+00
Joint	(1.14, 1.16]		3.42E+00	5.85E-01	4.88E+00
Joint	(1.16, 1.18]		2.79E+00	5.26E-01	4.94E+00
Joint	(1.18, 1.20]		4.96E+00	-1.05E-02	4.18E+00
Joint	(1.20, 1.28]		6.25E+00	9.40E-02	3.54E+00
Wing	(1.28,+∞)		3.16E+00	-4.03E-02	1.37E+00
Wing	(-∞, 0.74)	(0.12, 0.19]	6.36E+00	-1.50E-01	2.06E+00
Joint	[0.74, 0.81)		5.29E+00	-2.27E-02	3.40E+00
Joint	[0.81, 0.88)		5.75E+00	-6.70E-02	3.80E+00
Joint	[0.88, 0.90)		5.29E+00	-2.30E-02	4.63E+00
Joint	[0.90, 0.90)		4.50E+00	5.35E-01	6.66E+00
Joint	[0.90, 0.92)		8.81E+00	2.94E-01	4.86E+00
Joint	[0.92, 0.94)		9.20E+00	-1.31E-01	4.50E+00
Joint	[0.94, 0.94)		4.54E+00	4.52E-01	6.54E+00
Joint	[0.94, 0.98)		9.78E+00	-6.70E-02	3.97E+00
Joint	[0.98, 0.98)		5.34E+00	-1.21E-01	6.54E+00
Joint	[0.98, 1.00]		1.20E+01	8.16E-01	8.29E+00
Joint	(1.00, 1.02]		6.67E+00	2.37E-01	9.94E+00
Joint	(1.02, 1.04]		9.67E+00	1.41E-01	7.28E+00
Joint	(1.04, 1.06]		9.26E+00	4.22E-01	4.60E+00
Joint	(1.06, 1.06]		3.85E+00	5.20E-01	6.65E+00
Joint	(1.06, 1.08]		8.89E+00	5.31E-01	4.43E+00
Joint	(1.08, 1.10]		8.47E+00	5.67E-01	4.00E+00
Joint	(1.10, 1.12]		5.20E+00	1.59E-02	4.54E+00
Joint	(1.12, 1.16]		5.25E+00	1.87E-02	4.04E+00
Wing	(1.16,+∞)		7.84E+00	2.73E-01	7.04E-01
Wing	(-∞, 0.71)		3.66E+00	7.12E-01	3.07E+00

Joint	[0.71, 0.83)		5.71E+00	-6.14E-02	2.60E+00
Joint	[0.83, 0.85)		4.93E+00	2.76E-02	3.25E+00
Joint	[0.85, 0.89)		4.07E+00	1.22E-01	3.52E+00
Joint	[0.89, 0.91)		5.15E+00	3.54E-04	3.57E+00
Joint	[0.91, 0.93)		5.54E+00	-4.20E-02	4.40E+00
Joint	[0.93, 0.97)		7.21E+00	-8.88E-02	4.81E+00
Joint	[0.97, 0.99)		8.98E+00	1.04E-01	5.14E+00
Joint	[0.99, 1.01]		1.66E+01	8.04E-01	4.73E+00
Joint	(1.01, 1.06]		5.21E+00	3.60E-01	7.38E+00
Joint	(1.06, 1.12]		6.71E+00	1.44E-01	5.03E+00
Joint	(1.12, 1.22]		5.81E+00	7.08E-02	3.23E+00
Joint	(1.22, 1.52]		5.86E+00	8.58E-02	2.26E+00
Wing	(1.52,+∞)		5.37E+00	4.21E-02	1.37E+00
Wing	(-∞, 0.92)		7.80E+00	-2.49E-01	2.50E+00
Joint	[0.92, 1.00)		8.14E+00	9.66E-02	5.39E+00
Joint	[1.00, 1.02]	(0.44, 0.70]	2.18E+01	-3.17E-01	1.27E-01
Joint	(1.02, 1.04]		7.13E+00	3.48E-02	4.86E+00
Joint	(1.04, 1.12]		7.12E+00	1.11E-01	3.57E+00
Wing	(1.12,+∞)		9.99E+00	7.63E-01	5.45E-01
Wing	(-∞, 0.71)		9.64E+00	2.39E-01	1.43E+00
Joint	[0.71, 0.79)		5.53E+00	7.59E-01	3.99E+00
Joint	[0.79, 0.85)		8.05E+00	7.45E-01	4.29E+00
Joint	[0.85, 0.92)		5.95E+00	-1.12E-02	2.78E+00
Joint	[0.92, 0.94)		5.48E+00	3.82E-03	3.41E+00
Joint	[0.94, 0.98)		7.50E+00	-4.04E-02	3.95E+00
Joint	[0.98, 1.02]		1.88E+01	-7.47E-01	4.09E+00
Joint	(1.02, 1.02]	(0.70, 0.95]	2.50E+00	1.22E-01	8.39E+00

Joint	(1.02, 1.08]		8.38E+00	8.05E-02	4.95E+00
Joint	(1.08, 1.16]		9.90E+00	7.37E-01	2.63E+00
Joint	(1.16, 1.16]		2.06E+00	8.13E-01	7.49E+00
Joint	(1.16, 1.18]		2.83E+00	8.09E-01	6.26E+00
Joint	(1.18, 1.22]		5.22E-01	8.07E-01	6.37E+00
Joint	(1.22, 1.34]		3.71E+00	-1.15E-02	2.71E+00
Wing	(1.34,+∞)		5.46E+00	5.02E-02	1.29E+00
Wing	(-∞, 0.88)	(0.95, 1.45]	9.86E+00	-6.24E-01	6.39E-01
Joint	[0.88, 1.01]		5.10E+01	8.16E-01	4.91E+00
Joint	(1.01, 1.26]		9.20E+00	-7.51E-02	4.55E+00
Wing	(1.26,+∞)		5.73E+00	1.20E-01	1.77E+00
Wing	(-∞, 0.88)	(1.45, 1.95]	5.69E+00	-7.70E-02	2.34E+00
Joint	[0.88, 0.92)		4.43E+00	6.46E-02	3.64E+00
Joint	[0.92, 0.96)		5.16E+00	1.98E-02	2.99E+00
Joint	[0.96, 1.01]		2.16E+01	4.45E-01	1.96E-01
Joint	(1.01, 1.05]		8.00E+00	-3.14E-01	3.38E+00
Joint	(1.05, 1.05]		4.92E+00	-5.29E-03	4.81E+00
Joint	(1.05, 1.13]		7.92E+00	2.39E-03	3.68E+00
Joint	(1.13, 1.13]		1.44E+00	8.12E-01	6.93E+00
Joint	(1.13, 1.21]		7.31E+00	8.07E-01	2.66E+00
Joint	(1.21, 1.32]		1.31E+00	8.01E-01	3.86E+00
Joint	(1.32, 1.38]		1.17E+00	-7.89E-01	4.49E+00
Wing	(1.38,+∞)		7.45E+00	2.28E-01	8.94E-01
Wing	(-∞, 0.84)	(1.95, 2.45]	5.64E+00	-8.28E-02	2.26E+00
Joint	[0.84, 0.91)		6.68E+00	4.51E-01	4.36E+00
Joint	[0.91, 1.08]		2.79E+01	-7.36E-01	3.93E+00
Wing	(1.08,+∞)		8.26E+00	2.44E-01	9.96E-01

Wing	(-∞, 0.90)	(2.45,+∞]	5.90E+00	-7.88E-02	2.20E+00
Wing	[0.90,+∞)		6.67E+01	-3.98E-01	8.18E-01

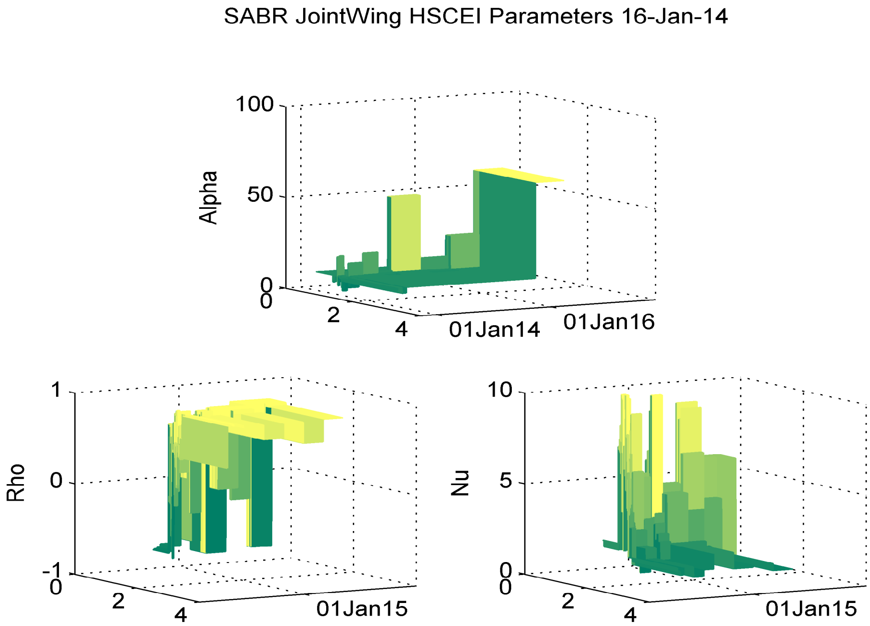


Figure B.17: SABR JointWing HSCEI Parameters

Table B.12: Heston JointWing HSCEI Parameters ($\kappa = 2.00\text{E}{+}00$, v0 = 1.00E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.87)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.87, 0.89)		1.00E-01	3.50E-01	-2.50E-01

Joint	[0.89, 0.91)	2.07E-01	3.60E+00	-6.67E-01
Joint	[0.91, 0.93)	8.77E-01	1.73E+00	-6.14E-01
Joint	[0.93, 0.93)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.93, 0.94)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.94, 0.96)	8.97E-01	1.40E+00	-4.46E-01
Joint	[0.96, 0.96)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.96, 0.98)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.98, 0.98)	8.90E-04	4.86E-01	5.60E-01
Joint	[0.98, 1.00]	1.79E+00	8.12E+00	9.99E-01
Joint	(1.00, 1.00]	1.26E+00	6.01E+00	-8.25E-02
Joint	(1.00, 1.02]	1.12E+00	6.97E+00	5.68E-01
Joint	(1.02, 1.02]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.02, 1.04]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.04, 1.04]	1.14E+00	5.13E+00	-8.54E-02
Joint	(1.04, 1.06]	1.39E+00	7.31E-01	-5.83E-01
Joint	(1.06, 1.06]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.06, 1.08]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.08, 1.10]	2.03E-01	3.31E+00	2.72E-01
Joint	(1.10, 1.12]	8.95E-01	1.19E+00	4.61E-01
Joint	(1.12, 1.14]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.14, 1.16]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.16, 1.18]	1.00E-01	3.50E-01	-2.50E-01
Wing	(1.18,+∞)	1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.83)	3.01E-01	1.93E+00	9.99E-01
Joint	[0.83, 0.85)	5.23E-02	3.16E+00	-6.87E-01
Joint	[0.85, 0.87)	8.69E-01	4.82E+00	6.51E-01
Joint	[0.87, 0.89)	4.99E-01	4.49E+00	4.43E-01

Joint	[0.89, 0.91)	9.33E-01	3.93E+00	7.01E-01
Joint	[0.91, 0.91)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.91, 0.93)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.93, 0.94)	8.58E-01	5.22E+00	5.95E-01
Joint	[0.94, 0.96)	8.00E-01	5.53E+00	5.09E-01
Joint	[0.96, 0.96)	3.99E-01	1.93E+00	7.55E-02
Joint	[0.96, 0.98)	5.43E-01	3.18E+00	3.06E-01
Joint	[0.98, 0.98)	7.28E-01	6.10E+00	8.18E-02
Joint	[0.98, 1.00]	7.61E-01	5.88E+00	3.34E-01
Joint	(1.00, 1.00]	1.00E-05	1.00E+01	-9.99E-01
Joint	(1.00, 1.02]	1.00E-05	1.00E+01	-9.99E-01
Joint	(1.02, 1.02]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.02, 1.04]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.04, 1.04]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.04, 1.06]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.06, 1.06]	7.10E-01	5.88E+00	-3.68E-01
Joint	(1.06, 1.08]	4.13E-01	1.47E+00	-2.70E-01
Joint	(1.08, 1.08]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.08, 1.10]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.10, 1.10]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.10, 1.12]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.12, 1.14]	4.08E-01	8.97E-01	-2.21E-01
Joint	(1.14, 1.16]	3.48E-02	1.21E+00	9.01E-01
Joint	(1.16, 1.18]	2.55E-01	1.37E+00	5.06E-02
Joint	(1.18, 1.20]	4.65E-01	5.84E-01	-1.01E-01
Joint	(1.20, 1.28]	2.07E-01	2.20E+00	-1.98E-01
Wing	(1.28,$+\infty$)	1.08E-01	3.90E-01	-6.23E-01

Wing	(-∞, 0.74)	(0.12, 0.19]	5.86E-01	8.29E-04	-5.05E-02
Joint	[0.74, 0.81)		1.17E-01	1.50E+00	-5.84E-01
Joint	[0.81, 0.88)		6.63E-01	3.90E+00	6.73E-01
Joint	[0.88, 0.90)		5.45E-01	4.50E+00	5.50E-01
Joint	[0.90, 0.90)		4.09E-01	5.86E+00	2.86E-01
Joint	[0.90, 0.92)		5.71E-01	3.86E+00	6.09E-01
Joint	[0.92, 0.94)		2.34E-01	8.16E-01	-1.66E-01
Joint	[0.94, 0.94)		3.94E-04	2.15E-01	9.87E-01
Joint	[0.94, 0.98)		3.94E-04	2.15E-01	9.87E-01
Joint	[0.98, 0.98)		1.79E-01	4.44E+00	4.69E-01
Joint	[0.98, 1.00]		1.94E-01	3.85E+00	9.95E-01
Joint	(1.00, 1.02]		4.92E-01	3.31E+00	6.17E-01
Joint	(1.02, 1.04]		1.00E-01	3.50E-01	-2.50E-01
Joint	(1.04, 1.06]		1.00E-01	3.50E-01	-2.50E-01
Joint	(1.06, 1.06]		5.31E-01	5.97E+00	-2.65E-01
Joint	(1.06, 1.08]		7.03E-01	5.91E+00	-5.31E-01
Joint	(1.08, 1.10]		2.50E-01	1.03E+00	-8.77E-02
Joint	(1.10, 1.12]		8.39E-01	5.62E+00	-7.10E-01
Joint	(1.12, 1.16]		9.26E-01	4.35E+00	-7.99E-01
Wing	(1.16,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.71)	(0.19, 0.44]	3.59E-01	1.32E+00	5.40E-01
Joint	[0.71, 0.83)		2.34E-01	2.28E+00	3.96E-01
Joint	[0.83, 0.85)		1.79E-01	1.29E+00	1.01E-01
Joint	[0.85, 0.89)		3.96E-04	1.58E-01	9.67E-01
Joint	[0.89, 0.91)		3.96E-04	1.58E-01	9.67E-01
Joint	[0.91, 0.93)		2.66E-01	2.89E+00	2.66E-01

Joint	[0.93, 0.97)		3.65E-01	5.78E+00	-3.96E-01
Joint	[0.97, 0.99)		9.81E-01	4.36E+00	-9.17E-02
Joint	[0.99, 1.01]		3.35E-01	4.23E+00	3.70E-02
Joint	(1.01, 1.06]		4.21E-01	2.58E+00	6.06E-02
Joint	(1.06, 1.12]		1.26E+00	6.66E+00	-8.21E-01
Joint	(1.12, 1.22]		4.00E-01	4.66E+00	-4.76E-01
Joint	(1.22, 1.52]		1.01E+00	4.60E+00	-8.78E-01
Wing	(1.52,+∞)		8.67E-02	1.76E-03	-3.40E-01
Wing	(-∞, 0.92)	(0.44, 0.70]	4.94E-01	4.23E-01	-9.98E-01
Joint	[0.92, 1.00)		6.75E-01	5.77E+00	-5.38E-01
Joint	[1.00, 1.02]		1.02E-01	4.81E-01	-2.62E-01
Joint	(1.02, 1.04]		1.80E-01	2.09E+00	3.01E-02
Joint	(1.04, 1.12]		4.20E-01	4.93E+00	-6.52E-01
Wing	(1.12,+∞)		1.39E+00	3.38E-01	-9.99E-01
Wing	(-∞, 0.71)	(0.70, 0.95]	9.61E-02	1.62E+00	-1.13E-01
Joint	[0.71, 0.79)		1.64E-01	1.15E+00	4.39E-01
Joint	[0.79, 0.85)		1.47E-01	4.29E+00	-5.10E-01
Joint	[0.85, 0.92)		9.27E-02	6.14E-01	-3.54E-02
Joint	[0.92, 0.94)		1.24E-01	1.82E+00	-6.11E-02
Joint	[0.94, 0.98)		1.53E-01	1.58E+00	-1.16E-01
Joint	[0.98, 1.02]		3.16E-01	4.79E+00	-5.08E-01
Joint	(1.02, 1.02]		5.92E-01	4.39E+00	-5.43E-01
Joint	(1.02, 1.08]		1.00E-01	3.50E-01	-2.50E-01
Joint	(1.08, 1.16]		1.00E-01	3.50E-01	-2.50E-01
Joint	(1.16, 1.16]		1.00E-01	3.50E-01	-2.50E-01
Joint	(1.16, 1.18]		1.00E-01	3.50E-01	-2.50E-01
Joint	(1.18, 1.22]		1.00E-01	3.50E-01	-2.50E-01

Joint	(1.22, 1.34]		1.00E-01	3.50E-01	-2.50E-01
Wing	(1.34,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.88)	(0.95, 1.45]	1.43E-02	2.86E-01	7.24E-01
Joint	[0.88, 1.01]		6.88E-01	1.96E+00	-3.76E-01
Joint	(1.01, 1.26]		1.00E-01	3.50E-01	-2.50E-01
Wing	(1.26,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.88)	(1.45, 1.95]	6.09E-01	1.68E+00	-5.04E-01
Joint	[0.88, 0.92)		4.11E-01	4.11E+00	-5.65E-01
Joint	[0.92, 0.96)		9.89E-02	1.71E+00	-3.60E-01
Joint	[0.96, 1.01]		6.77E-02	5.31E-01	2.32E-01
Joint	(1.01, 1.05]		8.62E-02	1.29E+00	-7.38E-02
Joint	(1.05, 1.05]		6.81E-01	4.75E+00	-6.47E-01
Joint	(1.05, 1.13]		3.54E-01	3.18E+00	-4.85E-01
Joint	(1.13, 1.13]		7.03E-02	3.75E-01	-2.52E-01
Joint	(1.13, 1.21]		6.46E-02	4.01E-01	5.22E-02
Joint	(1.21, 1.32]		5.90E-02	4.26E-01	3.23E-01
Joint	(1.32, 1.38]		6.68E-02	3.93E-01	5.26E-02
Wing	(1.38,+∞)		2.29E-01	5.48E-05	-1.94E-01
Wing	(-∞, 0.84)	(1.95, 2.45]	5.50E-01	1.17E+00	-3.51E-01
Joint	[0.84, 0.91)		5.46E-01	8.14E-01	-1.90E-01
Joint	[0.91, 1.08]		3.40E-01	2.43E+00	-4.12E-01
Wing	(1.08,+∞)		5.32E-01	2.17E+00	9.90E-01
Wing	(-∞, 0.90)	(2.45,+∞]	6.51E-01	7.09E-01	-6.25E-01
Wing	[0.90,+∞)		4.84E-01	4.70E-01	-9.97E-01

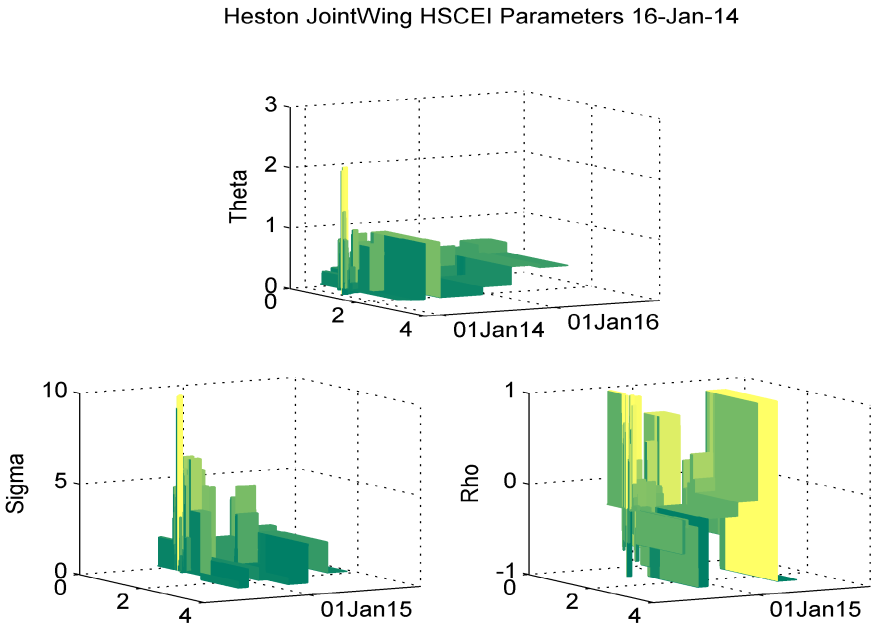


Figure B.18: Heston JointWing HSCEI Parameters

Table B.13: Elapsed Seconds HSCEI Calibration (02-Jan-14∼31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
6.564E+0	1.278E+1	3.145E+1	3.366E+2	7.450E+2	2.081E+3

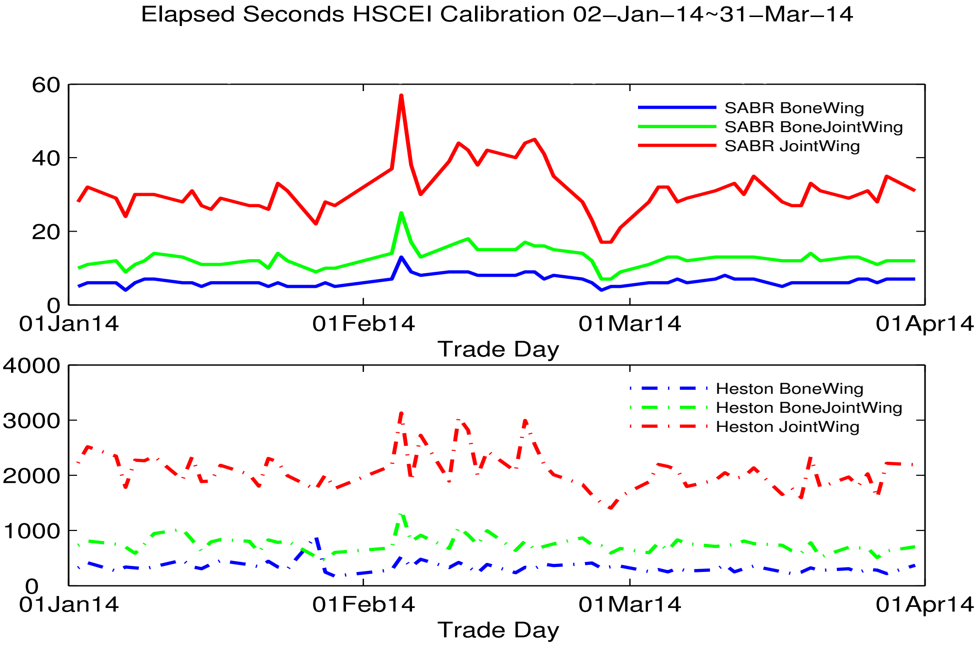


Figure B.19: Elapsed Seconds HSCEI Calibration (02-Jan-14∼31-Mar-14)

Table B.14: SABR BoneWing SnP500 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.94)		6.32E+00	1.75E-01	9.89E-01
Bone	[0.94, 1.62]	(0.00, 0.01]	3.68E+00	8.15E-01	1.00E+01
Wing	(1.62,+∞)		8.97E+00	6.09E-01	5.70E+00
Wing	(-∞, 0.65)		4.91E+00	2.60E-02	2.36E+00
Bone	[0.65, 1.14]	(0.01, 0.10]	4.22E+00	-3.58E-01	3.03E+00
Wing	(1.14,+∞)		7.37E+00	-2.02E-02	3.78E-01
Wing	(-∞, 0.58)		6.55E+00	-1.32E-01	1.66E+00
Bone	[0.58, 1.36]	(0.10, 0.18]	4.77E+00	-2.25E-01	2.28E+00
Wing	(1.36,+∞)		8.85E+00	4.51E-01	7.58E-01
Wing	(-∞, 0.58)		6.48E+00	-1.07E-01	1.32E+00
Bone	[0.58, 1.11]	(0.18, 0.25]	5.26E+00	-5.80E-01	1.39E+00
Wing	(1.11,+∞)		4.72E+00	-8.77E-02	2.93E-01
Wing	(-∞, 0.38)		4.15E+00	1.02E-01	1.52E+00
Bone	[0.38, 1.20]	(0.25, 0.43]	5.76E+00	-5.79E-01	1.06E+00
Wing	(1.20,+∞)		5.26E+00	-7.07E-02	3.02E-01
Wing	(-∞, 0.44)		3.89E+00	1.18E-01	1.31E+00
Bone	[0.44, 1.26]	(0.43, 0.68]	6.18E+00	-6.97E-01	7.33E-01
Wing	(1.26,+∞)		4.69E+00	-9.13E-02	3.11E-01
Wing	(-∞, 0.36)		3.91E+00	1.23E-01	1.06E+00
Bone	[0.36, 1.34]	(0.68, 0.93]	6.38E+00	-5.77E-01	5.88E-01
Wing	(1.34,+∞)		5.31E+00	-5.11E-02	2.97E-01
Wing	(-∞, 0.36)		3.93E+00	1.27E-01	1.04E+00
Bone	[0.36, 1.21]	(0.93, 1.00]	6.49E+00	-7.43E-01	5.36E-01

Wing	(1.21,+∞)		4.92E+00	-7.51E-02	3.03E-01
Wing	(-∞, 0.36)		3.86E+00	1.28E-01	9.32E-01
Bone	[0.36, 1.38]	(1.00, 1.42]	6.92E+00	-6.33E-01	4.37E-01
Wing	(1.38,+∞)		5.24E+00	-5.86E-02	3.07E-01
Wing	(-∞, 0.36)		3.83E+00	1.30E-01	8.60E-01
Bone	[0.36, 1.66]	(1.42, 1.92]	7.21E+00	-5.97E-01	3.73E-01
Wing	(1.66,+∞)		4.85E+00	-5.30E-02	3.38E-01
Wing	(-∞, 0.36)		3.79E+00	1.32E-01	7.62E-01
Bone	[0.36, 1.68]	(1.92,+∞]	7.92E+00	-6.07E-01	2.50E-01
Wing	(1.68,+∞)		5.13E+00	-3.53E-02	3.36E-01

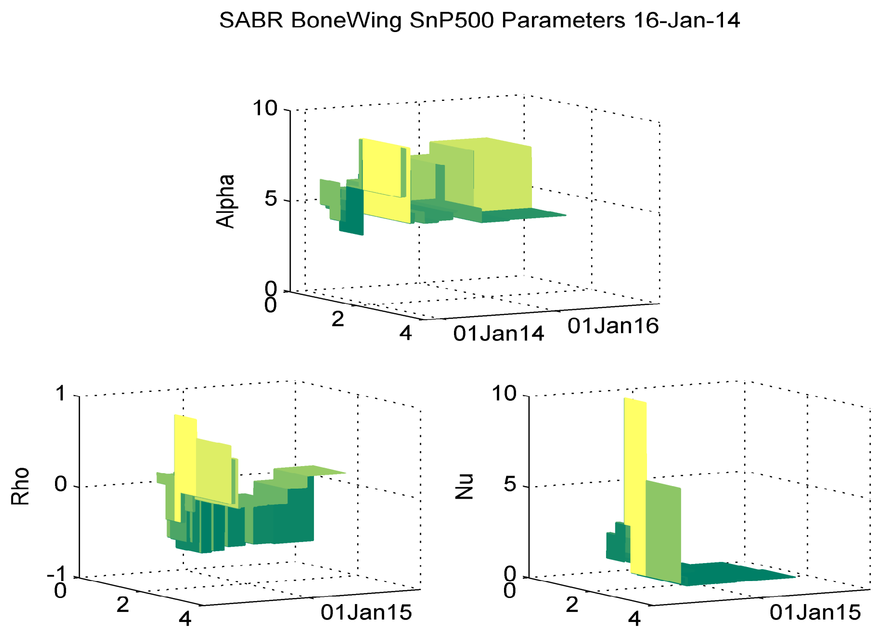


Figure B.20: SABR BoneWing SnP500 Parameters

Table B.15: Heston BoneWing SnP500 Parameters (κ = 5.12E-01, v0 = 2.11E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.94)		5.93E-02	2.43E-01	-7.69E-01
Bone	[0.94, 1.62]	(0.00, 0.01]	5.93E-02	2.43E-01	-7.69E-01
Wing	(1.62,+∞)		2.53E-01	3.00E-03	-7.06E-01
Wing	(-∞, 0.65)		1.04E-05	8.48E-01	9.99E-01
Bone	[0.65, 1.14]	(0.01, 0.10]	2.88E-01	1.02E+00	-7.71E-01
Wing	(1.14,+∞)		5.93E-02	2.43E-01	-7.69E-01
Wing	(-∞, 0.58)		7.93E-02	1.71E-01	-7.43E-01
Bone	[0.58, 1.36]	(0.10, 0.18]	1.09E-03	1.13E+00	-5.66E-01
Wing	(1.36,+∞)		6.05E-02	1.22E-03	-7.48E-01
Wing	(-∞, 0.58)		3.34E-01	3.41E-04	-2.75E-01
Bone	[0.58, 1.11]	(0.18, 0.25]	6.57E-03	7.97E-01	-6.76E-01
Wing	(1.11,+∞)		5.93E-02	2.43E-01	-7.69E-01
Wing	(-∞, 0.38)		6.68E-02	9.25E-01	-4.78E-01
Bone	[0.38, 1.20]	(0.25, 0.43]	1.06E-03	8.47E-01	-6.42E-01
Wing	(1.20,+∞)		5.93E-02	2.43E-01	-7.69E-01
Wing	(-∞, 0.44)		2.52E-01	1.11E+00	4.29E-01
Bone	[0.44, 1.26]	(0.43, 0.68]	7.71E-02	5.33E-01	-7.65E-01
Wing	(1.26,+∞)		6.47E-02	1.22E-03	-8.68E-01
Wing	(-∞, 0.36)		6.26E-01	5.90E-01	4.62E-01
Bone	[0.36, 1.34]	(0.68, 0.93]	9.25E-02	4.07E-01	-8.06E-01
Wing	(1.34,+∞)		5.93E-02	2.43E-01	-7.69E-01
Wing	(-∞, 0.36)		5.76E-01	5.78E-01	4.24E-01
Bone	[0.36, 1.21]	(0.93, 1.00]	6.56E-02	5.12E-01	-6.38E-01

Wing	(1.21,+∞)		1.05E-01	3.48E-01	-9.98E-01
Wing	(-∞, 0.36)		3.54E-01	4.59E-01	3.26E-01
Bone	[0.36, 1.38]	(1.00, 1.42]	8.08E-02	3.94E-01	-7.58E-01
Wing	(1.38,+∞)		8.99E-02	1.98E+00	1.83E-01
Wing	(-∞, 0.36)		2.39E-01	3.96E-01	2.29E-01
Bone	[0.36, 1.66]	(1.42, 1.92]	6.73E-02	4.05E-01	-7.04E-01
Wing	(1.66,+∞)		1.00E-05	4.92E-01	-1.85E-01
Wing	(-∞, 0.36)		1.49E-01	3.43E-01	4.60E-02
Bone	[0.36, 1.68]	(1.92,+∞]	6.71E-02	4.20E-01	-6.39E-01
Wing	(1.68,+∞)		5.93E-02	2.43E-01	-7.69E-01

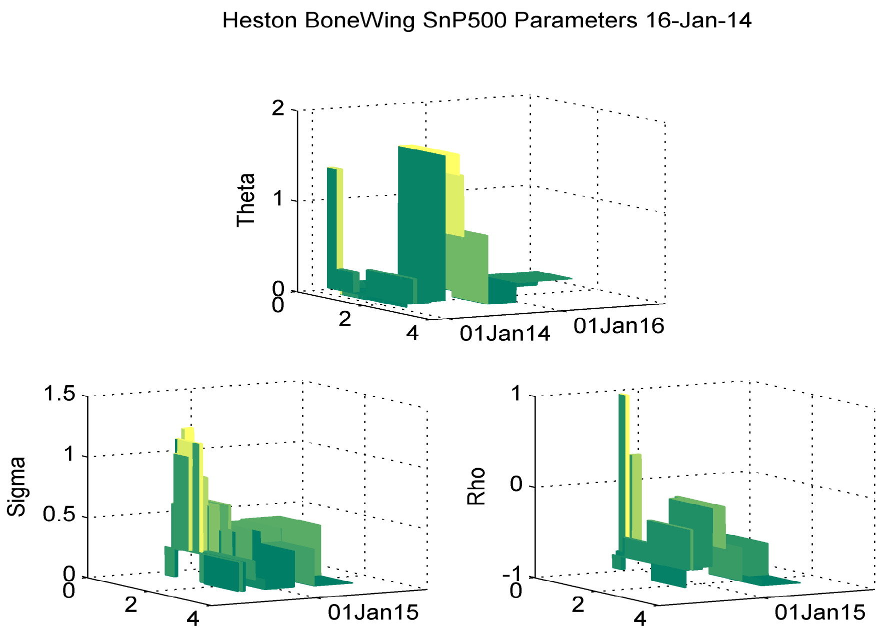


Figure B.21: Heston BoneWing SnP500 Parameters

Table B.16: SABR BoneJointWing SnP500 Parameters ($\beta = 5.00\text{E-}01$, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.94)		9.99E+00	-7.43E-01	4.70E-01
Bone	[0.94, 0.95]		6.33E+00	-3.78E-01	9.12E+00
Joint	(0.95, 0.95)		5.34E+00	-5.80E-02	7.87E+00
Bone	[0.95, 1.05]	(0.00, 0.01]	3.60E+00	5.98E-01	9.92E+00
Joint	(1.05, 1.05)		8.18E+00	4.68E-01	8.73E+00
Bone	[1.05, 1.62]		1.00E+01	8.15E-01	1.00E+01
Wing	(1.62,+∞)		8.82E+00	5.79E-01	6.28E+00
Wing	(-∞, 0.65)		4.90E+00	2.73E-02	2.34E+00
Bone	[0.65, 0.95]		4.66E+00	4.55E-02	3.17E+00
Joint	(0.95, 0.95)		6.70E+00	4.44E-01	2.01E+00
Bone	[0.95, 1.05]	(0.01, 0.10]	4.41E+00	-6.68E-01	2.58E+00
Joint	(1.05, 1.05)		1.54E+00	-3.40E-02	3.73E+00
Bone	[1.05, 1.14]		7.51E-01	-1.98E-01	5.22E+00
Wing	(1.14,+∞)		7.82E+00	1.71E-02	3.67E-01
Wing	(-∞, 0.58)		4.66E+00	5.23E-02	2.00E+00
Bone	[0.58, 0.95]		5.01E+00	3.06E-02	2.21E+00
Joint	(0.95, 0.95)		6.66E+00	5.18E-01	2.01E+00
Bone	[0.95, 1.05]	(0.10, 0.18]	5.03E+00	-6.48E-01	1.78E+00
Joint	(1.05, 1.05)		3.64E+00	-2.89E-01	1.53E+00
Bone	[1.05, 1.36]		1.40E+00	-6.52E-01	4.84E+00
Wing	(1.36,+∞)		8.85E+00	4.57E-01	7.98E-01
Wing	(-∞, 0.58)	(0.18, 0.25]	6.49E+00	-1.06E-01	1.32E+00
Bone	[0.58, 0.95]		6.12E+00	9.05E-04	1.57E+00

Joint	(0.95, 0.96)		2.21E+00	-2.34E-01	4.55E+00
Bone	[0.96, 1.05]		5.28E+00	-6.53E-01	1.48E+00
Joint	(1.05, 1.05)		4.68E+00	-4.08E-01	1.08E+00
Bone	[1.05, 1.11]		4.91E+00	-5.66E-01	1.28E+00
Wing	(1.11,+∞)		4.53E+00	-2.25E-01	3.09E-01
Wing	(-∞, 0.38)		4.25E+00	9.28E-02	1.51E+00
Bone	[0.38, 0.94]		6.29E+00	-5.78E-02	1.21E+00
Joint	(0.94, 0.95)		5.33E+00	3.30E-01	2.77E+00
Bone	[0.95, 1.05]	(0.25, 0.43]	5.80E+00	-6.70E-01	1.07E+00
Joint	(1.05, 1.06)		5.85E+00	-6.62E-01	1.59E+00
Bone	[1.06, 1.20]		5.39E+00	-5.83E-01	1.01E+00
Wing	(1.20,+∞)		4.90E+00	-7.55E-02	3.09E-01
Wing	(-∞, 0.44)		3.91E+00	1.25E-01	1.32E+00
Bone	[0.44, 0.94]		6.66E+00	-6.93E-02	9.05E-01
Joint	(0.94, 0.96)		4.87E+00	3.17E-01	2.76E+00
Bone	[0.96, 1.04]	(0.43, 0.68]	6.14E+00	-6.70E-01	8.79E-01
Joint	(1.04, 1.05)		6.67E+00	-7.39E-01	1.96E+00
Bone	[1.05, 1.26]		6.00E+00	-6.96E-01	8.81E-01
Wing	(1.26,+∞)		4.73E+00	-7.13E-02	3.08E-01
Wing	(-∞, 0.36)		3.94E+00	1.27E-01	1.07E+00
Bone	[0.36, 0.95]		6.66E+00	-3.24E-02	7.41E-01
Joint	(0.95, 0.96)		5.56E+00	5.04E-01	2.73E+00
Bone	[0.96, 1.04]	(0.68, 0.93]	6.41E+00	-6.89E-01	7.35E-01
Joint	(1.04, 1.06)		6.06E+00	-4.49E-01	6.76E-01
Bone	[1.06, 1.34]		6.45E+00	-7.28E-01	7.61E-01
Wing	(1.34,+∞)		4.77E+00	-7.22E-02	3.08E-01
Wing	(-∞, 0.36)		3.93E+00	1.23E-01	1.06E+00

(0.93, 1.00]

Bone	[0.36, 0.95]		7.04E+00	-4.42E-02	6.98E-01
Joint	(0.95, 0.96)		4.18E+00	3.44E-01	2.96E+00
Bone	[0.96, 1.04]		6.44E+00	-6.83E-01	7.02E-01
Joint	(1.04, 1.06)		6.26E+00	-5.50E-01	1.12E+00
Bone	[1.06, 1.21]		6.57E+00	-7.26E-01	7.70E-01
Wing	(1.21,+∞)		4.86E+00	-7.60E-02	3.04E-01
Wing	(-∞, 0.36)		3.86E+00	1.28E-01	9.35E-01
Bone	[0.36, 0.94]		7.20E+00	-7.07E-02	5.65E-01
Joint	(0.94, 0.95)		5.57E+00	5.37E-01	2.60E+00
Bone	[0.95, 1.05]	(1.00, 1.42]	6.82E+00	-5.80E-01	6.76E-01
Joint	(1.05, 1.06)		6.33E+00	-5.39E-01	1.55E+00
Bone	[1.06, 1.38]		6.79E+00	-6.75E-01	5.75E-01
Wing	(1.38,+∞)		5.01E+00	-6.42E-02	3.11E-01
Wing	(-∞, 0.36)		3.83E+00	1.30E-01	8.60E-01
Bone	[0.36, 0.94]		7.56E+00	-1.52E-02	4.96E-01
Joint	(0.94, 0.96)		4.74E+00	5.35E-01	2.81E+00
Bone	[0.96, 1.04]	(1.42, 1.92]	7.18E+00	-6.04E-01	5.03E-01
Joint	(1.04, 1.05)		6.84E+00	-4.29E-01	7.15E-01
Bone	[1.05, 1.66]		7.18E+00	-6.43E-01	5.37E-01
Wing	(1.66,+∞)		5.24E+00	-3.07E-02	3.29E-01
Wing	(-∞, 0.36)		3.81E+00	1.30E-01	7.79E-01
Bone	[0.36, 0.95]		7.47E+00	1.60E-02	4.57E-01
Joint	(0.95, 0.96)		5.54E+00	6.40E-01	2.60E+00
Bone	[0.96, 1.05]	(1.92,+∞]	7.77E+00	-1.19E-01	2.77E-01
Joint	(1.05, 1.06)		4.47E+00	-1.82E-01	1.71E+00
Bone	[1.06, 1.68]		7.93E+00	-6.19E-01	4.46E-01
Wing	(1.68,+∞)		6.57E+00	-3.38E-01	3.49E-01

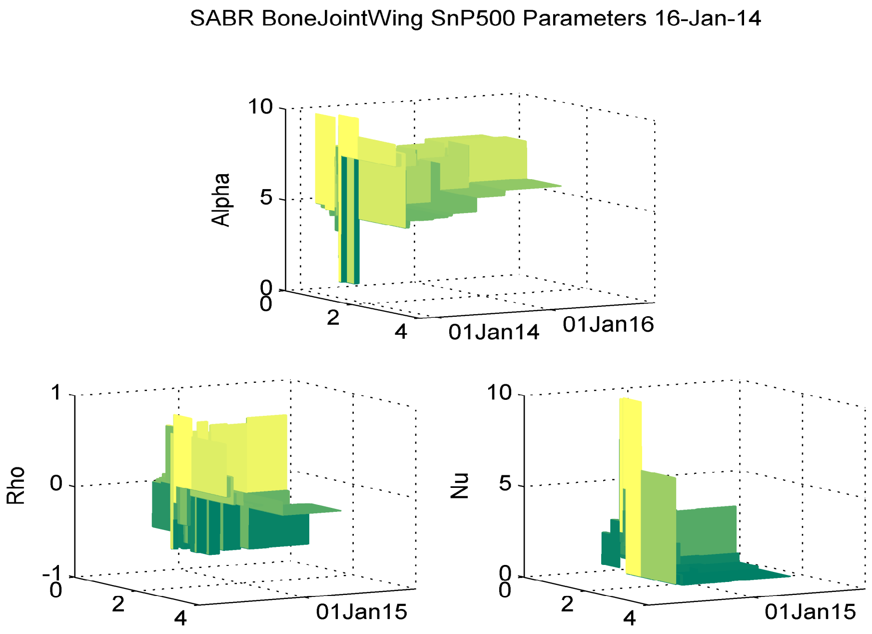


Figure B.22: SABR BoneJointWing SnP500 Parameters

Table B.17: Heston BoneJointWing SnP500 Parameters (κ = 5.12E-01, v0 = 2.11E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.94)		5.93E-02	2.43E-01	-7.69E-01
Bone	[0.94, 0.95]		1.23E+00	4.30E+00	-3.97E-01
Joint	(0.95, 0.95)		3.01E+00	2.53E+00	-4.83E-01
Bone	[0.95, 1.05]	(0.00, 0.01]	4.51E-01	1.77E+00	-3.96E-01
Joint	(1.05, 1.05)		5.93E-02	2.43E-01	-7.69E-01
Bone	[1.05, 1.62]		5.93E-02	2.43E-01	-7.69E-01

Wing	(1.62,+∞)		2.53E-01	3.00E-03	-7.06E-01
Wing	(-∞, 0.65)		1.04E-05	8.48E-01	9.99E-01
Bone	[0.65, 0.95]		6.56E-02	2.46E-01	-7.67E-01
Joint	(0.95, 0.95)		2.13E+00	5.30E+00	6.79E-01
Bone	[0.95, 1.05]	(0.01, 0.10]	6.13E-01	6.61E+00	-8.37E-01
Joint	(1.05, 1.05)		1.25E-01	1.12E+00	-8.44E-01
Bone	[1.05, 1.14]		1.14E-01	7.85E+00	-8.73E-01
Wing	(1.14,+∞)		5.93E-02	2.43E-01	-7.69E-01
Wing	(-∞, 0.58)		7.93E-02	1.71E-01	-7.43E-01
Bone	[0.58, 0.95]		2.37E-02	2.00E+00	9.48E-02
Joint	(0.95, 0.95)		9.73E-03	4.17E-02	2.85E-01
Bone	[0.95, 1.05]	(0.10, 0.18]	6.89E-01	5.96E+00	-7.17E-01
Joint	(1.05, 1.05)		1.24E+00	6.18E+00	-8.35E-01
Bone	[1.05, 1.36]		1.29E-01	1.42E+00	-6.59E-01
Wing	(1.36,+∞)		6.05E-02	1.22E-03	-7.48E-01
Wing	(-∞, 0.58)		3.34E-01	3.41E-04	-2.75E-01
Bone	[0.58, 0.95]		1.24E-02	8.05E-01	-6.54E-01
Joint	(0.95, 0.96)		9.53E-02	1.88E-01	3.96E-01
Bone	[0.96, 1.05]	(0.18, 0.25]	4.45E-01	3.73E+00	-6.31E-01
Joint	(1.05, 1.05)		2.60E-01	1.90E+00	-6.91E-01
Bone	[1.05, 1.11]		3.36E-01	1.23E+00	-8.90E-01
Wing	(1.11,+∞)		5.93E-02	2.43E-01	-7.69E-01
Wing	(-∞, 0.38)		2.25E+00	3.80E-01	4.13E-01
Bone	[0.38, 0.94]		1.10E-02	1.56E+00	3.66E-02
Joint	(0.94, 0.95)		5.06E-01	1.73E+00	9.99E-01
Bone	[0.95, 1.05]	(0.25, 0.43]	6.27E-01	5.67E+00	-5.26E-01
Joint	(1.05, 1.06)		9.23E-01	5.67E+00	-7.62E-01

Bone	[1.06, 1.20]		4.47E-01	1.64E+00	-9.11E-01
Wing	(1.20,+∞)		5.93E-02	2.43E-01	-7.69E-01
Wing	(-∞, 0.44)		6.17E-02	1.28E+00	2.15E-01
Bone	[0.44, 0.94]		2.18E-01	1.18E+00	3.27E-01
Joint	(0.94, 0.96)		7.85E-01	7.12E+00	-7.26E-01
Bone	[0.96, 1.04]	(0.43, 0.68]	3.39E-01	3.21E+00	-4.11E-01
Joint	(1.04, 1.05)		7.38E-01	5.49E+00	-6.74E-01
Bone	[1.05, 1.26]		1.43E-01	6.72E-01	-8.62E-01
Wing	(1.26,+∞)		6.47E-02	1.22E-03	-8.68E-01
Wing	(-∞, 0.36)		6.26E-01	5.91E-01	4.63E-01
Bone	[0.36, 0.95]		1.68E-01	5.91E-01	-1.36E-01
Joint	(0.95, 0.96)		6.04E-01	5.36E+00	-6.53E-01
Bone	[0.96, 1.04]	(0.68, 0.93]	7.53E-01	6.48E+00	-4.65E-01
Joint	(1.04, 1.06)		1.13E-01	1.06E+00	-3.21E-01
Bone	[1.06, 1.34]		1.85E-01	9.30E-01	-8.82E-01
Wing	(1.34,+∞)		5.93E-02	2.43E-01	-7.69E-01
Wing	(-∞, 0.36)		5.96E-01	5.90E-01	4.00E-01
Bone	[0.36, 0.95]		1.96E-01	8.22E-01	2.16E-01
Joint	(0.95, 0.96)		4.66E-01	4.63E+00	-6.31E-01
Bone	[0.96, 1.04]	(0.93, 1.00]	6.91E-01	6.06E+00	-4.62E-01
Joint	(1.04, 1.06)		1.44E-01	1.56E+00	-1.58E-01
Bone	[1.06, 1.21]		9.98E-01	5.24E+00	-9.17E-01
Wing	(1.21,+∞)		1.05E-01	3.48E-01	-9.98E-01
Wing	(-∞, 0.36)		3.54E-01	4.62E-01	3.27E-01
Bone	[0.36, 0.94]		1.48E-01	6.23E-01	3.37E-03
Joint	(0.94, 0.95)		5.78E-01	4.71E+00	-4.84E-01
Bone	[0.95, 1.05]	(1.00, 1.42]	8.40E-01	6.82E+00	-4.07E-01

Joint	(1.05, 1.06)		5.15E-01	3.36E+00	2.98E-02
Bone	[1.06, 1.38]		9.84E-02	4.96E-01	-8.19E-01
Wing	(1.38,+∞)		8.99E-02	1.98E+00	1.83E-01
Wing	(-∞, 0.36)		2.40E-01	3.95E-01	2.26E-01
Bone	[0.36, 0.94]		9.97E-02	5.82E-01	-1.16E-01
Joint	(0.94, 0.96)		2.98E-01	2.39E+00	-4.81E-01
Bone	[0.96, 1.04]	(1.42, 1.92]	7.43E-02	4.49E-01	-5.35E-01
Joint	(1.04, 1.05)		3.28E-01	2.70E+00	-2.86E-01
Bone	[1.05, 1.66]		1.01E-01	5.51E-01	-7.81E-01
Wing	(1.66,+∞)		1.00E-05	4.92E-01	-1.85E-01
Wing	(-∞, 0.36)		1.50E-01	3.43E-01	4.23E-02
Bone	[0.36, 0.95]		9.36E-02	4.84E-01	-2.07E-01
Joint	(0.95, 0.96)		1.76E-01	1.48E+00	-5.71E-01
Bone	[0.96, 1.05]	(1.92,+∞]	6.78E-01	6.65E+00	-3.57E-01
Joint	(1.05, 1.06)		1.66E+00	6.20E+00	-2.29E-01
Bone	[1.06, 1.68]		1.53E-01	9.33E-01	-7.96E-01
Wing	(1.68,+∞)		5.93E-02	2.43E-01	-7.69E-01

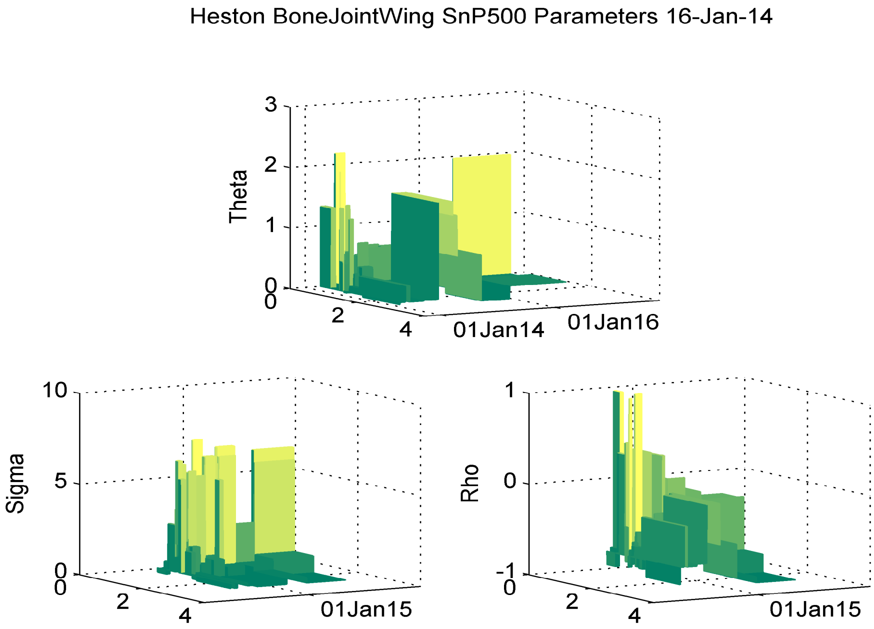


Figure B.23: Heston BoneJointWing SnP500 Parameters

Table B.18: SABR JointWing SnP500 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.94)		9.80E+00	-4.45E-01	6.42E-01
Joint	[0.94, 0.94)		7.17E+00	-3.63E-01	8.44E+00
Joint	[0.94, 0.94)		7.03E+00	-3.41E-01	8.39E+00
Joint	[0.94, 0.95)		6.82E+00	-3.33E-01	8.34E+00
Joint	[0.95, 0.95)		8.71E+00	-3.48E-01	5.70E+00
Joint	[0.95, 0.95)		6.77E+00	-3.04E-01	8.33E+00
Joint	[0.95, 0.96)		6.35E+00	-3.02E-01	8.24E+00
Joint	[0.96, 0.96)		6.08E+00	-2.39E-01	8.18E+00
Joint	[0.96, 0.96)		7.13E+00	-4.79E-02	7.36E+00
Joint	[0.96, 0.96)		6.03E+00	-1.61E-01	8.22E+00
Joint	[0.96, 0.97)		6.15E+00	-4.88E-02	8.31E+00
Joint	[0.97, 0.97)		6.10E+00	5.67E-03	8.07E+00
Joint	[0.97, 0.97)		5.89E+00	5.74E-02	8.01E+00
Joint	[0.97, 0.97)		4.97E+00	-1.81E-01	8.08E+00
Joint	[0.97, 0.98)		4.79E+00	-1.69E-01	8.08E+00
Joint	[0.98, 0.98)		4.43E+00	-2.31E-01	8.00E+00
Joint	[0.98, 0.98)		4.28E+00	-1.96E-01	8.10E+00
Joint	[0.98, 0.99)		4.05E+00	-2.90E-01	8.06E+00
Joint	[0.99, 0.99)		3.70E+00	-3.04E-01	8.22E+00
Joint	[0.99, 0.99)		4.86E+00	3.55E-01	4.30E+00
Joint	[0.99, 0.99)		4.32E+00	-1.28E-01	5.36E+00
Joint	[0.99, 1.00)		3.61E+00	-5.38E-01	8.35E+00
Joint	[1.00, 1.00)		3.89E+00	-9.12E-02	5.11E+00

Joint	[1.00, 1.00]	3.62E+00	-5.52E-01	8.49E+00
Joint	(1.00, 1.00]	3.74E+00	-5.74E-01	6.69E+00
Joint	(1.00, 1.01]	3.40E+00	2.73E-02	5.10E+00
Joint	(1.01, 1.01]	3.42E+00	1.81E-02	4.94E+00
Joint	(1.01, 1.01]	2.77E+00	3.45E-01	8.14E+00
Joint	(1.01, 1.01]	3.04E+00	2.69E-01	8.13E+00
Joint	(1.01, 1.02]	3.49E+00	1.20E-01	7.98E+00
Joint	(1.02, 1.02]	3.68E+00	1.23E-01	7.95E+00
Joint	(1.02, 1.02]	3.95E+00	1.41E-01	7.95E+00
Joint	(1.02, 1.03]	4.16E+00	2.08E-01	8.00E+00
Joint	(1.03, 1.03]	6.34E+00	2.36E-01	2.39E+00
Joint	(1.03, 1.03]	5.78E+00	3.21E-01	8.20E+00
Joint	(1.03, 1.03]	7.31E+00	4.14E-01	8.46E+00
Joint	(1.03, 1.04]	7.59E+00	4.57E-01	8.53E+00
Joint	(1.04, 1.04]	7.94E+00	4.59E-01	8.62E+00
Joint	(1.04, 1.04]	8.20E+00	4.88E-01	8.71E+00
Joint	(1.04, 1.04]	8.47E+00	5.05E-01	8.81E+00
Joint	(1.04, 1.05]	8.70E+00	5.27E-01	8.92E+00
Joint	(1.05, 1.05]	8.90E+00	5.52E-01	9.04E+00
Joint	(1.05, 1.05]	9.08E+00	5.78E-01	9.16E+00
Joint	(1.05, 1.06]	9.25E+00	6.06E-01	9.28E+00
Joint	(1.06, 1.06]	9.40E+00	6.35E-01	9.40E+00
Joint	(1.06, 1.06]	9.53E+00	6.64E-01	9.51E+00
Joint	(1.06, 1.06]	9.65E+00	6.95E-01	9.63E+00
Joint	(1.06, 1.07]	9.77E+00	7.27E-01	9.74E+00
Joint	(1.07, 1.07]	9.84E+00	7.65E-01	9.89E+00
Joint	(1.07, 1.07]	9.97E+00	8.00E-01	9.96E+00

Joint	(1.07, 1.08]	1.00E+01	8.16E-01	1.00E+01
Joint	(1.08, 1.08]	1.00E+01	8.16E-01	1.00E+01
Joint	(1.08, 1.11]	1.00E+01	8.16E-01	1.00E+01
Joint	(1.11, 1.14]	1.00E+01	8.16E-01	1.00E+01
Joint	(1.14, 1.22]	1.00E+01	8.16E-01	1.00E+01
Joint	(1.22, 1.35]	1.00E+01	8.16E-01	1.00E+01
Joint	(1.35, 1.62]	1.00E+01	8.15E-01	1.00E+01
Wing	(1.62,+∞)	9.80E+00	7.73E-01	7.95E+00
Wing	(-∞, 0.65)	4.88E+00	2.98E-02	2.29E+00
Joint	[0.65, 0.66)	4.25E+00	8.43E-02	3.14E+00
Joint	[0.66, 0.68)	4.44E+00	6.82E-02	3.08E+00
Joint	[0.68, 0.68)	4.38E+00	7.53E-02	3.12E+00
Joint	[0.68, 0.68)	4.51E+00	6.30E-02	3.26E+00
Joint	[0.68, 0.69)	4.49E+00	6.15E-02	3.44E+00
Joint	[0.69, 0.69)	4.45E+00	6.61E-02	3.33E+00
Joint	[0.69, 0.69)	4.42E+00	6.99E-02	3.18E+00
Joint	[0.69, 0.69)	4.44E+00	6.83E-02	3.19E+00
Joint	[0.69, 0.70)	4.52E+00	6.03E-02	3.32E+00
Joint	[0.70, 0.70)	4.48E+00	6.25E-02	3.44E+00
Joint	[0.70, 0.70)	4.47E+00	6.37E-02	3.43E+00
Joint	[0.70, 0.70)	4.43E+00	6.80E-02	3.33E+00
Joint	[0.70, 0.71)	4.50E+00	6.26E-02	3.29E+00
Joint	[0.71, 0.71)	4.53E+00	5.79E-02	3.40E+00
Joint	[0.71, 0.71)	4.48E+00	6.19E-02	3.41E+00
Joint	[0.71, 0.72)	4.45E+00	6.70E-02	3.40E+00
Joint	[0.72, 0.72)	4.41E+00	7.40E-02	3.30E+00
Joint	[0.72, 0.72)	4.44E+00	6.74E-02	3.27E+00

Joint	[0.72, 0.72)	4.43E+00	6.93E-02	3.38E+00
Joint	[0.72, 0.73)	4.37E+00	7.32E-02	3.25E+00
Joint	[0.73, 0.73)	4.44E+00	6.85E-02	3.21E+00
Joint	[0.73, 0.73)	4.37E+00	7.48E-02	3.26E+00
Joint	[0.73, 0.73)	4.44E+00	6.65E-02	3.23E+00
Joint	[0.73, 0.74)	4.41E+00	7.08E-02	3.37E+00
Joint	[0.74, 0.74)	4.40E+00	7.08E-02	3.36E+00
Joint	[0.74, 0.74)	4.39E+00	7.01E-02	3.35E+00
Joint	[0.74, 0.75)	4.34E+00	8.00E-02	3.21E+00
Joint	[0.75, 0.75)	4.41E+00	7.21E-02	3.17E+00
Joint	[0.75, 0.75)	4.37E+00	7.35E-02	3.33E+00
Joint	[0.75, 0.75)	4.46E+00	6.74E-02	3.30E+00
Joint	[0.75, 0.76)	4.38E+00	7.55E-02	3.34E+00
Joint	[0.76, 0.76)	4.34E+00	7.67E-02	3.27E+00
Joint	[0.76, 0.76)	4.41E+00	7.10E-02	3.22E+00
Joint	[0.76, 0.76)	4.35E+00	7.55E-02	3.31E+00
Joint	[0.76, 0.77)	4.43E+00	6.91E-02	3.28E+00
Joint	[0.77, 0.77)	4.34E+00	7.55E-02	3.32E+00
Joint	[0.77, 0.77)	4.34E+00	7.70E-02	3.32E+00
Joint	[0.77, 0.78)	4.33E+00	7.76E-02	3.31E+00
Joint	[0.78, 0.78)	4.42E+00	7.27E-02	3.27E+00
Joint	[0.78, 0.78)	4.32E+00	7.83E-02	3.31E+00
Joint	[0.78, 0.78)	4.31E+00	7.90E-02	3.31E+00
Joint	[0.78, 0.79)	4.28E+00	8.25E-02	3.24E+00
Joint	[0.79, 0.79)	6.69E+00	1.95E-01	2.69E+00
Joint	[0.79, 0.79)	4.29E+00	8.02E-02	3.30E+00
Joint	[0.79, 0.79)	4.39E+00	7.26E-02	3.26E+00

Joint	[0.79, 0.80)	4.29E+00	8.14E-02	3.31E+00
Joint	[0.80, 0.80)	4.28E+00	8.34E-02	3.30E+00
Joint	[0.80, 0.80)	4.34E+00	4.70E-02	3.22E+00
Joint	[0.80, 0.81)	4.26E+00	8.25E-02	3.30E+00
Joint	[0.81, 0.81)	4.36E+00	7.52E-02	3.25E+00
Joint	[0.81, 0.81)	4.36E+00	7.62E-02	3.26E+00
Joint	[0.81, 0.81)	4.36E+00	7.64E-02	3.27E+00
Joint	[0.81, 0.82)	4.25E+00	8.35E-02	3.32E+00
Joint	[0.82, 0.82)	5.84E+00	1.51E-01	2.89E+00
Joint	[0.82, 0.82)	6.81E+00	2.50E-01	2.78E+00
Joint	[0.82, 0.82)	5.91E+00	1.64E-01	2.89E+00
Joint	[0.82, 0.83)	4.11E+00	2.87E-01	3.71E+00
Joint	[0.83, 0.83)	5.83E+00	1.80E-01	2.87E+00
Joint	[0.83, 0.83)	5.40E+00	2.46E-01	3.25E+00
Joint	[0.83, 0.83)	5.11E+00	2.36E-01	3.33E+00
Joint	[0.83, 0.84)	5.92E+00	1.68E-01	2.91E+00
Joint	[0.84, 0.84)	4.05E+00	2.83E-01	3.76E+00
Joint	[0.84, 0.84)	6.81E+00	2.77E-01	2.73E+00
Joint	[0.84, 0.85)	5.70E+00	1.47E-01	2.94E+00
Joint	[0.85, 0.85)	6.12E+00	1.34E-01	2.76E+00
Joint	[0.85, 0.85)	3.98E+00	3.00E-01	3.96E+00
Joint	[0.85, 0.85)	5.96E+00	1.23E-01	2.84E+00
Joint	[0.85, 0.86)	4.24E+00	2.63E-01	3.84E+00
Joint	[0.86, 0.86)	5.91E+00	2.51E-01	3.07E+00
Joint	[0.86, 0.86)	6.03E+00	2.33E-01	2.98E+00
Joint	[0.86, 0.86)	6.46E+00	1.12E-01	2.53E+00
Joint	[0.86, 0.87)	3.60E+00	2.38E-01	4.00E+00

Joint	[0.87, 0.87)	6.52E+00	1.70E-01	2.57E+00
Joint	[0.87, 0.87)	5.50E+00	3.68E-01	3.59E+00
Joint	[0.87, 0.88)	6.32E+00	1.65E-01	2.68E+00
Joint	[0.88, 0.88)	3.35E+00	2.01E-01	4.12E+00
Joint	[0.88, 0.88)	6.82E+00	2.73E-01	2.69E+00
Joint	[0.88, 0.88)	3.88E+00	1.66E-01	3.83E+00
Joint	[0.88, 0.89)	3.67E+00	1.66E-01	3.94E+00
Joint	[0.89, 0.89)	3.45E+00	1.41E-01	4.00E+00
Joint	[0.89, 0.89)	6.98E+00	1.95E-01	2.31E+00
Joint	[0.89, 0.89)	4.47E+00	1.39E-01	3.49E+00
Joint	[0.89, 0.90)	5.47E+00	1.84E-01	3.08E+00
Joint	[0.90, 0.90)	4.03E+00	1.33E-01	3.72E+00
Joint	[0.90, 0.90)	5.92E+00	2.17E-01	2.91E+00
Joint	[0.90, 0.91)	3.02E+00	1.47E-01	4.34E+00
Joint	[0.91, 0.91)	6.57E+00	7.81E-02	2.06E+00
Joint	[0.91, 0.91)	2.92E+00	7.13E-02	4.36E+00
Joint	[0.91, 0.91)	6.52E+00	2.12E-01	2.46E+00
Joint	[0.91, 0.92)	3.51E+00	1.07E-02	3.90E+00
Joint	[0.92, 0.92)	6.03E+00	2.37E-01	2.84E+00
Joint	[0.92, 0.92)	2.86E+00	2.17E-02	4.41E+00
Joint	[0.92, 0.92)	6.40E+00	3.04E-01	2.73E+00
Joint	[0.92, 0.93)	7.02E+00	2.46E-01	1.83E+00
Joint	[0.93, 0.93)	2.82E+00	4.27E-02	4.63E+00
Joint	[0.93, 0.93)	4.33E+00	1.10E-02	3.44E+00
Joint	[0.93, 0.94)	5.65E+00	2.10E-01	2.95E+00
Joint	[0.94, 0.94)	6.47E+00	3.41E-01	2.67E+00
Joint	[0.94, 0.94)	2.91E+00	-1.64E-01	4.39E+00

Joint	[0.94, 0.94)	4.53E+00	-2.75E-02	3.24E+00
Joint	[0.94, 0.95)	5.34E+00	1.61E-01	2.99E+00
Joint	[0.95, 0.95)	5.37E+00	1.43E-01	2.89E+00
Joint	[0.95, 0.95)	3.86E+00	-7.61E-02	3.84E+00
Joint	[0.95, 0.95)	4.73E+00	1.57E-02	3.17E+00
Joint	[0.95, 0.96)	4.78E+00	4.49E-02	3.19E+00
Joint	[0.96, 0.96)	4.77E+00	3.40E-02	3.17E+00
Joint	[0.96, 0.96)	4.74E+00	3.18E-02	3.21E+00
Joint	[0.96, 0.97)	5.10E+00	1.29E-01	3.00E+00
Joint	[0.97, 0.97)	4.44E+00	-1.77E-02	3.52E+00
Joint	[0.97, 0.97)	4.34E+00	-9.97E-02	3.46E+00
Joint	[0.97, 0.97)	4.71E+00	3.38E-02	3.22E+00
Joint	[0.97, 0.98)	4.08E+00	-6.73E-02	4.02E+00
Joint	[0.98, 0.98)	4.86E+00	1.48E-01	3.19E+00
Joint	[0.98, 0.98)	4.42E+00	-8.86E-02	3.42E+00
Joint	[0.98, 0.98)	4.27E+00	-1.01E-01	3.72E+00
Joint	[0.98, 0.99)	4.24E+00	-1.77E-02	4.01E+00
Joint	[0.99, 0.99)	4.17E+00	-7.99E-02	4.05E+00
Joint	[0.99, 0.99)	4.14E+00	1.95E-02	4.38E+00
Joint	[0.99, 0.99)	3.80E+00	-1.01E-02	5.33E+00
Joint	[0.99, 1.00)	3.82E+00	-1.29E-01	5.12E+00
Joint	[1.00, 1.00]	4.45E+00	-7.92E-01	4.81E+00
Joint	(1.00, 1.00]	4.00E+00	-3.55E-01	4.26E+00
Joint	(1.00, 1.01]	3.96E+00	-3.11E-01	4.27E+00
Joint	(1.01, 1.01]	4.24E+00	-5.71E-01	5.05E+00
Joint	(1.01, 1.01]	4.08E+00	-4.44E-01	4.19E+00
Joint	(1.01, 1.01]	4.36E+00	-5.71E-01	3.94E+00

Joint	(1.01, 1.02]	4.37E+00	-5.70E-01	3.22E+00
Joint	(1.02, 1.02]	4.28E+00	-5.25E-01	2.74E+00
Joint	(1.02, 1.02]	4.18E+00	-4.87E-01	2.77E+00
Joint	(1.02, 1.02]	3.96E+00	-3.79E-01	2.41E+00
Joint	(1.02, 1.03]	3.92E+00	-3.81E-01	2.24E+00
Joint	(1.03, 1.03]	3.86E+00	-3.63E-01	2.10E+00
Joint	(1.03, 1.03]	3.82E+00	-3.47E-01	1.99E+00
Joint	(1.03, 1.04]	3.81E+00	-3.54E-01	1.90E+00
Joint	(1.04, 1.04]	3.78E+00	-3.51E-01	1.82E+00
Joint	(1.04, 1.04]	3.76E+00	-3.21E-01	1.54E+00
Joint	(1.04, 1.04]	2.92E+00	-3.45E-02	2.32E+00
Joint	(1.04, 1.05]	3.77E+00	-3.41E-01	1.63E+00
Joint	(1.05, 1.05]	3.69E+00	-3.62E-01	1.73E+00
Joint	(1.05, 1.05]	1.39E+00	6.36E-02	3.75E+00
Joint	(1.05, 1.05]	3.81E+00	-3.97E-01	1.87E+00
Joint	(1.05, 1.06]	1.50E+00	4.52E-03	3.64E+00
Joint	(1.06, 1.06]	3.92E+00	-3.03E-01	1.53E+00
Joint	(1.06, 1.06]	2.61E+00	-6.79E-02	2.44E+00
Joint	(1.06, 1.07]	1.80E+00	-1.16E-01	3.40E+00
Joint	(1.07, 1.07]	3.84E+00	-2.66E-01	1.49E+00
Joint	(1.07, 1.07]	1.84E+00	-1.60E-01	3.11E+00
Joint	(1.07, 1.07]	1.76E+00	-1.49E-02	3.21E+00
Joint	(1.07, 1.08]	2.10E+00	6.64E-02	2.90E+00
Joint	(1.08, 1.08]	2.56E+00	2.25E-01	2.30E+00
Joint	(1.08, 1.08]	2.29E+00	4.34E-02	2.80E+00
Joint	(1.08, 1.08]	4.30E+00	-2.15E-01	1.45E+00
Joint	(1.08, 1.10]	1.81E+00	3.13E-01	2.58E+00

Joint	(1.10, 1.11]	1.50E+00	-2.58E-01	4.46E+00
Joint	(1.11, 1.13]	1.94E+00	-4.15E-01	5.33E+00
Joint	(1.13, 1.14]	5.70E+00	-6.68E-02	2.49E+00
Wing	(1.14,+∞)	8.69E+00	1.27E-01	3.44E-01
Wing	(-∞, 0.58)	4.65E+00	5.37E-02	1.97E+00
Joint	[0.58, 0.62)	3.82E+00	1.32E-01	2.47E+00
Joint	[0.62, 0.63)	4.27E+00	8.93E-02	2.33E+00
Joint	[0.63, 0.64)	4.12E+00	1.03E-01	2.39E+00
Joint	[0.64, 0.64)	4.27E+00	7.20E-02	2.41E+00
Joint	[0.64, 0.65)	4.07E+00	4.18E-01	2.98E+00
Joint	[0.65, 0.66)	7.10E+00	2.87E-01	2.15E+00
Joint	[0.66, 0.66)	6.67E+00	4.35E-01	2.47E+00
Joint	[0.66, 0.67)	6.04E+00	3.26E-01	2.38E+00
Joint	[0.67, 0.67)	4.82E+00	3.84E-01	2.79E+00
Joint	[0.67, 0.68)	4.93E+00	3.65E-01	2.73E+00
Joint	[0.68, 0.68)	4.73E+00	3.83E-01	2.74E+00
Joint	[0.68, 0.68)	5.60E+00	3.38E-01	2.48E+00
Joint	[0.68, 0.68)	4.86E+00	3.73E-01	2.76E+00
Joint	[0.68, 0.69)	6.68E+00	4.26E-01	2.50E+00
Joint	[0.69, 0.69)	4.73E+00	3.71E-01	2.77E+00
Joint	[0.69, 0.69)	4.62E+00	3.85E-01	2.75E+00
Joint	[0.69, 0.69)	5.57E+00	3.57E-01	2.49E+00
Joint	[0.69, 0.70)	4.83E+00	3.77E-01	2.76E+00
Joint	[0.70, 0.70)	4.70E+00	3.93E-01	2.81E+00
Joint	[0.70, 0.70)	6.42E+00	4.01E-01	2.48E+00
Joint	[0.70, 0.71)	4.93E+00	3.98E-01	2.72E+00
Joint	[0.71, 0.71)	5.59E+00	3.59E-01	2.49E+00

Joint	[0.71, 0.71)	5.46E+00	3.21E-01	2.52E+00
Joint	[0.71, 0.71)	4.77E+00	3.86E-01	2.79E+00
Joint	[0.71, 0.72)	4.64E+00	3.87E-01	2.82E+00
Joint	[0.72, 0.72)	5.48E+00	3.42E-01	2.55E+00
Joint	[0.72, 0.72)	4.60E+00	3.67E-01	2.81E+00
Joint	[0.72, 0.72)	4.44E+00	3.79E-01	2.87E+00
Joint	[0.72, 0.73)	5.40E+00	3.38E-01	2.58E+00
Joint	[0.73, 0.73)	4.60E+00	3.69E-01	2.83E+00
Joint	[0.73, 0.73)	5.55E+00	2.92E-01	2.48E+00
Joint	[0.73, 0.74)	4.64E+00	3.75E-01	2.85E+00
Joint	[0.74, 0.74)	5.69E+00	3.30E-01	2.52E+00
Joint	[0.74, 0.74)	5.61E+00	3.28E-01	2.54E+00
Joint	[0.74, 0.74)	5.64E+00	3.24E-01	2.53E+00
Joint	[0.74, 0.75)	6.68E+00	4.20E-01	2.51E+00
Joint	[0.75, 0.75)	5.59E+00	3.04E-01	2.53E+00
Joint	[0.75, 0.75)	4.54E+00	3.81E-01	2.93E+00
Joint	[0.75, 0.75)	5.47E+00	3.10E-01	2.57E+00
Joint	[0.75, 0.76)	5.62E+00	3.17E-01	2.55E+00
Joint	[0.76, 0.76)	4.21E+00	3.75E-01	2.95E+00
Joint	[0.76, 0.76)	5.65E+00	3.60E-01	2.57E+00
Joint	[0.76, 0.77)	4.47E+00	3.73E-01	2.96E+00
Joint	[0.77, 0.77)	5.51E+00	3.52E-01	2.66E+00
Joint	[0.77, 0.77)	5.51E+00	3.28E-01	2.62E+00
Joint	[0.77, 0.77)	4.28E+00	3.66E-01	3.02E+00
Joint	[0.77, 0.78)	5.70E+00	3.44E-01	2.61E+00
Joint	[0.78, 0.78)	4.17E+00	3.74E-01	3.03E+00
Joint	[0.78, 0.78)	5.55E+00	3.62E-01	2.58E+00

Joint	[0.78, 0.78)	5.53E+00	2.91E-01	2.50E+00
Joint	[0.78, 0.79)	4.28E+00	3.83E-01	3.04E+00
Joint	[0.79, 0.79)	4.19E+00	4.05E-01	3.06E+00
Joint	[0.79, 0.79)	5.47E+00	2.89E-01	2.47E+00
Joint	[0.79, 0.80)	5.14E+00	3.26E-01	2.64E+00
Joint	[0.80, 0.80)	4.96E+00	4.17E-01	2.90E+00
Joint	[0.80, 0.80)	5.40E+00	3.53E-01	2.64E+00
Joint	[0.80, 0.80)	5.31E+00	3.15E-01	2.59E+00
Joint	[0.80, 0.81)	5.67E+00	3.33E-01	2.54E+00
Joint	[0.81, 0.81)	5.73E+00	3.09E-01	2.47E+00
Joint	[0.81, 0.81)	5.71E+00	3.30E-01	2.58E+00
Joint	[0.81, 0.81)	5.94E+00	3.80E-01	2.71E+00
Joint	[0.81, 0.82)	4.01E+00	3.56E-01	3.23E+00
Joint	[0.82, 0.82)	5.49E+00	3.01E-01	2.64E+00
Joint	[0.82, 0.82)	3.99E+00	3.95E-01	3.28E+00
Joint	[0.82, 0.83)	5.67E+00	3.10E-01	2.53E+00
Joint	[0.83, 0.83)	5.52E+00	3.19E-01	2.59E+00
Joint	[0.83, 0.83)	6.23E+00	3.09E-01	2.40E+00
Joint	[0.83, 0.83)	5.91E+00	2.71E-01	2.47E+00
Joint	[0.83, 0.84)	4.68E+00	3.17E-01	2.97E+00
Joint	[0.84, 0.84)	6.64E+00	2.83E-01	2.25E+00
Joint	[0.84, 0.84)	4.35E+00	3.28E-01	3.11E+00
Joint	[0.84, 0.84)	5.54E+00	3.54E-01	2.78E+00
Joint	[0.84, 0.85)	5.43E+00	3.64E-01	2.85E+00
Joint	[0.85, 0.85)	6.54E+00	2.39E-01	2.19E+00
Joint	[0.85, 0.85)	5.56E+00	2.53E-01	2.57E+00
Joint	[0.85, 0.85)	5.32E+00	3.80E-01	2.96E+00

Joint	[0.85, 0.86)	5.49E+00	3.19E-01	2.75E+00
Joint	[0.86, 0.86)	5.53E+00	3.88E-01	2.92E+00
Joint	[0.86, 0.86)	5.71E+00	2.62E-01	2.54E+00
Joint	[0.86, 0.87)	6.35E+00	2.68E-01	2.32E+00
Joint	[0.87, 0.87)	6.41E+00	2.64E-01	2.30E+00
Joint	[0.87, 0.87)	3.28E+00	3.30E-01	3.70E+00
Joint	[0.87, 0.87)	7.02E+00	2.16E-01	1.88E+00
Joint	[0.87, 0.88)	4.87E+00	3.41E-01	3.10E+00
Joint	[0.88, 0.88)	3.24E+00	3.08E-01	3.69E+00
Joint	[0.88, 0.88)	7.37E+00	2.70E-01	1.74E+00
Joint	[0.88, 0.88)	3.16E+00	2.81E-01	3.70E+00
Joint	[0.88, 0.89)	7.34E+00	2.87E-01	1.76E+00
Joint	[0.89, 0.89)	3.13E+00	2.59E-01	3.72E+00
Joint	[0.89, 0.89)	7.26E+00	2.75E-01	1.71E+00
Joint	[0.89, 0.90)	6.29E+00	1.10E-01	1.94E+00
Joint	[0.90, 0.90)	3.04E+00	2.16E-01	3.79E+00
Joint	[0.90, 0.90)	3.48E+00	2.64E-01	3.67E+00
Joint	[0.90, 0.90)	6.06E+00	3.25E-01	2.56E+00
Joint	[0.90, 0.91)	5.18E+00	3.12E-01	2.99E+00
Joint	[0.91, 0.91)	6.38E+00	1.24E-01	1.79E+00
Joint	[0.91, 0.91)	6.06E+00	3.72E-01	2.76E+00
Joint	[0.91, 0.91)	5.95E+00	3.57E-01	2.77E+00
Joint	[0.91, 0.92)	6.09E+00	3.25E-01	2.56E+00
Joint	[0.92, 0.92)	5.43E+00	3.02E-01	2.91E+00
Joint	[0.92, 0.92)	6.05E+00	3.67E-01	2.74E+00
Joint	[0.92, 0.93)	5.98E+00	3.59E-01	2.75E+00
Joint	[0.93, 0.93)	5.71E+00	3.25E-01	2.82E+00

Joint	[0.93, 0.93)	6.27E+00	3.95E-01	2.67E+00
Joint	[0.93, 0.93)	3.42E+00	9.42E-02	3.73E+00
Joint	[0.93, 0.94)	6.79E+00	4.46E-01	2.25E+00
Joint	[0.94, 0.94)	5.78E+00	3.32E-01	2.79E+00
Joint	[0.94, 0.94)	5.52E+00	2.92E-01	2.87E+00
Joint	[0.94, 0.94)	5.82E+00	3.41E-01	2.78E+00
Joint	[0.94, 0.95)	5.29E+00	2.51E-01	2.94E+00
Joint	[0.95, 0.95)	5.97E+00	3.73E-01	2.73E+00
Joint	[0.95, 0.95)	5.02E+00	1.74E-01	2.98E+00
Joint	[0.95, 0.96)	5.42E+00	2.66E-01	2.87E+00
Joint	[0.96, 0.96)	5.17E+00	2.13E-01	2.96E+00
Joint	[0.96, 0.96)	4.94E+00	1.59E-01	3.04E+00
Joint	[0.96, 0.96)	4.88E+00	1.47E-01	3.07E+00
Joint	[0.96, 0.97)	5.08E+00	2.03E-01	3.01E+00
Joint	[0.97, 0.97)	4.85E+00	1.25E-01	3.06E+00
Joint	[0.97, 0.97)	4.87E+00	1.48E-01	3.10E+00
Joint	[0.97, 0.97)	4.96E+00	1.81E-01	3.09E+00
Joint	[0.97, 0.98)	4.68E+00	5.54E-02	3.16E+00
Joint	[0.98, 0.98)	4.49E+00	2.13E-02	3.38E+00
Joint	[0.98, 0.98)	4.58E+00	6.95E-02	3.34E+00
Joint	[0.98, 0.99)	4.52E+00	5.16E-02	3.40E+00
Joint	[0.99, 0.99)	4.37E+00	2.97E-02	3.65E+00
Joint	[0.99, 0.99)	4.31E+00	3.92E-02	3.80E+00
Joint	[0.99, 0.99)	4.16E+00	-1.20E-02	4.04E+00
Joint	[0.99, 1.00)	3.89E+00	-2.95E-02	4.62E+00
Joint	[1.00, 1.00)	3.87E+00	-9.64E-02	4.63E+00
Joint	[1.00, 1.00]	5.03E+00	-7.59E-01	3.18E+00

Joint	(1.00, 1.00]	4.40E+00	-3.77E-01	3.48E+00
Joint	(1.00, 1.01]	4.17E+00	-3.61E-01	4.19E+00
Joint	(1.01, 1.01]	4.49E+00	-3.74E-01	3.09E+00
Joint	(1.01, 1.01]	4.47E+00	-5.32E-01	4.57E+00
Joint	(1.01, 1.01]	4.53E+00	-5.02E-01	3.86E+00
Joint	(1.01, 1.02]	4.76E+00	-5.50E-01	3.13E+00
Joint	(1.02, 1.02]	4.79E+00	-5.57E-01	2.97E+00
Joint	(1.02, 1.02]	4.63E+00	-4.48E-01	2.25E+00
Joint	(1.02, 1.03]	4.45E+00	-3.91E-01	2.45E+00
Joint	(1.03, 1.03]	4.51E+00	-3.97E-01	1.94E+00
Joint	(1.03, 1.03]	4.49E+00	-4.13E-01	1.97E+00
Joint	(1.03, 1.03]	4.44E+00	-4.25E-01	2.09E+00
Joint	(1.03, 1.04]	4.39E+00	-4.12E-01	1.98E+00
Joint	(1.04, 1.04]	4.44E+00	-4.45E-01	1.90E+00
Joint	(1.04, 1.04]	4.71E+00	-5.69E-01	1.97E+00
Joint	(1.04, 1.04]	4.25E+00	-3.94E-01	1.82E+00
Joint	(1.04, 1.05]	4.09E+00	-3.11E-01	1.68E+00
Joint	(1.05, 1.05]	4.14E+00	-3.70E-01	1.76E+00
Joint	(1.05, 1.05]	3.96E+00	-2.89E-01	1.60E+00
Joint	(1.05, 1.06]	3.95E+00	-3.04E-01	1.55E+00
Joint	(1.06, 1.06]	3.88E+00	-2.84E-01	1.52E+00
Joint	(1.06, 1.06]	3.86E+00	-2.78E-01	1.50E+00
Joint	(1.06, 1.06]	3.88E+00	-2.95E-01	1.46E+00
Joint	(1.06, 1.07]	3.75E+00	-2.53E-01	1.46E+00
Joint	(1.07, 1.07]	2.17E+00	-3.02E-01	3.03E+00
Joint	(1.07, 1.07]	3.81E+00	-2.74E-01	1.40E+00
Joint	(1.07, 1.07]	3.33E+00	-5.87E-01	2.96E+00

Joint	(1.07, 1.08]	3.87E+00	-1.92E-01	1.21E+00
Joint	(1.08, 1.08]	3.75E+00	-2.64E-01	1.37E+00
Joint	(1.08, 1.08]	1.35E+00	8.06E-02	2.78E+00
Joint	(1.08, 1.09]	3.72E+00	-2.64E-01	1.50E+00
Joint	(1.09, 1.09]	3.70E+00	-1.16E-01	1.20E+00
Joint	(1.09, 1.09]	2.98E+00	-3.00E-02	1.63E+00
Joint	(1.09, 1.09]	1.81E+00	3.07E-01	2.47E+00
Joint	(1.09, 1.10]	4.15E+00	-1.00E-01	1.62E+00
Joint	(1.10, 1.10]	4.53E+00	-8.38E-02	9.00E-01
Joint	(1.10, 1.11]	2.03E+00	-4.11E-01	3.29E+00
Joint	(1.11, 1.11]	6.14E+00	-5.82E-01	9.42E-01
Joint	(1.11, 1.13]	2.69E+00	-6.95E-01	3.97E+00
Joint	(1.13, 1.14]	2.28E+00	-6.20E-01	4.20E+00
Joint	(1.14, 1.15]	5.52E+00	-2.53E-01	1.77E+00
Joint	(1.15, 1.17]	2.43E+00	-4.91E-01	3.68E+00
Joint	(1.17, 1.19]	4.15E+00	-2.68E-01	2.55E+00
Joint	(1.19, 1.22]	2.01E+00	-2.55E-01	3.45E+00
Joint	(1.22, 1.25]	5.68E+00	-1.10E-01	2.07E+00
Joint	(1.25, 1.28]	2.03E+00	-2.33E-01	3.50E+00
Joint	(1.28, 1.30]	7.02E+00	1.76E-01	1.61E+00
Joint	(1.30, 1.33]	2.61E+00	-2.25E-01	3.33E+00
Joint	(1.33, 1.36]	8.41E+00	3.02E-01	1.39E+00
Wing	(1.36,+∞)	8.85E+00	4.63E-01	8.08E-01
Wing	(-∞, 0.58)	4.55E+00	6.54E-02	1.70E+00
Joint	[0.58, 0.61)	3.60E+00	6.14E-01	2.83E+00
Joint	[0.61, 0.62)	6.66E+00	5.16E-01	2.08E+00
Joint	[0.62, 0.63)	4.48E+00	5.67E-01	2.54E+00

Joint	[0.63, 0.64)	5.83E+00	5.51E-01	2.28E+00
Joint	[0.64, 0.64)	5.92E+00	5.89E-01	2.44E+00
Joint	[0.64, 0.64)	4.81E+00	5.17E-01	2.54E+00
Joint	[0.64, 0.65)	4.63E+00	5.39E-01	2.62E+00
Joint	[0.65, 0.65)	6.73E+00	6.21E-01	2.48E+00
Joint	[0.65, 0.66)	5.92E+00	4.86E-01	2.30E+00
Joint	[0.66, 0.66)	6.03E+00	5.19E-01	2.36E+00
Joint	[0.66, 0.67)	4.80E+00	5.42E-01	2.65E+00
Joint	[0.67, 0.67)	4.95E+00	5.24E-01	2.57E+00
Joint	[0.67, 0.67)	5.88E+00	4.76E-01	2.30E+00
Joint	[0.67, 0.68)	5.96E+00	5.16E-01	2.39E+00
Joint	[0.68, 0.68)	5.87E+00	4.71E-01	2.32E+00
Joint	[0.68, 0.69)	5.84E+00	4.76E-01	2.35E+00
Joint	[0.69, 0.69)	5.61E+00	5.20E-01	2.50E+00
Joint	[0.69, 0.70)	5.38E+00	4.69E-01	2.44E+00
Joint	[0.70, 0.70)	6.02E+00	4.82E-01	2.35E+00
Joint	[0.70, 0.71)	5.88E+00	4.14E-01	2.25E+00
Joint	[0.71, 0.71)	6.18E+00	4.49E-01	2.27E+00
Joint	[0.71, 0.72)	6.06E+00	4.04E-01	2.21E+00
Joint	[0.72, 0.72)	6.73E+00	5.97E-01	2.52E+00
Joint	[0.72, 0.72)	5.51E+00	4.86E-01	2.46E+00
Joint	[0.72, 0.73)	6.19E+00	3.38E-01	2.08E+00
Joint	[0.73, 0.73)	4.01E+00	5.12E-01	2.88E+00
Joint	[0.73, 0.74)	6.09E+00	3.29E-01	2.07E+00
Joint	[0.74, 0.74)	6.68E+00	3.48E-01	2.02E+00
Joint	[0.74, 0.75)	4.43E+00	4.17E-01	2.60E+00
Joint	[0.75, 0.75)	5.26E+00	4.51E-01	2.45E+00

Joint	[0.75, 0.76)	6.22E+00	4.68E-01	2.29E+00
Joint	[0.76, 0.76)	6.31E+00	4.17E-01	2.17E+00
Joint	[0.76, 0.77)	6.49E+00	3.53E-01	2.04E+00
Joint	[0.77, 0.77)	3.90E+00	4.53E-01	2.88E+00
Joint	[0.77, 0.77)	5.45E+00	3.86E-01	2.37E+00
Joint	[0.77, 0.78)	4.36E+00	4.34E-01	2.74E+00
Joint	[0.78, 0.78)	6.38E+00	3.61E-01	2.07E+00
Joint	[0.78, 0.79)	7.31E+00	2.39E-01	1.63E+00
Joint	[0.79, 0.79)	6.53E+00	4.40E-01	2.24E+00
Joint	[0.79, 0.80)	7.87E+00	2.31E-01	1.48E+00
Joint	[0.80, 0.80)	5.58E+00	3.44E-01	2.34E+00
Joint	[0.80, 0.80)	4.28E+00	4.47E-01	2.91E+00
Joint	[0.80, 0.81)	6.91E+00	3.51E-01	1.99E+00
Joint	[0.81, 0.81)	7.37E+00	3.98E-01	1.98E+00
Joint	[0.81, 0.82)	6.90E+00	3.77E-01	2.06E+00
Joint	[0.82, 0.83)	6.68E+00	4.03E-01	2.18E+00
Joint	[0.83, 0.83)	5.19E+00	3.23E-01	2.44E+00
Joint	[0.83, 0.83)	5.88E+00	3.09E-01	2.22E+00
Joint	[0.83, 0.84)	6.20E+00	4.45E-01	2.46E+00
Joint	[0.84, 0.84)	7.09E+00	3.63E-01	1.96E+00
Joint	[0.84, 0.85)	6.64E+00	3.41E-01	2.06E+00
Joint	[0.85, 0.85)	6.64E+00	4.18E-01	2.26E+00
Joint	[0.85, 0.86)	5.61E+00	2.94E-01	2.32E+00
Joint	[0.86, 0.86)	5.59E+00	3.15E-01	2.38E+00
Joint	[0.86, 0.86)	6.14E+00	3.26E-01	2.22E+00
Joint	[0.86, 0.87)	5.68E+00	4.17E-01	2.63E+00
Joint	[0.87, 0.87)	6.92E+00	3.82E-01	2.04E+00

Joint	[0.87, 0.88)	6.83E+00	2.76E-01	1.78E+00
Joint	[0.88, 0.88)	5.72E+00	2.82E-01	2.31E+00
Joint	[0.88, 0.89)	4.73E+00	3.96E-01	3.01E+00
Joint	[0.89, 0.89)	7.10E+00	3.63E-01	1.85E+00
Joint	[0.89, 0.90)	6.66E+00	3.95E-01	2.21E+00
Joint	[0.90, 0.90)	6.30E+00	4.36E-01	2.55E+00
Joint	[0.90, 0.91)	6.64E+00	4.18E-01	2.29E+00
Joint	[0.91, 0.91)	5.84E+00	3.52E-01	2.52E+00
Joint	[0.91, 0.91)	6.84E+00	4.65E-01	2.37E+00
Joint	[0.91, 0.92)	6.36E+00	3.93E-01	2.38E+00
Joint	[0.92, 0.92)	5.46E+00	3.70E-01	2.85E+00
Joint	[0.92, 0.93)	6.44E+00	3.89E-01	2.24E+00
Joint	[0.93, 0.93)	6.13E+00	3.87E-01	2.50E+00
Joint	[0.93, 0.94)	6.07E+00	4.18E-01	2.70E+00
Joint	[0.94, 0.94)	5.69E+00	3.24E-01	2.62E+00
Joint	[0.94, 0.95)	5.61E+00	3.20E-01	2.67E+00
Joint	[0.95, 0.95)	5.60E+00	2.95E-01	2.59E+00
Joint	[0.95, 0.96)	5.37E+00	3.08E-01	2.87E+00
Joint	[0.96, 0.96)	5.35E+00	3.04E-01	2.88E+00
Joint	[0.96, 0.96)	5.28E+00	2.93E-01	2.92E+00
Joint	[0.96, 0.97)	4.97E+00	1.91E-01	2.93E+00
Joint	[0.97, 0.97)	4.98E+00	2.19E-01	3.01E+00
Joint	[0.97, 0.98)	4.84E+00	1.10E-01	2.89E+00
Joint	[0.98, 0.98)	4.64E+00	1.11E-01	3.18E+00
Joint	[0.98, 0.99)	4.48E+00	1.05E-03	3.21E+00
Joint	[0.99, 0.99)	4.15E+00	1.31E-02	3.80E+00
Joint	[0.99, 0.99)	4.08E+00	2.17E-02	3.93E+00

Joint	[0.99, 1.00)	4.65E+00	-2.33E-01	2.78E+00
Joint	[1.00, 1.00]	5.33E+00	-7.69E-01	2.02E+00
Joint	(1.00, 1.01]	4.53E+00	-3.57E-01	3.16E+00
Joint	(1.01, 1.02]	4.99E+00	-4.91E-01	2.24E+00
Joint	(1.02, 1.02]	4.92E+00	-5.09E-01	2.76E+00
Joint	(1.02, 1.02]	4.91E+00	-4.99E-01	2.66E+00
Joint	(1.02, 1.03]	5.15E+00	-5.93E-01	2.83E+00
Joint	(1.03, 1.03]	5.17E+00	-5.92E-01	1.92E+00
Joint	(1.03, 1.04]	5.15E+00	-5.96E-01	1.85E+00
Joint	(1.04, 1.04]	4.80E+00	-4.36E-01	1.20E+00
Joint	(1.04, 1.05]	4.48E+00	-3.17E-01	1.56E+00
Joint	(1.05, 1.05]	4.42E+00	-2.99E-01	1.52E+00
Joint	(1.05, 1.05]	4.70E+00	-4.39E-01	1.15E+00
Joint	(1.05, 1.06]	4.08E+00	-1.15E-01	1.24E+00
Joint	(1.06, 1.06]	4.60E+00	-4.30E-01	1.12E+00
Joint	(1.06, 1.07]	4.36E+00	-3.33E-01	1.15E+00
Joint	(1.07, 1.07]	3.99E+00	-2.78E-01	1.47E+00
Joint	(1.07, 1.08]	3.98E+00	-2.75E-01	1.45E+00
Joint	(1.08, 1.08]	4.51E+00	-4.03E-01	1.05E+00
Joint	(1.08, 1.09]	4.50E+00	-4.12E-01	1.12E+00
Joint	(1.09, 1.09]	4.71E+00	-4.92E-01	1.14E+00
Joint	(1.09, 1.10]	4.72E+00	-4.84E-01	1.07E+00
Joint	(1.10, 1.10]	3.76E+00	-1.89E-01	1.22E+00
Joint	(1.10, 1.10]	3.61E+00	-4.60E-01	1.91E+00
Joint	(1.10, 1.11]	5.35E+00	-6.96E-01	1.74E+00
Joint	(1.11, 1.11]	4.36E+00	-3.19E-01	9.41E-01
Wing	(1.11,+∞)	4.51E+00	-2.34E-01	3.10E-01

Wing	(-∞, 0.38)	4.15E+00	1.03E-01	1.51E+00
Joint	[0.38, 0.44)	4.22E+00	9.51E-02	1.54E+00
Joint	[0.44, 0.46)	4.17E+00	1.00E-01	1.60E+00
Joint	[0.46, 0.49)	4.05E+00	1.12E-01	1.63E+00
Joint	[0.49, 0.54)	4.18E+00	9.96E-02	1.62E+00
Joint	[0.54, 0.56)	3.92E+00	1.22E-01	1.69E+00
Joint	[0.56, 0.56)	3.98E+00	1.19E-01	1.66E+00
Joint	[0.56, 0.57)	4.04E+00	1.11E-01	1.66E+00
Joint	[0.57, 0.59)	3.89E+00	1.24E-01	1.71E+00
Joint	[0.59, 0.60)	4.17E+00	1.05E-01	1.66E+00
Joint	[0.60, 0.61)	4.14E+00	6.29E-01	2.41E+00
Joint	[0.61, 0.63)	4.07E+00	6.46E-01	2.44E+00
Joint	[0.63, 0.64)	6.33E+00	5.26E-01	1.81E+00
Joint	[0.64, 0.65)	6.68E+00	6.91E-01	2.24E+00
Joint	[0.65, 0.67)	7.05E+00	4.53E-01	1.67E+00
Joint	[0.67, 0.68)	7.52E+00	5.47E-01	1.81E+00
Joint	[0.68, 0.69)	5.95E+00	5.82E-01	2.16E+00
Joint	[0.69, 0.71)	8.58E+00	4.52E-01	1.46E+00
Joint	[0.71, 0.72)	3.73E+00	6.16E-01	2.74E+00
Joint	[0.72, 0.74)	9.14E+00	4.89E-01	1.38E+00
Joint	[0.74, 0.75)	7.96E+00	2.71E-01	1.24E+00
Joint	[0.75, 0.76)	7.99E+00	4.01E-01	1.47E+00
Joint	[0.76, 0.78)	7.72E+00	4.75E-01	1.69E+00
Joint	[0.78, 0.79)	8.22E+00	4.80E-01	1.58E+00
Joint	[0.79, 0.80)	8.00E+00	4.08E-01	1.46E+00
Joint	[0.80, 0.82)	7.84E+00	4.49E-01	1.61E+00

(0.25, 0.43]

Joint	[0.82, 0.83)	7.47E+00	4.19E-01	1.65E+00
Joint	[0.83, 0.84)	7.10E+00	3.96E-01	1.71E+00
Joint	[0.84, 0.85)	7.47E+00	4.69E-01	1.79E+00
Joint	[0.85, 0.86)	7.41E+00	4.92E-01	1.89E+00
Joint	[0.86, 0.87)	6.73E+00	2.98E-01	1.62E+00
Joint	[0.87, 0.89)	6.80E+00	3.39E-01	1.69E+00
Joint	[0.89, 0.90)	6.70E+00	4.08E-01	1.94E+00
Joint	[0.90, 0.91)	6.46E+00	3.83E-01	2.00E+00
Joint	[0.91, 0.93)	6.33E+00	4.06E-01	2.16E+00
Joint	[0.93, 0.94)	6.42E+00	5.25E-01	2.67E+00
Joint	[0.94, 0.95)	5.86E+00	4.49E-01	2.72E+00
Joint	[0.95, 0.97)	5.49E+00	3.90E-01	2.80E+00
Joint	[0.97, 0.98)	4.67E+00	2.32E-01	3.09E+00
Joint	[0.98, 0.99)	4.46E+00	1.87E-02	2.92E+00
Joint	[0.99, 1.01]	5.76E+00	-6.04E-01	1.13E+00
Joint	(1.01, 1.02]	5.31E+00	-4.54E-01	1.99E+00
Joint	(1.02, 1.04]	5.51E+00	-5.60E-01	2.13E+00
Joint	(1.04, 1.05]	5.77E+00	-6.33E-01	1.77E+00
Joint	(1.05, 1.06]	5.90E+00	-6.75E-01	1.61E+00
Joint	(1.06, 1.08]	5.44E+00	-5.74E-01	1.17E+00
Joint	(1.08, 1.09]	5.33E+00	-5.67E-01	9.79E-01
Joint	(1.09, 1.10]	4.99E+00	-4.58E-01	5.38E-01
Joint	(1.10, 1.12]	5.03E+00	-4.77E-01	7.28E-01
Joint	(1.12, 1.13]	4.58E+00	-1.80E-01	4.95E-01
Joint	(1.13, 1.14]	4.24E+00	-9.06E-02	6.88E-01
Joint	(1.14, 1.20]	3.95E+00	-1.07E-02	7.67E-01
Wing	(1.20,+∞)	5.01E+00	-7.96E-02	3.07E-01

Wing	(-∞, 0.44)	3.91E+00	1.24E-01	1.32E+00
Joint	[0.44, 0.46)	4.03E+00	1.14E-01	1.30E+00
Joint	[0.46, 0.48)	3.94E+00	1.22E-01	1.31E+00
Joint	[0.48, 0.49)	4.06E+00	1.09E-01	1.28E+00
Joint	[0.49, 0.51)	3.97E+00	1.19E-01	1.32E+00
Joint	[0.51, 0.52)	4.06E+00	1.10E-01	1.32E+00
Joint	[0.52, 0.53)	3.96E+00	1.20E-01	1.35E+00
Joint	[0.53, 0.55)	4.06E+00	1.13E-01	1.34E+00
Joint	[0.55, 0.56)	4.06E+00	1.11E-01	1.36E+00
Joint	[0.56, 0.57)	4.06E+00	1.11E-01	1.38E+00
Joint	[0.57, 0.59)	3.78E+00	1.36E-01	1.44E+00
Joint	[0.59, 0.60)	3.94E+00	1.22E-01	1.43E+00
Joint	[0.60, 0.62)	3.76E+00	1.37E-01	1.48E+00
Joint	[0.62, 0.63)	4.03E+00	1.17E-01	1.43E+00
Joint	[0.63, 0.64)	4.03E+00	1.16E-01	1.44E+00
Joint	[0.64, 0.66)	4.03E+00	1.13E-01	1.47E+00
Joint	[0.66, 0.67)	4.04E+00	1.14E-01	1.50E+00
Joint	[0.67, 0.68)	8.89E+00	4.85E-01	1.18E+00
Joint	[0.68, 0.70)	8.01E+00	6.23E-01	1.64E+00
Joint	[0.70, 0.71)	6.60E+00	6.49E-01	1.96E+00
Joint	[0.71, 0.72)	6.69E+00	6.69E-01	2.04E+00
Joint	[0.72, 0.74)	7.44E+00	6.07E-01	1.76E+00
Joint	[0.74, 0.75)	6.69E+00	6.76E-01	2.15E+00
Joint	[0.75, 0.77)	7.75E+00	4.54E-01	1.37E+00
Joint	[0.77, 0.78)	7.92E+00	4.26E-01	1.29E+00
Joint	[0.78, 0.79)	8.63E+00	5.54E-01	1.42E+00

(0.43, 0.68]

Joint	[0.79, 0.81)	8.38E+00	5.35E-01	1.44E+00
Joint	[0.81, 0.82)	7.74E+00	4.28E-01	1.35E+00
Joint	[0.82, 0.83)	8.19E+00	5.52E-01	1.56E+00
Joint	[0.83, 0.85)	7.86E+00	5.31E-01	1.61E+00
Joint	[0.85, 0.86)	7.82E+00	5.34E-01	1.64E+00
Joint	[0.86, 0.87)	7.41E+00	5.15E-01	1.74E+00
Joint	[0.87, 0.89)	7.11E+00	4.37E-01	1.61E+00
Joint	[0.89, 0.90)	7.14E+00	5.22E-01	1.92E+00
Joint	[0.90, 0.92)	6.89E+00	5.28E-01	2.10E+00
Joint	[0.92, 0.93)	6.60E+00	5.08E-01	2.19E+00
Joint	[0.93, 0.94)	6.42E+00	5.20E-01	2.36E+00
Joint	[0.94, 0.96)	5.77E+00	4.63E-01	2.57E+00
Joint	[0.96, 0.97)	5.16E+00	3.82E-01	2.74E+00
Joint	[0.97, 0.98)	4.64E+00	2.11E-01	2.72E+00
Joint	[0.98, 1.00)	3.96E+00	7.90E-02	3.18E+00
Joint	[1.00, 1.01]	6.16E+00	-6.87E-01	8.46E-01
Joint	(1.01, 1.03]	5.48E+00	-4.41E-01	1.91E+00
Joint	(1.03, 1.04]	5.90E+00	-5.41E-01	1.45E+00
Joint	(1.04, 1.05]	5.98E+00	-5.96E-01	1.88E+00
Joint	(1.05, 1.07]	6.22E+00	-6.58E-01	1.29E+00
Joint	(1.07, 1.08]	5.81E+00	-5.72E-01	7.99E-01
Joint	(1.08, 1.09]	5.68E+00	-5.39E-01	9.82E-01
Joint	(1.09, 1.11]	5.61E+00	-5.52E-01	7.01E-01
Joint	(1.11, 1.12]	5.60E+00	-5.67E-01	7.14E-01
Joint	(1.12, 1.13]	5.71E+00	-6.18E-01	7.24E-01
Joint	(1.13, 1.15]	5.46E+00	-5.48E-01	6.67E-01
Joint	(1.15, 1.16]	5.36E+00	-5.18E-01	6.22E-01

Joint	(1.16, 1.18]	4.75E+00	-2.50E-01	5.39E-01
Joint	(1.18, 1.19]	4.26E+00	-1.46E-02	5.51E-01
Joint	(1.19, 1.20]	5.23E+00	-4.65E-01	5.05E-01
Joint	(1.20, 1.23]	5.28E+00	-5.12E-01	3.99E-01
Joint	(1.23, 1.26]	2.21E+00	-6.35E-01	1.72E+00
Wing	(1.26,+∞)	4.89E+00	-7.30E-02	3.06E-01
Wing	(-∞, 0.22)	4.02E+00	1.13E-01	1.04E+00
Joint	[0.22, 0.25)	3.96E+00	1.21E-01	1.04E+00
Joint	[0.25, 0.27)	3.93E+00	1.23E-01	1.03E+00
Joint	[0.27, 0.30)	3.89E+00	1.26E-01	1.02E+00
Joint	[0.30, 0.33)	3.97E+00	1.19E-01	1.02E+00
Joint	[0.33, 0.36)	3.87E+00	1.31E-01	1.04E+00
Joint	[0.36, 0.38)	4.02E+00	1.15E-01	1.03E+00
Joint	[0.38, 0.41)	3.85E+00	1.29E-01	1.09E+00
Joint	[0.41, 0.43)	3.91E+00	1.24E-01	1.10E+00
Joint	[0.43, 0.44)	3.91E+00	1.24E-01	1.11E+00
Joint	[0.44, 0.45)	4.00E+00	1.16E-01	1.11E+00
Joint	[0.45, 0.47)	4.01E+00	1.16E-01	1.13E+00
Joint	[0.47, 0.48)	3.91E+00	1.21E-01	1.15E+00
Joint	[0.48, 0.49)	4.02E+00	1.17E-01	1.16E+00
Joint	[0.49, 0.51)	3.86E+00	1.28E-01	1.20E+00
Joint	[0.51, 0.52)	4.01E+00	1.17E-01	1.18E+00
Joint	[0.52, 0.54)	4.00E+00	1.18E-01	1.20E+00
Joint	[0.54, 0.55)	4.02E+00	1.17E-01	1.22E+00
Joint	[0.55, 0.56)	3.85E+00	1.29E-01	1.27E+00
Joint	[0.56, 0.58)	3.89E+00	1.26E-01	1.27E+00
Joint	[0.58, 0.59)	4.00E+00	1.20E-01	1.26E+00

Joint	[0.59, 0.60)	3.89E+00	1.29E-01	1.30E+00
Joint	[0.60, 0.62)	4.00E+00	1.16E-01	1.30E+00
Joint	[0.62, 0.63)	4.00E+00	1.19E-01	1.32E+00
Joint	[0.63, 0.64)	4.00E+00	1.15E-01	1.34E+00
Joint	[0.64, 0.66)	4.00E+00	1.18E-01	1.36E+00
Joint	[0.66, 0.67)	4.01E+00	1.18E-01	1.38E+00
Joint	[0.67, 0.69)	4.01E+00	1.18E-01	1.40E+00
Joint	[0.69, 0.70)	4.01E+00	1.19E-01	1.42E+00
Joint	[0.70, 0.71)	4.01E+00	1.19E-01	1.45E+00
Joint	[0.71, 0.73)	8.99E+00	5.23E-01	1.10E+00
Joint	[0.73, 0.74)	8.69E+00	4.69E-01	1.05E+00
Joint	[0.74, 0.75)	9.29E+00	5.94E-01	1.23E+00
Joint	[0.75, 0.77)	8.51E+00	5.04E-01	1.18E+00
Joint	[0.77, 0.78)	7.96E+00	4.82E-01	1.27E+00
Joint	[0.78, 0.80)	7.90E+00	4.29E-01	1.17E+00
Joint	[0.80, 0.81)	8.07E+00	4.56E-01	1.18E+00
Joint	[0.81, 0.82)	7.91E+00	4.81E-01	1.30E+00
Joint	[0.82, 0.84)	7.60E+00	4.44E-01	1.31E+00
Joint	[0.84, 0.85)	7.49E+00	4.46E-01	1.35E+00
Joint	[0.85, 0.86)	7.52E+00	4.82E-01	1.45E+00
Joint	[0.86, 0.88)	7.28E+00	4.79E-01	1.55E+00
Joint	[0.88, 0.89)	7.16E+00	4.85E-01	1.62E+00
Joint	[0.89, 0.91)	6.93E+00	4.86E-01	1.74E+00
Joint	[0.91, 0.92)	6.62E+00	5.35E-01	2.12E+00
Joint	[0.92, 0.93)	6.70E+00	5.31E-01	2.05E+00
Joint	[0.93, 0.95)	6.19E+00	5.35E-01	2.43E+00
Joint	[0.95, 0.96)	5.66E+00	5.11E-01	2.69E+00

Joint	[0.96, 0.97)	4.91E+00	3.06E-01	2.48E+00
Joint	[0.97, 0.99)	4.30E+00	2.30E-01	2.79E+00
Joint	[0.99, 1.00]	6.41E+00	-6.91E-01	7.14E-01
Joint	(1.00, 1.02]	5.87E+00	-3.75E-01	1.32E+00
Joint	(1.02, 1.03]	5.96E+00	-4.61E-01	1.38E+00
Joint	(1.03, 1.04]	6.09E+00	-5.15E-01	1.35E+00
Joint	(1.04, 1.06]	6.28E+00	-6.09E-01	1.82E+00
Joint	(1.06, 1.07]	6.52E+00	-6.45E-01	1.42E+00
Joint	(1.07, 1.08]	5.95E+00	-4.95E-01	1.10E+00
Joint	(1.08, 1.10]	5.78E+00	-4.54E-01	9.27E-01
Joint	(1.10, 1.11]	5.51E+00	-3.66E-01	7.84E-01
Joint	(1.11, 1.12]	5.64E+00	-4.50E-01	5.42E-01
Joint	(1.12, 1.14]	4.52E+00	1.02E-01	6.31E-01
Joint	(1.14, 1.15]	5.40E+00	-4.06E-01	4.90E-01
Joint	(1.15, 1.17]	5.39E+00	-4.34E-01	5.16E-01
Joint	(1.17, 1.18]	5.31E+00	-4.18E-01	5.00E-01
Joint	(1.18, 1.19]	5.05E+00	-3.32E-01	5.23E-01
Joint	(1.19, 1.21]	5.24E+00	-4.17E-01	5.03E-01
Joint	(1.21, 1.23]	5.41E+00	-5.04E-01	3.26E-01
Joint	(1.23, 1.29]	5.79E+00	-6.26E-01	6.24E-01
Joint	(1.29, 1.34]	4.82E+00	-5.01E-01	7.04E-01
Wing	(1.34,+∞)	4.83E+00	-7.04E-02	3.07E-01
Wing	(-∞, 0.22)	4.06E+00	1.10E-01	1.06E+00
Joint	[0.22, 0.25)	4.10E+00	1.05E-01	1.06E+00
Joint	[0.25, 0.27)	3.98E+00	1.18E-01	1.08E+00
Joint	[0.27, 0.30)	3.95E+00	1.21E-01	1.07E+00
Joint	[0.30, 0.33)	4.01E+00	1.14E-01	1.05E+00

Joint	[0.33, 0.36)	3.88E+00	1.24E-01	1.06E+00
Joint	[0.36, 0.38)	3.94E+00	1.22E-01	1.06E+00
Joint	[0.38, 0.41)	3.69E+00	1.45E-01	1.11E+00
Joint	[0.41, 0.43)	3.77E+00	1.37E-01	1.09E+00
Joint	[0.43, 0.44)	3.89E+00	1.27E-01	1.07E+00
Joint	[0.44, 0.45)	3.98E+00	1.16E-01	1.07E+00
Joint	[0.45, 0.47)	3.98E+00	1.17E-01	1.09E+00
Joint	[0.47, 0.48)	3.99E+00	1.19E-01	1.10E+00
Joint	[0.48, 0.49)	3.99E+00	1.19E-01	1.12E+00
Joint	[0.49, 0.51)	3.99E+00	1.19E-01	1.14E+00
Joint	[0.51, 0.52)	3.98E+00	1.16E-01	1.15E+00
Joint	[0.52, 0.54)	3.99E+00	1.19E-01	1.17E+00
Joint	[0.54, 0.55)	3.98E+00	1.19E-01	1.18E+00
Joint	[0.55, 0.56)	3.99E+00	1.19E-01	1.20E+00
Joint	[0.56, 0.58)	3.97E+00	1.11E-01	1.21E+00
Joint	[0.58, 0.59)	3.98E+00	1.18E-01	1.23E+00
Joint	[0.59, 0.60)	3.99E+00	1.20E-01	1.25E+00
Joint	[0.60, 0.62)	3.98E+00	1.20E-01	1.26E+00
Joint	[0.62, 0.63)	3.98E+00	1.17E-01	1.28E+00
Joint	[0.63, 0.65)	3.98E+00	1.18E-01	1.30E+00
Joint	[0.65, 0.66)	3.99E+00	1.21E-01	1.32E+00
Joint	[0.66, 0.67)	3.98E+00	1.17E-01	1.34E+00
Joint	[0.67, 0.69)	3.99E+00	1.19E-01	1.36E+00
Joint	[0.69, 0.70)	3.98E+00	1.17E-01	1.38E+00
Joint	[0.70, 0.71)	3.99E+00	1.20E-01	1.41E+00
Joint	[0.71, 0.73)	8.93E+00	5.17E-01	1.05E+00
Joint	[0.73, 0.74)	9.12E+00	6.04E-01	1.24E+00

Joint	[0.74, 0.75)	8.61E+00	5.44E-01	1.19E+00
Joint	[0.75, 0.77)	8.06E+00	4.57E-01	1.13E+00
Joint	[0.77, 0.78)	8.50E+00	5.24E-01	1.18E+00
Joint	[0.78, 0.80)	8.24E+00	5.05E-01	1.20E+00
Joint	[0.80, 0.81)	7.87E+00	4.42E-01	1.17E+00
Joint	[0.81, 0.82)	8.60E+00	6.07E-01	1.43E+00
Joint	[0.82, 0.84)	8.31E+00	5.89E-01	1.47E+00
Joint	[0.84, 0.85)	7.63E+00	4.85E-01	1.36E+00
Joint	[0.85, 0.86)	7.83E+00	5.68E-01	1.58E+00
Joint	[0.86, 0.88)	7.25E+00	4.71E-01	1.48E+00
Joint	[0.88, 0.89)	7.22E+00	5.16E-01	1.65E+00
Joint	[0.89, 0.91)	7.25E+00	5.72E-01	1.89E+00
Joint	[0.91, 0.92)	6.75E+00	5.29E-01	1.97E+00
Joint	[0.92, 0.93)	6.54E+00	5.46E-01	2.18E+00
Joint	[0.93, 0.95)	6.27E+00	5.83E-01	2.59E+00
Joint	[0.95, 0.96)	5.52E+00	5.00E-01	2.64E+00
Joint	[0.96, 0.97)	4.88E+00	4.50E-01	2.89E+00
Joint	[0.97, 0.99)	3.99E+00	3.12E-01	3.14E+00
Joint	[0.99, 1.00]	6.44E+00	-6.76E-01	6.66E-01
Joint	(1.00, 1.02]	5.72E+00	-3.59E-01	1.47E+00
Joint	(1.02, 1.03]	5.99E+00	-4.58E-01	1.31E+00
Joint	(1.03, 1.04]	6.13E+00	-2.82E-01	4.50E-01
Joint	(1.04, 1.06]	6.26E+00	-5.69E-01	1.39E+00
Joint	(1.06, 1.07]	6.50E+00	-6.32E-01	1.18E+00
Joint	(1.07, 1.08]	6.25E+00	-5.73E-01	1.19E+00
Joint	(1.08, 1.10]	6.04E+00	-5.24E-01	9.78E-01
Joint	(1.10, 1.15]	6.64E+00	-7.34E-01	8.07E-01

Joint	(1.15, 1.21]	5.45E+00	-5.40E-01	2.06E-01
Wing	(1.21,+∞)	4.86E+00	-7.08E-02	3.04E-01
Wing	(-∞, 0.22)	3.87E+00	1.26E-01	8.25E-01
Joint	[0.22, 0.25)	3.90E+00	1.23E-01	8.22E-01
Joint	[0.25, 0.28)	3.90E+00	1.23E-01	8.44E-01
Joint	[0.28, 0.30)	3.92E+00	1.22E-01	8.83E-01
Joint	[0.30, 0.33)	3.83E+00	1.31E-01	9.15E-01
Joint	[0.33, 0.36)	3.89E+00	1.25E-01	9.08E-01
Joint	[0.36, 0.39)	3.86E+00	1.27E-01	9.24E-01
Joint	[0.39, 0.41)	4.09E+00	1.08E-01	9.39E-01
Joint	[0.41, 0.43)	3.75E+00	1.37E-01	1.00E+00
Joint	[0.43, 0.44)	3.96E+00	1.21E-01	9.71E-01
Joint	[0.44, 0.46)	3.86E+00	1.30E-01	9.97E-01
Joint	[0.46, 0.47)	3.86E+00	1.28E-01	1.02E+00
Joint	[0.47, 0.48)	3.72E+00	1.41E-01	1.05E+00
Joint	[0.48, 0.50)	3.95E+00	1.23E-01	1.02E+00
Joint	[0.50, 0.51)	3.95E+00	1.24E-01	1.03E+00
Joint	[0.51, 0.52)	3.95E+00	1.23E-01	1.04E+00
Joint	[0.52, 0.54)	3.95E+00	1.23E-01	1.06E+00
Joint	[0.54, 0.55)	3.94E+00	1.22E-01	1.07E+00
Joint	[0.55, 0.57)	3.84E+00	1.30E-01	1.11E+00
Joint	[0.57, 0.58)	3.95E+00	1.21E-01	1.10E+00
Joint	[0.58, 0.59)	3.83E+00	1.31E-01	1.14E+00
Joint	[0.59, 0.61)	3.95E+00	1.23E-01	1.14E+00
Joint	[0.61, 0.62)	3.95E+00	1.23E-01	1.15E+00
Joint	[0.62, 0.63)	3.95E+00	1.22E-01	1.17E+00
Joint	[0.63, 0.65)	3.96E+00	1.22E-01	1.19E+00

Joint	[0.65, 0.66)	3.83E+00	1.30E-01	1.24E+00
Joint	[0.66, 0.68)	3.82E+00	1.31E-01	1.26E+00
Joint	[0.68, 0.69)	3.97E+00	1.21E-01	1.25E+00
Joint	[0.69, 0.70)	3.98E+00	1.21E-01	1.28E+00
Joint	[0.70, 0.72)	3.99E+00	1.22E-01	1.30E+00
Joint	[0.72, 0.73)	3.99E+00	1.14E-01	1.32E+00
Joint	[0.73, 0.75)	4.00E+00	1.22E-01	1.35E+00
Joint	[0.75, 0.76)	4.01E+00	1.19E-01	1.38E+00
Joint	[0.76, 0.77)	8.81E+00	5.74E-01	1.11E+00
Joint	[0.77, 0.79)	8.79E+00	5.87E-01	1.16E+00
Joint	[0.79, 0.80)	8.70E+00	5.51E-01	1.07E+00
Joint	[0.80, 0.81)	8.58E+00	5.98E-01	1.28E+00
Joint	[0.81, 0.83)	8.57E+00	6.20E-01	1.38E+00
Joint	[0.83, 0.84)	8.30E+00	6.31E-01	1.53E+00
Joint	[0.84, 0.86)	8.18E+00	6.15E-01	1.50E+00
Joint	[0.86, 0.87)	7.86E+00	5.63E-01	1.41E+00
Joint	[0.87, 0.88)	7.65E+00	5.90E-01	1.64E+00
Joint	[0.88, 0.90)	7.08E+00	5.12E-01	1.58E+00
Joint	[0.90, 0.91)	6.96E+00	5.42E-01	1.76E+00
Joint	[0.91, 0.92)	6.80E+00	5.88E-01	2.07E+00
Joint	[0.92, 0.94)	6.47E+00	5.92E-01	2.29E+00
Joint	[0.94, 0.95)	5.97E+00	5.90E-01	2.59E+00
Joint	[0.95, 0.97)	5.24E+00	5.35E-01	2.74E+00
Joint	[0.97, 0.98)	4.46E+00	3.59E-01	2.59E+00
Joint	[0.98, 0.99)	6.61E+00	-4.45E-01	7.56E-01
Joint	[0.99, 1.01]	6.84E+00	-6.59E-01	8.17E-01
Joint	(1.01, 1.02]	6.08E+00	-3.58E-01	1.18E+00

Joint	(1.02, 1.03]	6.16E+00	-4.10E-01	1.19E+00
Joint	(1.03, 1.05]	6.23E+00	-4.60E-01	1.26E+00
Joint	(1.05, 1.06]	6.26E+00	-4.69E-01	1.23E+00
Joint	(1.06, 1.08]	6.64E+00	-5.71E-01	1.19E+00
Joint	(1.08, 1.09]	6.48E+00	-4.81E-01	6.60E-01
Joint	(1.09, 1.10]	6.47E+00	-5.21E-01	9.53E-01
Joint	(1.10, 1.12]	6.32E+00	-4.48E-01	5.72E-01
Joint	(1.12, 1.13]	6.00E+00	-3.74E-01	7.38E-01
Joint	(1.13, 1.15]	5.96E+00	-3.73E-01	6.69E-01
Joint	(1.15, 1.16]	3.52E+00	-2.05E-01	1.35E+00
Joint	(1.16, 1.19]	3.53E+00	-2.38E-01	1.26E+00
Joint	(1.19, 1.21]	3.38E+00	-1.82E-01	1.11E+00
Joint	(1.21, 1.24]	4.61E+00	-2.01E-01	7.01E-01
Joint	(1.24, 1.27]	3.19E+00	-1.90E-01	9.67E-01
Joint	(1.27, 1.32]	3.31E+00	-1.89E-01	8.43E-01
Joint	(1.32, 1.38]	5.41E+00	-3.28E-01	4.04E-01
Wing	(1.38,+∞)	5.03E+00	-6.45E-02	3.11E-01
Wing	(-∞, 0.11)	3.87E+00	1.22E-01	7.10E-01
Joint	[0.11, 0.17)	3.84E+00	1.26E-01	7.13E-01
Joint	[0.17, 0.22)	3.90E+00	1.20E-01	7.12E-01
Joint	[0.22, 0.28)	3.90E+00	1.22E-01	7.51E-01
Joint	[0.28, 0.31)	3.87E+00	1.25E-01	7.98E-01
Joint	[0.31, 0.33)	3.86E+00	1.25E-01	8.16E-01
Joint	[0.33, 0.36)	4.01E+00	1.12E-01	8.16E-01
Joint	[0.36, 0.39)	3.87E+00	1.33E-01	8.51E-01
Joint	[0.39, 0.42)	4.05E+00	1.12E-01	8.51E-01
Joint	[0.42, 0.44)	3.85E+00	1.12E-01	8.91E-01

Joint	[0.44, 0.47)	4.05E+00	1.13E-01	8.93E-01
Joint	[0.47, 0.50)	4.06E+00	1.16E-01	9.17E-01
Joint	[0.50, 0.53)	3.84E+00	1.28E-01	9.73E-01
Joint	[0.53, 0.54)	3.92E+00	1.23E-01	9.83E-01
Joint	[0.54, 0.55)	3.82E+00	1.31E-01	1.01E+00
Joint	[0.55, 0.57)	3.93E+00	1.24E-01	1.01E+00
Joint	[0.57, 0.58)	3.93E+00	1.23E-01	1.03E+00
Joint	[0.58, 0.60)	3.94E+00	1.23E-01	1.04E+00
Joint	[0.60, 0.61)	3.81E+00	1.30E-01	1.08E+00
Joint	[0.61, 0.62)	3.94E+00	1.23E-01	1.08E+00
Joint	[0.62, 0.64)	3.95E+00	1.23E-01	1.10E+00
Joint	[0.64, 0.65)	3.95E+00	1.23E-01	1.11E+00
Joint	[0.65, 0.67)	3.96E+00	1.22E-01	1.13E+00
Joint	[0.67, 0.68)	3.97E+00	1.24E-01	1.15E+00
Joint	[0.68, 0.69)	3.97E+00	1.21E-01	1.18E+00
Joint	[0.69, 0.71)	3.98E+00	1.23E-01	1.20E+00
Joint	[0.71, 0.72)	3.99E+00	1.20E-01	1.22E+00
Joint	[0.72, 0.74)	4.00E+00	1.18E-01	1.25E+00
Joint	[0.74, 0.75)	4.00E+00	1.15E-01	1.27E+00
Joint	[0.75, 0.76)	4.01E+00	1.15E-01	1.30E+00
Joint	[0.76, 0.78)	4.03E+00	1.18E-01	1.33E+00
Joint	[0.78, 0.79)	8.75E+00	6.03E-01	1.13E+00
Joint	[0.79, 0.80)	8.54E+00	5.77E-01	1.11E+00
Joint	[0.80, 0.82)	8.33E+00	5.76E-01	1.18E+00
Joint	[0.82, 0.83)	8.10E+00	5.56E-01	1.20E+00
Joint	[0.83, 0.85)	8.21E+00	6.03E-01	1.33E+00
Joint	[0.85, 0.86)	8.22E+00	6.41E-01	1.51E+00

Joint	[0.86, 0.87)	7.59E+00	5.67E-01	1.43E+00
Joint	[0.87, 0.89)	7.41E+00	5.68E-01	1.52E+00
Joint	[0.89, 0.90)	7.03E+00	5.34E-01	1.56E+00
Joint	[0.90, 0.92)	6.90E+00	5.82E-01	1.82E+00
Joint	[0.92, 0.93)	6.82E+00	6.35E-01	2.18E+00
Joint	[0.93, 0.94)	6.65E+00	6.78E-01	2.66E+00
Joint	[0.94, 0.96)	5.67E+00	6.02E-01	2.62E+00
Joint	[0.96, 0.97)	4.86E+00	4.93E-01	2.53E+00
Joint	[0.97, 0.98)	6.87E+00	-3.92E-01	6.99E-01
Joint	[0.98, 1.00)	6.78E+00	-3.50E-01	7.56E-01
Joint	[1.00, 1.01]	7.21E+00	-6.81E-01	6.21E-01
Joint	(1.01, 1.03]	6.86E+00	7.24E-01	3.38E-01
Joint	(1.03, 1.04]	6.52E+00	-3.96E-01	1.01E+00
Joint	(1.04, 1.05]	6.77E+00	4.03E-01	1.81E-01
Joint	(1.05, 1.07]	6.79E+00	-2.70E-02	1.74E-01
Joint	(1.07, 1.08]	6.70E+00	-4.91E-01	1.11E+00
Joint	(1.08, 1.10]	6.78E+00	-3.75E-01	5.19E-01
Joint	(1.10, 1.11]	7.19E+00	-5.85E-01	9.19E-01
Joint	(1.11, 1.12]	6.76E+00	-4.84E-01	8.61E-01
Joint	(1.12, 1.14]	6.67E+00	-4.07E-01	5.59E-01
Joint	(1.14, 1.17]	3.92E+00	-1.96E-01	1.31E+00
Joint	(1.17, 1.22]	4.09E+00	-1.56E-01	1.04E+00
Joint	(1.22, 1.25]	3.49E+00	-1.77E-01	1.04E+00
Joint	(1.25, 1.39]	6.50E+00	-4.85E-01	3.33E-01
Joint	(1.39, 1.66]	4.55E+00	-1.59E-01	5.00E-01
Wing	(1.66,+∞)	5.31E+00	-2.54E-02	3.27E-01
Wing	(-∞, 0.06)	3.75E+00	1.87E-01	6.18E-01

Joint	[0.06, 0.11)	3.92E+00	1.08E-01	5.95E-01
Joint	[0.11, 0.17)	3.86E+00	1.20E-01	6.12E-01
Joint	[0.17, 0.22)	3.85E+00	1.22E-01	6.31E-01
Joint	[0.22, 0.28)	3.85E+00	1.24E-01	6.65E-01
Joint	[0.28, 0.31)	3.80E+00	1.30E-01	6.95E-01
Joint	[0.31, 0.34)	3.80E+00	1.28E-01	7.15E-01
Joint	[0.34, 0.36)	3.81E+00	1.29E-01	7.36E-01
Joint	[0.36, 0.39)	3.81E+00	1.29E-01	7.64E-01
Joint	[0.39, 0.42)	3.83E+00	1.27E-01	7.90E-01
Joint	[0.42, 0.45)	3.82E+00	1.28E-01	8.08E-01
Joint	[0.45, 0.47)	4.06E+00	8.04E-02	7.91E-01
Joint	[0.47, 0.50)	4.05E+00	1.13E-01	8.28E-01
Joint	[0.50, 0.52)	3.91E+00	1.20E-01	8.64E-01
Joint	[0.52, 0.53)	3.92E+00	1.24E-01	8.79E-01
Joint	[0.53, 0.54)	3.92E+00	1.20E-01	8.93E-01
Joint	[0.54, 0.56)	3.93E+00	1.19E-01	9.09E-01
Joint	[0.56, 0.57)	3.93E+00	1.19E-01	9.23E-01
Joint	[0.57, 0.59)	3.93E+00	1.17E-01	9.36E-01
Joint	[0.59, 0.60)	3.94E+00	1.19E-01	9.51E-01
Joint	[0.60, 0.61)	3.95E+00	1.24E-01	9.70E-01
Joint	[0.61, 0.63)	3.82E+00	1.29E-01	1.01E+00
Joint	[0.63, 0.64)	3.96E+00	1.17E-01	1.00E+00
Joint	[0.64, 0.66)	3.97E+00	1.21E-01	1.02E+00
Joint	[0.66, 0.67)	3.99E+00	1.27E-01	1.04E+00
Joint	[0.67, 0.68)	3.83E+00	1.27E-01	1.09E+00
Joint	[0.68, 0.70)	4.00E+00	1.20E-01	1.08E+00
Joint	[0.70, 0.71)	4.01E+00	1.17E-01	1.10E+00

Joint	[0.71, 0.73)	4.02E+00	1.21E-01	1.12E+00
Joint	[0.73, 0.74)	4.04E+00	1.22E-01	1.15E+00
Joint	[0.74, 0.75)	4.03E+00	1.21E-01	1.18E+00
Joint	[0.75, 0.77)	4.06E+00	1.14E-01	1.20E+00
Joint	[0.77, 0.78)	4.07E+00	1.17E-01	1.22E+00
Joint	[0.78, 0.80)	4.08E+00	1.14E-01	1.24E+00
Joint	[0.80, 0.81)	4.13E+00	1.34E-01	1.28E+00
Joint	[0.81, 0.82)	9.14E+00	6.56E-01	1.06E+00
Joint	[0.82, 0.84)	8.22E+00	6.66E-01	1.48E+00
Joint	[0.84, 0.85)	7.88E+00	5.69E-01	1.20E+00
Joint	[0.85, 0.87)	7.78E+00	5.93E-01	1.32E+00
Joint	[0.87, 0.88)	7.59E+00	5.98E-01	1.41E+00
Joint	[0.88, 0.89)	7.40E+00	6.18E-01	1.58E+00
Joint	[0.89, 0.91)	7.22E+00	6.41E-01	1.78E+00
Joint	[0.91, 0.92)	6.83E+00	6.28E-01	1.88E+00
Joint	[0.92, 0.94)	6.45E+00	6.43E-01	2.16E+00
Joint	[0.94, 0.95)	5.88E+00	6.32E-01	2.38E+00
Joint	[0.95, 0.96)	5.15E+00	6.16E-01	2.67E+00
Joint	[0.96, 0.98)	4.16E+00	4.78E-01	2.52E+00
Joint	[0.98, 0.99)	7.07E+00	-1.80E-01	6.67E-01
Joint	[0.99, 1.01]	7.80E+00	6.24E-01	2.00E-01
Joint	(1.01, 1.02]	7.19E+00	-2.20E-01	7.06E-01
Joint	(1.02, 1.03]	6.61E+00	-3.00E-01	1.04E+00
Joint	(1.03, 1.05]	4.46E+00	-2.17E-01	1.82E+00
Joint	(1.05, 1.06]	4.46E+00	-1.85E-01	1.71E+00
Joint	(1.06, 1.07]	4.39E+00	-1.73E-01	1.64E+00
Joint	(1.07, 1.09]	4.38E+00	-1.91E-01	1.59E+00

Joint	(1.09, 1.10]	4.20E+00	-1.76E-01	1.55E+00
Joint	(1.10, 1.12]	4.09E+00	-1.38E-01	1.46E+00
Joint	(1.12, 1.13]	4.39E+00	-2.67E-01	1.46E+00
Joint	(1.13, 1.14]	4.00E+00	-1.67E-01	1.38E+00
Joint	(1.14, 1.17]	4.21E+00	-2.36E-01	1.31E+00
Joint	(1.17, 1.20]	4.12E+00	-1.81E-01	1.18E+00
Joint	(1.20, 1.23]	4.08E+00	-1.97E-01	1.11E+00
Joint	(1.23, 1.26]	3.82E+00	-2.04E-01	1.08E+00
Joint	(1.26, 1.28]	3.74E+00	-1.65E-01	9.91E-01
Joint	(1.28, 1.34]	3.93E+00	-2.01E-01	8.93E-01
Joint	(1.34, 1.40]	3.72E+00	-1.72E-01	8.06E-01
Joint	(1.40, 1.45]	3.58E+00	-1.74E-01	7.46E-01
Joint	(1.45, 1.56]	3.78E+00	-1.64E-01	6.52E-01
Joint	(1.56, 1.68]	3.68E+00	-1.69E-01	6.13E-01
Wing	(1.68,+∞)	5.52E+00	-9.57E-03	3.27E-01

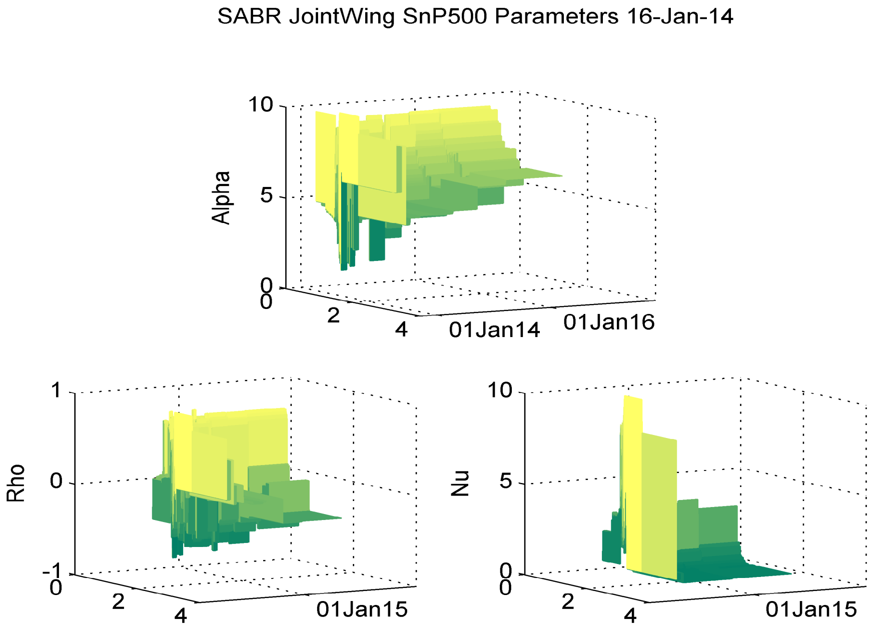


Figure B.24: SABR JointWing SnP500 Parameters

Table B.19: Heston JointWing SnP500 Parameters (κ = 5.12E-01, v0 = 2.11E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.94)		5.93E-02	2.43E-01	-7.69E-01
Joint	[0.94, 0.94)		9.26E-01	6.37E-01	-3.87E-01
Joint	[0.94, 0.94)		9.36E-01	3.13E+00	-8.54E-01
Joint	[0.94, 0.95)		6.56E-02	2.46E-01	-7.67E-01
Joint	[0.95, 0.95)		9.99E+00	2.68E+00	-5.77E-01
Joint	[0.95, 0.95)		1.56E+00	3.54E+00	-5.39E-01
Joint	[0.95, 0.96)		1.56E+00	2.74E+00	-5.59E-01
Joint	[0.96, 0.96)		4.52E+00	4.88E+00	2.20E-01
Joint	[0.96, 0.96)		8.73E+00	1.70E+00	-3.16E-01
Joint	[0.96, 0.96)		1.49E+00	2.16E+00	-4.38E-01
Joint	[0.96, 0.97)		8.28E-01	2.35E+00	-2.96E-01
Joint	[0.97, 0.97)		8.21E-01	1.76E+00	-3.13E-01
Joint	[0.97, 0.97)		6.00E-01	9.11E-01	-5.57E-01
Joint	[0.97, 0.97)		5.94E-01	9.10E-01	-3.69E-01
Joint	[0.97, 0.98)		4.56E+00	4.51E+00	5.09E-01
Joint	[0.98, 0.98)		9.06E+00	4.54E+00	6.56E-01
Joint	[0.98, 0.98)		3.79E-01	7.43E-01	4.92E-01
Joint	[0.98, 0.99)		2.27E-01	9.92E-01	6.31E-01
Joint	[0.99, 0.99)		6.07E-03	7.81E+00	6.38E-01
Joint	[0.99, 0.99)		3.22E+00	5.44E+00	7.17E-01
Joint	[0.99, 0.99)		4.31E+00	5.27E+00	7.61E-01
Joint	[0.99, 1.00)		3.19E-01	9.48E+00	3.39E-01
Joint	[1.00, 1.00)		1.43E-03	9.71E+00	3.97E-01

Joint	[1.00, 1.00]	1.97E-01	9.78E+00	7.35E-01
Joint	(1.00, 1.00]	2.72E+00	7.28E+00	-9.43E-01
Joint	(1.00, 1.01]	1.37E+00	8.66E+00	-7.98E-01
Joint	(1.01, 1.01]	2.98E+00	7.16E+00	-8.65E-01
Joint	(1.01, 1.01]	3.72E+00	6.71E+00	-8.66E-01
Joint	(1.01, 1.01]	3.83E+00	7.29E+00	-8.39E-01
Joint	(1.01, 1.02]	3.93E+00	6.00E+00	-8.11E-01
Joint	(1.02, 1.02]	3.98E+00	5.99E+00	-7.93E-01
Joint	(1.02, 1.02]	4.37E+00	4.55E+00	-7.31E-01
Joint	(1.02, 1.03]	4.49E+00	4.38E+00	-6.59E-01
Joint	(1.03, 1.03]	1.24E-01	4.62E-02	-6.73E-01
Joint	(1.03, 1.03]	2.51E+00	4.59E-01	8.98E-01
Joint	(1.03, 1.03]	1.67E+00	7.16E+00	3.27E-01
Joint	(1.03, 1.04]	1.85E+00	7.11E+00	4.53E-01
Joint	(1.04, 1.04]	1.99E+00	9.16E+00	3.94E-01
Joint	(1.04, 1.04]	2.17E+00	8.76E+00	5.24E-01
Joint	(1.04, 1.04]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.04, 1.05]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.05, 1.05]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.05, 1.05]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.05, 1.06]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.06, 1.06]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.06, 1.06]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.06, 1.06]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.06, 1.07]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.07, 1.07]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.07, 1.07]	5.93E-02	2.43E-01	-7.69E-01

Joint	(1.07, 1.08]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.08, 1.08]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.08, 1.11]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.11, 1.14]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.14, 1.22]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.22, 1.35]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.35, 1.62]	5.93E-02	2.43E-01	-7.69E-01
Wing	(1.62,+∞)	2.53E-01	3.00E-03	-7.06E-01
Wing	(-∞, 0.65)	1.04E-05	8.48E-01	9.99E-01
Joint	[0.65, 0.66)	5.93E-02	2.43E-01	-7.69E-01
Joint	[0.66, 0.68)	5.15E+00	2.36E-01	-1.41E-01
Joint	[0.68, 0.68)	1.21E+00	2.24E+00	1.41E-01
Joint	[0.68, 0.68)	6.56E-02	2.46E-01	-7.67E-01
Joint	[0.68, 0.69)	6.56E-02	2.46E-01	-7.67E-01
Joint	[0.69, 0.69)	1.17E+00	1.32E+00	-7.24E-01
Joint	[0.69, 0.69)	8.29E-01	1.59E+00	-3.47E-01
Joint	[0.69, 0.69)	8.67E+00	7.48E-01	8.93E-01
Joint	[0.69, 0.70)	1.58E+00	2.41E+00	4.59E-02
Joint	[0.70, 0.70)	7.38E-01	2.90E+00	-3.04E-02
Joint	[0.70, 0.70)	2.59E+00	3.19E+00	2.96E-01
Joint	[0.70, 0.70)	9.06E-01	1.74E+00	-3.54E-01
Joint	[0.70, 0.71)	7.92E+00	8.55E-01	7.19E-01
Joint	[0.71, 0.71)	5.60E+00	6.98E-01	9.24E-02
Joint	[0.71, 0.71)	1.89E+00	3.22E+00	2.70E-01
Joint	[0.71, 0.72)	1.83E+00	3.42E+00	3.26E-01
Joint	[0.72, 0.72)	1.88E+00	3.17E+00	3.79E-01
Joint	[0.72, 0.72)	9.47E+00	1.84E+00	7.81E-01

Joint	[0.72, 0.72)	1.80E+00	2.79E+00	2.34E-01
Joint	[0.72, 0.73)	6.92E-01	1.56E+00	-3.09E-01
Joint	[0.73, 0.73)	3.86E+00	2.15E-01	-4.73E-01
Joint	[0.73, 0.73)	5.68E-01	1.47E+00	-4.16E-01
Joint	[0.73, 0.73)	6.39E+00	6.18E-01	8.31E-01
Joint	[0.73, 0.74)	8.70E-01	2.05E+00	-1.11E-01
Joint	[0.74, 0.74)	6.26E-01	1.53E+00	-4.70E-01
Joint	[0.74, 0.74)	6.68E-01	1.85E+00	-2.18E-01
Joint	[0.74, 0.75)	6.69E-01	1.06E+00	-6.13E-01
Joint	[0.75, 0.75)	3.34E+00	1.76E-01	-3.78E-01
Joint	[0.75, 0.75)	6.05E-01	1.20E+00	-6.81E-01
Joint	[0.75, 0.75)	2.52E+00	5.53E-01	-6.16E-01
Joint	[0.75, 0.76)	5.79E-01	1.29E+00	-5.71E-01
Joint	[0.76, 0.76)	4.29E-01	1.07E+00	-7.50E-01
Joint	[0.76, 0.76)	5.20E+00	5.21E-01	8.72E-01
Joint	[0.76, 0.76)	5.39E-01	1.22E+00	-5.65E-01
Joint	[0.76, 0.77)	1.69E+00	7.88E-01	-5.27E-01
Joint	[0.77, 0.77)	5.23E-01	1.21E+00	-5.55E-01
Joint	[0.77, 0.77)	5.23E-01	1.18E+00	-5.60E-01
Joint	[0.77, 0.78)	3.77E-01	1.04E+00	-7.67E-01
Joint	[0.78, 0.78)	9.70E-01	8.74E-01	-6.57E-01
Joint	[0.78, 0.78)	3.55E-01	1.02E+00	-7.72E-01
Joint	[0.78, 0.78)	4.59E-01	1.09E+00	-5.93E-01
Joint	[0.78, 0.79)	2.61E-01	9.09E-01	-8.07E-01
Joint	[0.79, 0.79)	4.08E+00	4.73E-01	8.66E-01
Joint	[0.79, 0.79)	4.72E-01	1.04E+00	-5.64E-01
Joint	[0.79, 0.79)	4.69E-01	1.01E+00	-5.82E-01

Joint	[0.79, 0.80)	4.16E-01	9.75E-01	-6.40E-01
Joint	[0.80, 0.80)	3.49E-01	8.95E-01	-7.49E-01
Joint	[0.80, 0.80)	1.38E-01	8.84E-01	-8.82E-01
Joint	[0.80, 0.81)	1.87E-01	8.95E-01	-8.06E-01
Joint	[0.81, 0.81)	1.16E-01	8.69E-01	-8.71E-01
Joint	[0.81, 0.81)	1.27E-01	8.67E-01	-8.58E-01
Joint	[0.81, 0.81)	1.36E-01	8.57E-01	-8.58E-01
Joint	[0.81, 0.82)	1.29E-01	8.42E-01	-8.60E-01
Joint	[0.82, 0.82)	1.55E+00	5.79E-01	-2.14E-01
Joint	[0.82, 0.82)	1.01E-01	8.28E-01	-8.75E-01
Joint	[0.82, 0.82)	1.07E+00	6.64E-01	-3.95E-01
Joint	[0.82, 0.83)	2.43E+00	3.00E+00	6.47E-01
Joint	[0.83, 0.83)	2.20E+00	1.83E+00	5.15E-01
Joint	[0.83, 0.83)	2.07E+00	2.88E+00	5.85E-01
Joint	[0.83, 0.83)	2.36E+00	3.15E+00	6.38E-01
Joint	[0.83, 0.84)	3.00E+00	3.44E+00	7.06E-01
Joint	[0.84, 0.84)	2.66E+00	3.41E+00	7.10E-01
Joint	[0.84, 0.84)	2.80E+00	2.94E+00	7.01E-01
Joint	[0.84, 0.85)	2.37E+00	3.38E+00	6.68E-01
Joint	[0.85, 0.85)	2.46E+00	3.58E+00	6.86E-01
Joint	[0.85, 0.85)	2.46E+00	3.54E+00	6.90E-01
Joint	[0.85, 0.85)	2.69E+00	2.92E+00	6.71E-01
Joint	[0.85, 0.86)	2.72E+00	3.83E+00	7.16E-01
Joint	[0.86, 0.86)	1.90E+00	3.47E+00	6.52E-01
Joint	[0.86, 0.86)	1.92E+00	3.63E+00	6.65E-01
Joint	[0.86, 0.86)	2.51E+00	3.58E+00	7.14E-01
Joint	[0.86, 0.87)	2.04E+00	3.96E+00	7.06E-01

Joint	[0.87, 0.87)	2.66E+00	3.63E+00	7.46E-01
Joint	[0.87, 0.87)	2.05E+00	4.28E+00	7.10E-01
Joint	[0.87, 0.88)	2.03E+00	4.41E+00	7.14E-01
Joint	[0.88, 0.88)	1.97E+00	4.45E+00	7.17E-01
Joint	[0.88, 0.88)	2.62E+00	3.95E+00	7.57E-01
Joint	[0.88, 0.88)	2.01E+00	4.67E+00	7.24E-01
Joint	[0.88, 0.89)	1.67E+00	4.42E+00	6.90E-01
Joint	[0.89, 0.89)	1.60E+00	4.47E+00	6.89E-01
Joint	[0.89, 0.89)	1.83E+00	2.99E+00	6.72E-01
Joint	[0.89, 0.89)	1.56E+00	4.01E+00	6.73E-01
Joint	[0.89, 0.90)	1.23E+00	3.58E+00	6.15E-01
Joint	[0.90, 0.90)	2.13E+00	4.88E+00	7.53E-01
Joint	[0.90, 0.90)	1.91E+00	4.32E+00	7.26E-01
Joint	[0.90, 0.91)	1.17E+00	4.09E+00	6.37E-01
Joint	[0.91, 0.91)	7.30E-01	6.26E-01	2.88E-01
Joint	[0.91, 0.91)	9.62E-01	4.23E+00	5.83E-01
Joint	[0.91, 0.91)	2.19E+00	4.62E+00	7.62E-01
Joint	[0.91, 0.92)	1.64E+00	4.74E+00	7.04E-01
Joint	[0.92, 0.92)	2.11E+00	4.75E+00	7.59E-01
Joint	[0.92, 0.92)	1.12E+00	5.18E+00	6.26E-01
Joint	[0.92, 0.92)	1.34E+00	3.19E+00	6.58E-01
Joint	[0.92, 0.93)	5.80E-01	6.75E-01	3.82E-01
Joint	[0.93, 0.93)	1.24E+00	5.54E+00	6.29E-01
Joint	[0.93, 0.93)	4.94E-02	9.05E-01	-7.05E-02
Joint	[0.93, 0.94)	8.12E-04	9.34E-01	-9.86E-02
Joint	[0.94, 0.94)	8.42E-02	7.56E-01	-5.97E-02
Joint	[0.94, 0.94)	3.99E-02	1.14E+00	4.09E-03

Joint	[0.94, 0.94)	5.00E-02	1.69E+00	7.44E-02
Joint	[0.94, 0.95)	1.46E-01	1.33E+00	1.59E-01
Joint	[0.95, 0.95)	1.80E-01	1.32E+00	1.93E-01
Joint	[0.95, 0.95)	6.95E-01	4.75E+00	4.03E-01
Joint	[0.95, 0.95)	5.67E-01	2.66E+00	4.37E-01
Joint	[0.95, 0.96)	2.71E-01	1.87E+00	2.75E-01
Joint	[0.96, 0.96)	2.88E+00	5.24E+00	8.83E-01
Joint	[0.96, 0.96)	2.30E-01	1.76E+00	2.45E-01
Joint	[0.96, 0.97)	4.18E-02	1.74E-01	8.15E-01
Joint	[0.97, 0.97)	4.66E-02	2.87E-01	6.39E-01
Joint	[0.97, 0.97)	4.54E-01	2.82E+00	3.07E-01
Joint	[0.97, 0.97)	1.33E+00	3.61E+00	7.53E-01
Joint	[0.97, 0.98)	9.17E-01	3.73E+00	5.62E-01
Joint	[0.98, 0.98)	1.37E-01	7.21E-01	6.95E-01
Joint	[0.98, 0.98)	1.48E+00	7.10E+00	5.30E-01
Joint	[0.98, 0.98)	1.10E-01	9.84E-01	5.22E-01
Joint	[0.98, 0.99)	6.08E-02	1.53E+00	-4.35E-02
Joint	[0.99, 0.99)	6.89E-02	1.58E+00	-5.05E-02
Joint	[0.99, 0.99)	4.34E-02	1.56E+00	-2.41E-01
Joint	[0.99, 0.99)	9.87E-01	6.57E+00	-2.19E-03
Joint	[0.99, 1.00)	9.80E-01	6.58E+00	-1.71E-01
Joint	[1.00, 1.00]	1.26E+00	6.79E+00	-7.51E-01
Joint	(1.00, 1.00]	9.86E-01	6.60E+00	-4.02E-01
Joint	(1.00, 1.01]	9.31E-01	6.00E+00	-4.82E-01
Joint	(1.01, 1.01]	1.40E-01	1.55E+00	-7.79E-01
Joint	(1.01, 1.01]	2.39E-01	2.25E+00	-5.91E-01
Joint	(1.01, 1.01]	1.40E+00	6.40E+00	-7.72E-01

Joint	(1.01, 1.02]	3.49E-02	1.21E+00	-7.22E-01
Joint	(1.02, 1.02]	3.89E-02	1.14E+00	-7.54E-01
Joint	(1.02, 1.02]	1.04E+00	6.26E+00	-7.12E-01
Joint	(1.02, 1.02]	8.15E-01	6.38E+00	-6.75E-01
Joint	(1.02, 1.03]	1.18E+00	6.19E+00	-8.09E-01
Joint	(1.03, 1.03]	1.30E+00	6.15E+00	-8.47E-01
Joint	(1.03, 1.03]	1.36E+00	6.03E+00	-8.69E-01
Joint	(1.03, 1.04]	1.45E+00	6.01E+00	-8.89E-01
Joint	(1.04, 1.04]	1.49E+00	5.88E+00	-9.04E-01
Joint	(1.04, 1.04]	1.58E+00	5.80E+00	-9.18E-01
Joint	(1.04, 1.04]	1.53E+00	5.17E+00	-9.25E-01
Joint	(1.04, 1.05]	1.16E+00	3.30E+00	-9.30E-01
Joint	(1.05, 1.05]	5.11E-01	1.74E+00	-9.12E-01
Joint	(1.05, 1.05]	1.66E+00	5.86E+00	-9.40E-01
Joint	(1.05, 1.05]	4.63E-01	1.73E+00	-8.98E-01
Joint	(1.05, 1.06]	1.41E+00	5.88E+00	-9.32E-01
Joint	(1.06, 1.06]	1.36E+00	3.87E+00	-9.36E-01
Joint	(1.06, 1.06]	1.88E+00	5.52E+00	-9.49E-01
Joint	(1.06, 1.07]	1.91E+00	5.67E+00	-9.46E-01
Joint	(1.07, 1.07]	1.80E+00	4.01E+00	-9.57E-01
Joint	(1.07, 1.07]	1.91E+00	5.64E+00	-9.60E-01
Joint	(1.07, 1.07]	1.69E+00	5.51E+00	-9.43E-01
Joint	(1.07, 1.08]	2.02E+00	5.27E+00	-9.40E-01
Joint	(1.08, 1.08]	1.96E+00	5.43E+00	-9.38E-01
Joint	(1.08, 1.08]	1.80E+00	5.36E+00	-9.33E-01
Joint	(1.08, 1.08]	1.04E+00	2.22E+00	-9.14E-01
Joint	(1.08, 1.10]	2.03E+00	5.19E+00	-9.55E-01

Joint	(1.10, 1.11]	8.08E-02	2.46E-01	-7.35E-01
Joint	(1.11, 1.13]	2.03E+00	8.90E+00	-7.86E-01
Joint	(1.13, 1.14]	1.14E+00	2.88E+00	-6.25E-01
Wing	(1.14,+∞)	5.93E-02	2.43E-01	-7.69E-01
Wing	(-∞, 0.58)	7.93E-02	1.71E-01	-7.43E-01
Joint	[0.58, 0.62)	3.47E-02	1.54E+00	-1.15E-01
Joint	[0.62, 0.63)	4.01E+00	3.97E-01	9.27E-01
Joint	[0.63, 0.64)	1.68E-01	1.05E+00	-4.98E-01
Joint	[0.64, 0.64)	2.33E+00	1.75E-01	-6.22E-01
Joint	[0.64, 0.65)	1.09E-01	1.25E+00	-4.85E-01
Joint	[0.65, 0.66)	3.84E+00	4.67E-01	8.54E-01
Joint	[0.66, 0.66)	3.28E-01	8.51E-01	-7.60E-01
Joint	[0.66, 0.67)	7.29E+00	1.64E+00	9.51E-01
Joint	[0.67, 0.67)	3.16E-01	9.49E-01	-6.95E-01
Joint	[0.67, 0.68)	3.08E-01	9.25E-01	-7.11E-01
Joint	[0.68, 0.68)	2.61E-01	8.20E-01	-7.84E-01
Joint	[0.68, 0.68)	3.21E+00	3.94E-01	8.58E-01
Joint	[0.68, 0.68)	2.92E-01	8.56E-01	-7.95E-01
Joint	[0.68, 0.69)	3.86E-01	9.31E-01	-5.78E-01
Joint	[0.69, 0.69)	2.73E-01	8.31E-01	-7.95E-01
Joint	[0.69, 0.69)	2.26E-01	7.65E-01	-7.90E-01
Joint	[0.69, 0.69)	2.75E+00	3.65E-01	7.75E-01
Joint	[0.69, 0.70)	3.80E-01	8.61E-01	-6.11E-01
Joint	[0.70, 0.70)	3.13E-01	7.51E-01	-8.27E-01
Joint	[0.70, 0.70)	1.03E+00	5.31E-01	-5.72E-01
Joint	[0.70, 0.71)	1.28E-01	7.67E-01	-8.24E-01
Joint	[0.71, 0.71)	1.82E+00	6.91E-04	-6.97E-01

Joint	[0.71, 0.71)	7.44E-01	6.14E-01	-6.11E-01
Joint	[0.71, 0.71)	1.28E-01	8.09E-01	-8.12E-01
Joint	[0.71, 0.72)	1.18E-01	7.82E-01	-8.40E-01
Joint	[0.72, 0.72)	1.54E+00	1.39E-01	-6.73E-01
Joint	[0.72, 0.72)	1.16E-01	7.78E-01	-8.58E-01
Joint	[0.72, 0.72)	1.11E-01	7.87E-01	-8.13E-01
Joint	[0.72, 0.73)	2.46E+00	4.03E-01	7.68E-01
Joint	[0.73, 0.73)	7.46E-02	8.15E-01	-7.98E-01
Joint	[0.73, 0.73)	1.44E+00	1.52E-01	-6.03E-01
Joint	[0.73, 0.74)	6.95E-02	8.16E-01	-7.97E-01
Joint	[0.74, 0.74)	1.04E-01	7.63E-01	-8.40E-01
Joint	[0.74, 0.74)	2.02E+00	2.66E+00	6.70E-01
Joint	[0.74, 0.74)	1.74E+00	2.44E+00	6.18E-01
Joint	[0.74, 0.75)	2.30E+00	2.98E+00	7.23E-01
Joint	[0.75, 0.75)	2.16E+00	2.00E+00	6.32E-01
Joint	[0.75, 0.75)	2.05E+00	2.82E+00	6.91E-01
Joint	[0.75, 0.75)	2.07E+00	2.89E+00	7.02E-01
Joint	[0.75, 0.76)	1.60E+00	2.42E+00	6.10E-01
Joint	[0.76, 0.76)	1.77E+00	2.60E+00	6.80E-01
Joint	[0.76, 0.76)	1.54E+00	1.29E+00	4.85E-01
Joint	[0.76, 0.77)	2.07E+00	3.02E+00	7.19E-01
Joint	[0.77, 0.77)	2.40E+00	2.92E+00	7.45E-01
Joint	[0.77, 0.77)	2.34E+00	2.83E+00	7.35E-01
Joint	[0.77, 0.77)	2.01E+00	3.33E+00	7.36E-01
Joint	[0.77, 0.78)	2.31E+00	3.13E+00	7.53E-01
Joint	[0.78, 0.78)	1.86E+00	3.23E+00	7.39E-01
Joint	[0.78, 0.78)	1.69E+00	3.76E+00	7.78E-01

Joint	[0.78, 0.78)	2.39E+00	3.10E+00	7.81E-01
Joint	[0.78, 0.79)	1.86E+00	3.57E+00	7.61E-01
Joint	[0.79, 0.79)	1.11E+00	2.35E+00	6.02E-01
Joint	[0.79, 0.79)	1.50E+00	2.56E+00	6.85E-01
Joint	[0.79, 0.80)	1.44E+00	2.74E+00	6.90E-01
Joint	[0.80, 0.80)	8.84E-01	2.03E+00	5.23E-01
Joint	[0.80, 0.80)	9.66E-01	2.49E+00	5.99E-01
Joint	[0.80, 0.80)	1.42E+00	2.79E+00	6.96E-01
Joint	[0.80, 0.81)	1.26E+00	2.68E+00	6.67E-01
Joint	[0.81, 0.81)	1.21E+00	2.68E+00	6.60E-01
Joint	[0.81, 0.81)	2.18E+00	3.28E+00	7.89E-01
Joint	[0.81, 0.81)	2.17E+00	3.36E+00	7.75E-01
Joint	[0.81, 0.82)	1.78E+00	4.11E+00	7.94E-01
Joint	[0.82, 0.82)	2.00E+00	3.55E+00	7.78E-01
Joint	[0.82, 0.82)	9.75E-01	2.41E+00	5.85E-01
Joint	[0.82, 0.83)	6.80E-01	2.36E+00	5.26E-01
Joint	[0.83, 0.83)	1.67E+00	3.43E+00	7.68E-01
Joint	[0.83, 0.83)	8.24E-01	6.51E-01	2.73E-01
Joint	[0.83, 0.83)	1.89E+00	3.44E+00	7.72E-01
Joint	[0.83, 0.84)	1.56E+00	3.61E+00	7.44E-01
Joint	[0.84, 0.84)	1.51E+00	3.40E+00	7.31E-01
Joint	[0.84, 0.84)	1.59E+00	3.73E+00	7.55E-01
Joint	[0.84, 0.84)	1.50E+00	3.70E+00	7.45E-01
Joint	[0.84, 0.85)	8.57E-01	3.45E+00	6.23E-01
Joint	[0.85, 0.85)	1.82E+00	3.56E+00	7.80E-01
Joint	[0.85, 0.85)	1.10E+00	3.81E+00	6.89E-01
Joint	[0.85, 0.85)	7.86E-01	4.02E+00	6.29E-01

Joint	[0.85, 0.86)	1.56E+00	3.80E+00	7.63E-01
Joint	[0.86, 0.86)	8.13E-01	4.03E+00	6.35E-01
Joint	[0.86, 0.86)	1.68E+00	3.67E+00	7.75E-01
Joint	[0.86, 0.87)	1.57E+00	3.68E+00	7.62E-01
Joint	[0.87, 0.87)	1.60E+00	3.71E+00	7.65E-01
Joint	[0.87, 0.87)	8.46E-01	3.99E+00	6.32E-01
Joint	[0.87, 0.87)	1.58E+00	3.78E+00	7.71E-01
Joint	[0.87, 0.88)	1.09E+00	4.11E+00	6.98E-01
Joint	[0.88, 0.88)	1.05E+00	4.24E+00	7.05E-01
Joint	[0.88, 0.88)	4.95E-01	6.47E-01	2.85E-01
Joint	[0.88, 0.88)	1.04E+00	4.13E+00	6.94E-01
Joint	[0.88, 0.89)	3.79E-01	5.65E-01	1.37E-01
Joint	[0.89, 0.89)	6.80E-01	4.39E+00	5.86E-01
Joint	[0.89, 0.89)	4.67E-01	6.50E-01	3.44E-01
Joint	[0.89, 0.90)	1.44E+00	3.81E+00	7.57E-01
Joint	[0.90, 0.90)	1.03E+00	4.33E+00	6.79E-01
Joint	[0.90, 0.90)	1.03E+00	4.26E+00	6.84E-01
Joint	[0.90, 0.90)	1.25E+00	4.06E+00	7.35E-01
Joint	[0.90, 0.91)	9.70E-01	4.51E+00	6.82E-01
Joint	[0.91, 0.91)	3.78E-01	7.77E-01	3.33E-01
Joint	[0.91, 0.91)	1.19E+00	4.17E+00	7.15E-01
Joint	[0.91, 0.91)	1.17E+00	4.16E+00	7.11E-01
Joint	[0.91, 0.92)	1.19E+00	4.10E+00	7.14E-01
Joint	[0.92, 0.92)	5.84E-02	2.35E-01	-7.52E-01
Joint	[0.92, 0.92)	5.69E-02	8.91E-01	-1.28E-01
Joint	[0.92, 0.93)	6.42E-02	1.00E+00	-9.65E-02
Joint	[0.93, 0.93)	5.35E-02	1.06E+00	-1.16E-01

Joint	[0.93, 0.93)	8.64E-02	8.39E-01	-5.84E-02
Joint	[0.93, 0.93)	5.18E-02	1.53E+00	-1.63E-01
Joint	[0.93, 0.94)	1.08E-01	4.90E-01	8.50E-03
Joint	[0.94, 0.94)	1.61E-02	1.89E-01	-7.40E-01
Joint	[0.94, 0.94)	1.04E-01	1.11E+00	-8.78E-03
Joint	[0.94, 0.94)	1.22E-01	9.77E-01	5.34E-02
Joint	[0.94, 0.95)	1.63E-01	1.37E+00	7.50E-02
Joint	[0.95, 0.95)	1.19E-01	8.25E-01	8.80E-02
Joint	[0.95, 0.95)	6.59E-03	1.06E-01	-7.14E-01
Joint	[0.95, 0.96)	8.19E-02	6.58E-01	5.21E-02
Joint	[0.96, 0.96)	1.80E-01	1.38E+00	9.47E-02
Joint	[0.96, 0.96)	1.80E-01	1.54E+00	3.43E-02
Joint	[0.96, 0.96)	8.72E-01	4.65E+00	5.14E-01
Joint	[0.96, 0.97)	3.28E-03	6.42E-02	7.52E-01
Joint	[0.97, 0.97)	1.70E-01	1.44E+00	3.92E-02
Joint	[0.97, 0.97)	9.82E-01	4.20E+00	6.62E-01
Joint	[0.97, 0.97)	2.73E-01	1.79E+00	2.11E-01
Joint	[0.97, 0.98)	8.73E-01	5.38E+00	4.21E-01
Joint	[0.98, 0.98)	8.55E-02	6.64E-01	4.46E-01
Joint	[0.98, 0.98)	8.55E-01	5.68E+00	3.43E-01
Joint	[0.98, 0.99)	8.34E-01	5.65E+00	2.79E-01
Joint	[0.99, 0.99)	8.25E-01	5.90E+00	1.35E-01
Joint	[0.99, 0.99)	4.38E-01	3.18E+00	1.26E-02
Joint	[0.99, 0.99)	8.95E-01	6.61E+00	-3.66E-02
Joint	[0.99, 1.00)	5.13E-01	3.75E+00	-2.13E-01
Joint	[1.00, 1.00)	8.99E-01	6.52E+00	-2.37E-01
Joint	[1.00, 1.00]	2.72E-01	1.79E+00	-7.88E-01

Joint	(1.00, 1.00]	1.34E+00	8.78E+00	-5.51E-01
Joint	(1.00, 1.01]	1.38E-01	1.40E+00	-4.95E-01
Joint	(1.01, 1.01]	7.18E-02	1.11E+00	-4.43E-01
Joint	(1.01, 1.01]	6.59E-01	3.82E+00	-6.69E-01
Joint	(1.01, 1.01]	2.18E-01	1.65E+00	-6.19E-01
Joint	(1.01, 1.02]	1.38E-01	1.25E+00	-6.38E-01
Joint	(1.02, 1.02]	1.18E+00	5.83E+00	-7.76E-01
Joint	(1.02, 1.02]	7.63E-02	9.53E-01	-6.67E-01
Joint	(1.02, 1.03]	8.88E-01	6.42E+00	-6.01E-01
Joint	(1.03, 1.03]	8.73E-01	6.34E+00	-6.30E-01
Joint	(1.03, 1.03]	9.14E-01	6.36E+00	-6.78E-01
Joint	(1.03, 1.03]	8.23E-01	6.23E+00	-6.75E-01
Joint	(1.03, 1.04]	7.80E-01	6.45E+00	-6.76E-01
Joint	(1.04, 1.04]	9.34E-01	6.19E+00	-7.63E-01
Joint	(1.04, 1.04]	9.52E-01	5.49E+00	-8.09E-01
Joint	(1.04, 1.04]	1.13E+00	6.17E+00	-8.38E-01
Joint	(1.04, 1.05]	1.16E+00	6.01E+00	-8.55E-01
Joint	(1.05, 1.05]	1.17E+00	6.12E+00	-8.65E-01
Joint	(1.05, 1.05]	1.29E+00	5.54E+00	-8.97E-01
Joint	(1.05, 1.06]	1.45E+00	6.07E+00	-9.10E-01
Joint	(1.06, 1.06]	1.50E+00	6.07E+00	-9.18E-01
Joint	(1.06, 1.06]	1.42E+00	6.05E+00	-9.17E-01
Joint	(1.06, 1.06]	1.37E+00	5.86E+00	-9.21E-01
Joint	(1.06, 1.07]	1.42E+00	5.81E+00	-9.28E-01
Joint	(1.07, 1.07]	1.35E+00	6.01E+00	-9.24E-01
Joint	(1.07, 1.07]	1.49E+00	5.74E+00	-9.37E-01
Joint	(1.07, 1.07]	1.40E+00	6.45E+00	-9.32E-01

Joint	(1.07, 1.08]	1.52E+00	5.49E+00	-9.39E-01
Joint	(1.08, 1.08]	1.76E-01	1.41E+00	-8.50E-01
Joint	(1.08, 1.08]	1.38E+00	5.93E+00	-9.42E-01
Joint	(1.08, 1.09]	3.66E-01	1.64E+00	-8.82E-01
Joint	(1.09, 1.09]	6.51E-01	2.36E+00	-9.16E-01
Joint	(1.09, 1.09]	1.56E+00	5.01E+00	-9.49E-01
Joint	(1.09, 1.09]	1.62E+00	5.91E+00	-9.33E-01
Joint	(1.09, 1.10]	1.62E+00	5.49E+00	-9.06E-01
Joint	(1.10, 1.10]	1.72E+00	5.46E+00	-9.39E-01
Joint	(1.10, 1.11]	5.86E-01	5.16E+00	-8.83E-01
Joint	(1.11, 1.11]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.11, 1.13]	1.38E+00	6.19E+00	-9.18E-01
Joint	(1.13, 1.14]	3.90E-01	3.39E+00	-7.61E-01
Joint	(1.14, 1.15]	2.50E-02	5.69E-02	9.97E-01
Joint	(1.15, 1.17]	7.05E-01	4.17E+00	-7.79E-01
Joint	(1.17, 1.19]	1.92E-02	4.42E-01	1.94E-01
Joint	(1.19, 1.22]	8.38E-01	2.25E+00	-6.65E-01
Joint	(1.22, 1.25]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.25, 1.28]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.28, 1.30]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.30, 1.33]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.33, 1.36]	5.93E-02	2.43E-01	-7.69E-01
Wing	(1.36,+∞)	6.05E-02	1.22E-03	-7.48E-01
Wing	(-∞, 0.58)	3.34E-01	3.41E-04	-2.75E-01
Joint	[0.58, 0.61)	3.46E-02	1.08E+00	-2.59E-01
Joint	[0.61, 0.62)	6.33E-02	8.11E-01	-5.29E-01
Joint	[0.62, 0.63)	9.00E-02	6.31E-01	-8.25E-01

Joint	[0.63, 0.64)	5.77E-01	4.79E-01	-5.59E-01
Joint	[0.64, 0.64)	1.46E+00	6.96E-04	-6.41E-01
Joint	[0.64, 0.64)	9.56E-02	7.47E-01	-8.34E-01
Joint	[0.64, 0.65)	1.51E-01	8.64E-01	-5.26E-01
Joint	[0.65, 0.65)	8.08E-01	4.81E-01	-4.56E-01
Joint	[0.65, 0.66)	1.48E-01	8.36E-01	-5.58E-01
Joint	[0.66, 0.66)	4.05E-01	6.30E-01	-6.18E-01
Joint	[0.66, 0.67)	1.77E+00	2.40E+00	6.99E-01
Joint	[0.67, 0.67)	1.87E+00	2.41E+00	7.20E-01
Joint	[0.67, 0.67)	1.65E+00	2.30E+00	6.82E-01
Joint	[0.67, 0.68)	1.67E+00	2.39E+00	6.95E-01
Joint	[0.68, 0.68)	1.90E+00	2.47E+00	7.35E-01
Joint	[0.68, 0.69)	1.74E+00	2.55E+00	7.30E-01
Joint	[0.69, 0.69)	1.63E+00	2.58E+00	7.22E-01
Joint	[0.69, 0.70)	1.63E+00	2.63E+00	7.36E-01
Joint	[0.70, 0.70)	1.65E+00	2.65E+00	7.44E-01
Joint	[0.70, 0.71)	1.53E+00	2.43E+00	6.95E-01
Joint	[0.71, 0.71)	1.83E+00	2.53E+00	7.48E-01
Joint	[0.71, 0.72)	1.36E+00	2.50E+00	6.83E-01
Joint	[0.72, 0.72)	1.54E+00	2.63E+00	7.48E-01
Joint	[0.72, 0.72)	1.06E+00	6.80E-01	3.24E-01
Joint	[0.72, 0.73)	1.10E+00	1.87E+00	5.70E-01
Joint	[0.73, 0.73)	1.50E+00	2.59E+00	7.38E-01
Joint	[0.73, 0.74)	1.13E+00	8.88E-01	4.54E-01
Joint	[0.74, 0.74)	1.23E+00	2.62E+00	6.93E-01
Joint	[0.74, 0.75)	1.19E+00	2.60E+00	7.02E-01
Joint	[0.75, 0.75)	8.25E-01	2.09E+00	5.92E-01

Joint	[0.75, 0.76)	8.60E-01	2.32E+00	6.29E-01
Joint	[0.76, 0.76)	1.35E+00	2.67E+00	7.57E-01
Joint	[0.76, 0.77)	1.16E+00	1.30E+00	5.91E-01
Joint	[0.77, 0.77)	1.25E+00	2.70E+00	7.36E-01
Joint	[0.77, 0.77)	7.74E-01	5.49E-01	2.73E-01
Joint	[0.77, 0.78)	1.44E+00	2.59E+00	7.57E-01
Joint	[0.78, 0.78)	1.02E+00	2.52E+00	6.92E-01
Joint	[0.78, 0.79)	1.03E+00	1.67E+00	6.28E-01
Joint	[0.79, 0.79)	1.19E+00	2.75E+00	7.46E-01
Joint	[0.79, 0.80)	5.79E-01	4.07E-01	8.99E-02
Joint	[0.80, 0.80)	4.15E-01	1.44E+00	3.10E-01
Joint	[0.80, 0.80)	1.19E+00	2.77E+00	7.36E-01
Joint	[0.80, 0.81)	1.05E+00	2.87E+00	7.24E-01
Joint	[0.81, 0.81)	9.13E-01	2.93E+00	7.01E-01
Joint	[0.81, 0.82)	9.52E-01	2.95E+00	7.17E-01
Joint	[0.82, 0.83)	8.91E-01	2.96E+00	7.03E-01
Joint	[0.83, 0.83)	1.19E+00	2.72E+00	7.49E-01
Joint	[0.83, 0.83)	9.35E-01	2.54E+00	6.80E-01
Joint	[0.83, 0.84)	9.08E-01	2.96E+00	7.09E-01
Joint	[0.84, 0.84)	9.62E-01	2.81E+00	7.09E-01
Joint	[0.84, 0.85)	6.62E-01	2.49E+00	5.95E-01
Joint	[0.85, 0.85)	6.34E-01	3.02E+00	6.15E-01
Joint	[0.85, 0.86)	1.05E+00	2.81E+00	7.40E-01
Joint	[0.86, 0.86)	9.63E-01	2.90E+00	7.20E-01
Joint	[0.86, 0.86)	7.05E-01	3.09E+00	6.47E-01
Joint	[0.86, 0.87)	6.06E-01	3.29E+00	6.42E-01
Joint	[0.87, 0.87)	9.30E-01	2.90E+00	7.22E-01

Joint	[0.87, 0.88)	9.66E-01	2.72E+00	7.27E-01
Joint	[0.88, 0.88)	6.57E-01	2.76E+00	6.03E-01
Joint	[0.88, 0.89)	5.10E-01	3.23E+00	5.25E-01
Joint	[0.89, 0.89)	9.93E-01	2.79E+00	7.44E-01
Joint	[0.89, 0.90)	7.99E-01	3.05E+00	6.76E-01
Joint	[0.90, 0.90)	6.95E-01	3.19E+00	6.29E-01
Joint	[0.90, 0.91)	8.53E-01	3.00E+00	7.01E-01
Joint	[0.91, 0.91)	6.87E-02	7.62E-01	-2.05E-01
Joint	[0.91, 0.91)	8.19E-02	7.03E-01	-1.74E-01
Joint	[0.91, 0.92)	1.01E-01	8.67E-01	-9.34E-02
Joint	[0.92, 0.92)	1.65E-02	5.87E-01	-4.36E-01
Joint	[0.92, 0.93)	3.36E-02	4.74E-01	-3.85E-01
Joint	[0.93, 0.93)	8.64E-02	7.67E-01	-1.23E-01
Joint	[0.93, 0.94)	1.07E-01	8.87E-01	-6.85E-02
Joint	[0.94, 0.94)	1.60E-02	2.19E-01	-7.39E-01
Joint	[0.94, 0.95)	2.17E-02	6.34E-01	-4.17E-01
Joint	[0.95, 0.95)	2.49E-02	5.69E-01	-3.77E-01
Joint	[0.95, 0.96)	2.84E-01	1.87E+00	1.97E-01
Joint	[0.96, 0.96)	4.83E-02	6.67E-01	-2.55E-01
Joint	[0.96, 0.96)	1.10E-01	1.14E+00	-2.58E-01
Joint	[0.96, 0.97)	5.69E-01	3.88E+00	3.42E-01
Joint	[0.97, 0.97)	5.47E-01	3.65E+00	3.31E-01
Joint	[0.97, 0.98)	5.48E-01	3.87E+00	2.57E-01
Joint	[0.98, 0.98)	5.70E-01	4.29E+00	1.67E-01
Joint	[0.98, 0.99)	5.97E-01	4.69E+00	4.08E-03
Joint	[0.99, 0.99)	6.90E-01	5.59E+00	-1.21E-01
Joint	[0.99, 0.99)	5.87E-01	4.61E+00	-1.41E-01

Joint	[0.99, 1.00)	6.67E-01	5.22E+00	-2.36E-01
Joint	[1.00, 1.00]	1.96E-01	1.43E+00	-6.45E-01
Joint	(1.00, 1.01]	7.27E-01	5.20E+00	-5.01E-01
Joint	(1.01, 1.02]	1.31E-01	1.13E+00	-5.58E-01
Joint	(1.02, 1.02]	1.45E-01	1.10E+00	-6.57E-01
Joint	(1.02, 1.02]	1.04E-01	9.80E-01	-5.79E-01
Joint	(1.02, 1.03]	8.24E-01	6.31E+00	-5.41E-01
Joint	(1.03, 1.03]	7.92E-01	6.30E+00	-5.69E-01
Joint	(1.03, 1.04]	8.84E-01	6.31E+00	-6.74E-01
Joint	(1.04, 1.04]	8.73E-01	6.18E+00	-7.15E-01
Joint	(1.04, 1.05]	2.61E-01	2.42E+00	-6.18E-01
Joint	(1.05, 1.05]	4.11E-01	3.02E+00	-7.12E-01
Joint	(1.05, 1.05]	4.23E-01	2.74E+00	-7.65E-01
Joint	(1.05, 1.06]	1.07E+00	6.06E+00	-8.48E-01
Joint	(1.06, 1.06]	6.42E-01	3.31E+00	-8.54E-01
Joint	(1.06, 1.07]	4.28E-01	2.25E+00	-8.48E-01
Joint	(1.07, 1.07]	6.84E-01	3.19E+00	-8.83E-01
Joint	(1.07, 1.08]	1.18E+00	6.03E+00	-9.00E-01
Joint	(1.08, 1.08]	1.21E+00	5.62E+00	-9.12E-01
Joint	(1.08, 1.09]	1.15E+00	5.38E+00	-9.16E-01
Joint	(1.09, 1.09]	1.40E+00	5.49E+00	-9.34E-01
Joint	(1.09, 1.10]	1.48E+00	5.45E+00	-9.41E-01
Joint	(1.10, 1.10]	1.46E+00	5.72E+00	-9.41E-01
Joint	(1.10, 1.10]	1.15E+00	5.73E+00	-9.30E-01
Joint	(1.10, 1.11]	1.49E+00	5.07E+00	-9.48E-01
Joint	(1.11, 1.11]	3.77E-01	1.83E+00	-8.93E-01
Wing	(1.11,+∞)	5.93E-02	2.43E-01	-7.69E-01

Wing	(-∞, 0.38)	5.61E-02	9.36E-01	-5.07E-01
Joint	[0.38, 0.44)	1.00E+00	4.59E-01	-5.05E-01
Joint	[0.44, 0.46)	2.03E-02	1.35E+00	-1.91E-01
Joint	[0.46, 0.49)	2.03E-02	1.28E+00	-1.96E-01
Joint	[0.49, 0.54)	3.30E-02	1.49E+00	3.38E-03
Joint	[0.54, 0.56)	1.32E+00	1.51E+00	6.71E-01
Joint	[0.56, 0.56)	1.19E+00	1.51E+00	6.61E-01
Joint	[0.56, 0.57)	1.07E+00	5.80E-01	3.61E-01
Joint	[0.57, 0.59)	1.21E+00	1.55E+00	7.01E-01
Joint	[0.59, 0.60)	1.04E+00	6.15E-01	4.14E-01
Joint	[0.60, 0.61)	1.04E+00	1.61E+00	6.90E-01
Joint	[0.61, 0.63)	1.02E+00	1.60E+00	7.21E-01
Joint	[0.63, 0.64)	4.90E-01	1.47E+00	4.82E-01
Joint	[0.64, 0.65)	6.43E-01	4.08E-01	1.46E-01
Joint	[0.65, 0.67)	4.54E-01	1.38E+00	4.05E-01
Joint	[0.67, 0.68)	1.10E+00	1.52E+00	7.08E-01
Joint	[0.68, 0.69)	6.92E-01	1.77E+00	6.92E-01
Joint	[0.69, 0.71)	6.89E-01	9.73E-01	4.89E-01
Joint	[0.71, 0.72)	7.01E-01	1.74E+00	6.62E-01
Joint	[0.72, 0.74)	6.91E-01	1.73E+00	6.75E-01
Joint	[0.74, 0.75)	7.92E-01	1.64E+00	6.92E-01
Joint	[0.75, 0.76)	5.59E-01	1.81E+00	6.24E-01
Joint	[0.76, 0.78)	5.69E-01	1.83E+00	6.48E-01
Joint	[0.78, 0.79)	4.41E-01	1.87E+00	5.63E-01
Joint	[0.79, 0.80)	5.71E-01	1.79E+00	6.41E-01
Joint	[0.80, 0.82)	4.90E-01	1.83E+00	5.91E-01

(0.25, 0.43]

Joint	[0.82, 0.83)	4.71E-01	1.92E+00	5.99E-01
Joint	[0.83, 0.84)	4.25E-01	1.96E+00	5.63E-01
Joint	[0.84, 0.85)	4.48E-01	1.94E+00	5.83E-01
Joint	[0.85, 0.86)	4.40E-01	1.98E+00	5.87E-01
Joint	[0.86, 0.87)	4.36E-01	1.95E+00	5.74E-01
Joint	[0.87, 0.89)	4.38E-01	1.95E+00	5.75E-01
Joint	[0.89, 0.90)	3.95E-01	2.03E+00	5.18E-01
Joint	[0.90, 0.91)	9.49E-02	7.34E-01	-2.20E-01
Joint	[0.91, 0.93)	9.58E-02	7.22E-01	-2.09E-01
Joint	[0.93, 0.94)	7.15E-02	6.02E-01	-3.55E-01
Joint	[0.94, 0.95)	1.63E-01	1.18E+00	-6.49E-03
Joint	[0.95, 0.97)	1.16E-01	8.42E-01	-1.01E-01
Joint	[0.97, 0.98)	3.23E-01	2.69E+00	-1.09E-01
Joint	[0.98, 0.99)	2.12E-01	1.70E+00	-2.60E-01
Joint	[0.99, 1.01]	2.06E-02	2.88E-01	-7.67E-01
Joint	(1.01, 1.02]	2.47E-01	1.76E+00	-5.57E-01
Joint	(1.02, 1.04]	8.28E-01	6.63E+00	8.51E-02
Joint	(1.04, 1.05]	1.00E+00	6.07E+00	-7.75E-01
Joint	(1.05, 1.06]	8.98E-01	5.67E+00	-7.81E-01
Joint	(1.06, 1.08]	9.65E-01	5.41E+00	-8.43E-01
Joint	(1.08, 1.09]	1.52E+00	5.52E+00	-9.37E-01
Joint	(1.09, 1.10]	1.65E+00	5.43E+00	-9.49E-01
Joint	(1.10, 1.12]	1.51E+00	4.69E+00	-9.54E-01
Joint	(1.12, 1.13]	1.42E+00	4.24E+00	-9.56E-01
Joint	(1.13, 1.14]	1.43E+00	4.35E+00	-9.55E-01
Joint	(1.14, 1.20]	5.79E-01	1.93E+00	-9.28E-01
Wing	(1.20,+∞)	5.93E-02	2.43E-01	-7.69E-01

Wing	(-∞, 0.44)	4.90E-03	8.79E-01	-5.45E-01
Joint	[0.44, 0.46)	8.32E-01	9.39E-01	5.93E-01
Joint	[0.46, 0.48)	8.11E-01	9.37E-01	6.13E-01
Joint	[0.48, 0.49)	8.00E-01	9.31E-01	6.20E-01
Joint	[0.49, 0.51)	6.88E-01	9.50E-01	5.41E-01
Joint	[0.51, 0.52)	7.25E-01	9.72E-01	6.18E-01
Joint	[0.52, 0.53)	7.00E-01	9.79E-01	6.12E-01
Joint	[0.53, 0.55)	6.53E-01	9.95E-01	5.90E-01
Joint	[0.55, 0.56)	6.30E-01	1.01E+00	5.93E-01
Joint	[0.56, 0.57)	6.20E-01	1.03E+00	6.06E-01
Joint	[0.57, 0.59)	5.68E-01	1.05E+00	6.12E-01
Joint	[0.59, 0.60)	4.33E-01	5.38E-01	2.58E-01
Joint	[0.60, 0.62)	5.31E-01	1.08E+00	6.09E-01
Joint	[0.62, 0.63)	5.08E-01	1.06E+00	5.98E-01
Joint	[0.63, 0.64)	4.40E-01	1.09E+00	5.57E-01
Joint	[0.64, 0.66)	5.67E-01	1.00E+00	6.29E-01
Joint	[0.66, 0.67)	3.65E-01	1.18E+00	5.22E-01
Joint	[0.67, 0.68)	2.84E-01	1.18E+00	4.09E-01
Joint	[0.68, 0.70)	3.99E-01	1.17E+00	5.76E-01
Joint	[0.70, 0.71)	3.53E-01	1.20E+00	5.37E-01
Joint	[0.71, 0.72)	3.73E-01	1.18E+00	5.62E-01
Joint	[0.72, 0.74)	3.69E-01	1.18E+00	5.62E-01
Joint	[0.74, 0.75)	3.36E-01	1.20E+00	5.24E-01
Joint	[0.75, 0.77)	3.39E-01	1.21E+00	5.43E-01
Joint	[0.77, 0.78)	3.00E-01	1.27E+00	5.10E-01
Joint	[0.78, 0.79)	2.90E-01	1.27E+00	4.90E-01

(0.43, 0.68]

Joint	[0.79, 0.81)	2.75E-01	1.31E+00	4.75E-01
Joint	[0.81, 0.82)	2.62E-01	1.31E+00	4.41E-01
Joint	[0.82, 0.83)	2.60E-01	1.34E+00	4.48E-01
Joint	[0.83, 0.85)	2.48E-01	1.37E+00	4.20E-01
Joint	[0.85, 0.86)	2.49E-01	1.35E+00	4.17E-01
Joint	[0.86, 0.87)	2.31E-01	1.43E+00	3.64E-01
Joint	[0.87, 0.89)	5.93E-02	2.43E-01	-7.69E-01
Joint	[0.89, 0.90)	9.80E-02	6.54E-01	-2.32E-01
Joint	[0.90, 0.92)	8.38E-02	5.88E-01	-3.65E-01
Joint	[0.92, 0.93)	6.00E-02	4.05E-01	-6.05E-01
Joint	[0.93, 0.94)	2.06E-01	1.61E+00	1.00E-01
Joint	[0.94, 0.96)	2.06E-01	1.66E+00	5.61E-02
Joint	[0.96, 0.97)	6.06E-02	4.15E-01	-6.16E-01
Joint	[0.97, 0.98)	2.45E-01	2.18E+00	-1.64E-01
Joint	[0.98, 1.00)	2.85E-01	2.54E+00	-2.97E-01
Joint	[1.00, 1.01]	1.09E-01	7.89E-01	-5.14E-01
Joint	(1.01, 1.03]	5.39E-01	2.39E+00	2.73E-03
Joint	(1.03, 1.04]	8.26E-01	6.58E+00	-6.61E-01
Joint	(1.04, 1.05]	8.36E-01	5.98E+00	-7.32E-01
Joint	(1.05, 1.07]	1.10E-01	1.08E+00	-4.32E-01
Joint	(1.07, 1.08]	1.07E+00	6.20E+00	-8.48E-01
Joint	(1.08, 1.09]	1.00E+00	6.06E+00	-8.46E-01
Joint	(1.09, 1.11]	1.11E+00	5.92E+00	-8.85E-01
Joint	(1.11, 1.12]	9.05E-01	5.69E+00	-8.66E-01
Joint	(1.12, 1.13]	1.92E-01	1.40E+00	-7.90E-01
Joint	(1.13, 1.15]	1.05E+00	5.29E+00	-9.15E-01
Joint	(1.15, 1.16]	1.12E+00	4.67E+00	-9.36E-01

Joint	(1.16, 1.18]	1.32E+00	4.79E+00	-9.50E-01
Joint	(1.18, 1.19]	1.26E+00	4.29E+00	-9.54E-01
Joint	(1.19, 1.20]	1.91E-01	1.29E+00	-8.63E-01
Joint	(1.20, 1.23]	3.22E-01	1.48E+00	-9.16E-01
Joint	(1.23, 1.26]	8.00E-01	6.35E+00	-9.33E-01
Wing	(1.26,+∞)	6.47E-02	1.22E-03	-8.68E-01
Wing	(-∞, 0.22)	2.39E-01	5.83E-01	-6.39E-01
Joint	[0.22, 0.25)	9.21E-03	8.67E-01	-3.67E-01
Joint	[0.25, 0.27)	7.64E-01	5.78E-01	3.76E-01
Joint	[0.27, 0.30)	6.85E-01	5.79E-01	4.97E-01
Joint	[0.30, 0.33)	7.22E-01	5.41E-01	4.00E-01
Joint	[0.33, 0.36)	6.21E-01	6.09E-01	5.85E-01
Joint	[0.36, 0.38)	6.19E-01	2.97E-01	3.36E-01
Joint	[0.38, 0.41)	5.57E-01	6.72E-01	5.62E-01
Joint	[0.41, 0.43)	5.33E-01	4.35E-01	3.25E-01
Joint	[0.43, 0.44)	5.10E-01	7.10E-01	5.19E-01
Joint	[0.44, 0.45)	5.19E-01	7.03E-01	5.12E-01
Joint	[0.45, 0.47)	4.83E-01	7.24E-01	4.97E-01
Joint	[0.47, 0.48)	4.90E-01	7.25E-01	5.19E-01
Joint	[0.48, 0.49)	5.48E-01	6.57E-01	5.27E-01
Joint	[0.49, 0.51)	4.62E-01	7.55E-01	5.25E-01
Joint	[0.51, 0.52)	2.67E-02	7.70E-01	-3.37E-01
Joint	[0.52, 0.54)	4.26E-01	7.76E-01	5.34E-01
Joint	[0.54, 0.55)	4.83E-01	7.03E-01	5.39E-01
Joint	[0.55, 0.56)	2.99E-01	8.71E-01	5.24E-01
Joint	[0.56, 0.58)	1.73E-01	7.49E-01	8.93E-02
Joint	[0.58, 0.59)	3.10E-01	8.57E-01	5.98E-01

Joint	[0.59, 0.60)	4.16E-01	7.44E-01	5.54E-01
Joint	[0.60, 0.62)	3.15E-01	8.72E-01	5.14E-01
Joint	[0.62, 0.63)	1.75E-01	9.47E-01	2.37E-01
Joint	[0.63, 0.64)	3.21E-01	8.54E-01	5.05E-01
Joint	[0.64, 0.66)	1.64E-01	1.01E+00	2.47E-01
Joint	[0.66, 0.67)	2.77E-01	8.95E-01	4.63E-01
Joint	[0.67, 0.69)	1.67E-01	1.00E+00	2.28E-01
Joint	[0.69, 0.70)	2.18E-01	9.70E-01	3.79E-01
Joint	[0.70, 0.71)	1.93E-01	1.01E+00	3.31E-01
Joint	[0.71, 0.73)	2.41E-01	9.60E-01	4.76E-01
Joint	[0.73, 0.74)	2.16E-01	9.91E-01	4.09E-01
Joint	[0.74, 0.75)	2.09E-01	1.00E+00	3.88E-01
Joint	[0.75, 0.77)	1.88E-01	1.02E+00	3.13E-01
Joint	[0.77, 0.78)	1.76E-01	1.02E+00	2.57E-01
Joint	[0.78, 0.80)	1.87E-01	1.05E+00	3.28E-01
Joint	[0.80, 0.81)	1.92E-01	1.02E+00	3.38E-01
Joint	[0.81, 0.82)	1.85E-01	1.08E+00	3.31E-01
Joint	[0.82, 0.84)	1.49E-01	1.29E+00	1.11E-01
Joint	[0.84, 0.85)	1.74E-01	1.11E+00	2.61E-01
Joint	[0.85, 0.86)	1.57E-01	1.29E+00	1.06E-01
Joint	[0.86, 0.88)	1.57E-01	1.34E+00	4.60E-02
Joint	[0.88, 0.89)	1.59E-01	1.34E+00	4.40E-02
Joint	[0.89, 0.91)	1.52E-01	1.11E+00	6.52E-02
Joint	[0.91, 0.92)	1.60E-01	1.40E+00	-9.99E-02
Joint	[0.92, 0.93)	5.40E-02	2.30E-01	-7.53E-01
Joint	[0.93, 0.95)	1.59E-01	1.29E+00	-3.89E-02
Joint	[0.95, 0.96)	1.62E-01	1.35E+00	-1.14E-01

Joint	[0.96, 0.97)	1.86E-01	1.64E+00	-2.47E-01
Joint	[0.97, 0.99)	2.04E-01	1.84E+00	-2.74E-01
Joint	[0.99, 1.00]	1.32E-01	1.01E+00	-4.59E-01
Joint	(1.00, 1.02]	9.13E-02	6.20E-01	-5.56E-01
Joint	(1.02, 1.03]	3.75E-01	3.82E+00	-5.56E-01
Joint	(1.03, 1.04]	4.83E-01	3.88E+00	-6.58E-01
Joint	(1.04, 1.06]	1.11E-01	1.07E+00	-3.19E-01
Joint	(1.06, 1.07]	8.81E-01	6.39E+00	-7.67E-01
Joint	(1.07, 1.08]	4.80E-01	3.17E+00	-7.91E-01
Joint	(1.08, 1.10]	8.56E-01	6.34E+00	-7.84E-01
Joint	(1.10, 1.11]	4.45E-01	3.18E+00	-7.88E-01
Joint	(1.11, 1.12]	7.33E-01	5.96E+00	-7.95E-01
Joint	(1.12, 1.14]	1.87E-01	1.51E+00	-7.42E-01
Joint	(1.14, 1.15]	8.31E-01	5.81E+00	-8.61E-01
Joint	(1.15, 1.17]	2.88E-01	2.03E+00	-8.32E-01
Joint	(1.17, 1.18]	1.49E-01	1.06E+00	-8.00E-01
Joint	(1.18, 1.19]	1.92E-01	1.21E+00	-8.40E-01
Joint	(1.19, 1.21]	9.06E-01	5.25E+00	-9.15E-01
Joint	(1.21, 1.23]	7.66E-01	2.53E+00	-9.58E-01
Joint	(1.23, 1.29]	1.94E+00	6.47E+00	-9.66E-01
Joint	(1.29, 1.34]	1.63E+00	6.14E+00	-9.60E-01
Wing	(1.34,+∞)	5.93E-02	2.43E-01	-7.69E-01
Wing	(-∞, 0.22)	1.07E-01	8.92E-01	-5.40E-01
Joint	[0.22, 0.25)	1.70E-02	1.13E+00	-4.29E-01
Joint	[0.25, 0.27)	3.71E-03	9.29E-01	-6.44E-01
Joint	[0.27, 0.30)	5.13E-03	1.07E+00	-2.66E-01
Joint	[0.30, 0.33)	6.29E-01	6.13E-01	4.06E-01

Joint	[0.33, 0.36)	5.61E-01	6.08E-01	5.30E-01
Joint	[0.36, 0.38)	5.73E-01	5.95E-01	4.29E-01
Joint	[0.38, 0.41)	5.33E-01	6.25E-01	5.93E-01
Joint	[0.41, 0.43)	4.66E-01	6.55E-01	5.43E-01
Joint	[0.43, 0.44)	4.86E-01	6.23E-01	4.81E-01
Joint	[0.44, 0.45)	4.50E-01	6.67E-01	5.12E-01
Joint	[0.45, 0.47)	4.29E-01	6.75E-01	4.84E-01
Joint	[0.47, 0.48)	4.28E-01	6.76E-01	4.82E-01
Joint	[0.48, 0.49)	4.11E-01	7.01E-01	5.04E-01
Joint	[0.49, 0.51)	4.07E-01	7.05E-01	5.06E-01
Joint	[0.51, 0.52)	3.93E-01	7.20E-01	5.11E-01
Joint	[0.52, 0.54)	3.71E-01	7.32E-01	4.92E-01
Joint	[0.54, 0.55)	3.57E-01	7.50E-01	5.12E-01
Joint	[0.55, 0.56)	2.39E-01	7.98E-01	3.10E-01
Joint	[0.56, 0.58)	3.11E-01	7.78E-01	4.63E-01
Joint	[0.58, 0.59)	9.91E-02	4.54E-01	-4.62E-01
Joint	[0.59, 0.60)	2.81E-01	8.09E-01	4.56E-01
Joint	[0.60, 0.62)	2.20E-01	8.33E-01	3.28E-01
Joint	[0.62, 0.63)	2.81E-01	8.14E-01	4.82E-01
Joint	[0.63, 0.65)	2.61E-01	8.37E-01	4.71E-01
Joint	[0.65, 0.66)	2.45E-01	8.59E-01	4.77E-01
Joint	[0.66, 0.67)	2.68E-01	8.21E-01	4.81E-01
Joint	[0.67, 0.69)	1.54E-01	8.66E-01	1.71E-01
Joint	[0.69, 0.70)	1.64E-01	9.80E-01	3.15E-01
Joint	[0.70, 0.71)	2.15E-01	8.79E-01	3.99E-01
Joint	[0.71, 0.73)	2.00E-01	9.22E-01	3.92E-01
Joint	[0.73, 0.74)	1.86E-01	9.51E-01	3.63E-01

Joint	[0.74, 0.75)	1.71E-01	9.65E-01	2.96E-01
Joint	[0.75, 0.77)	2.03E-01	8.96E-01	4.00E-01
Joint	[0.77, 0.78)	1.76E-01	9.71E-01	3.16E-01
Joint	[0.78, 0.80)	1.64E-01	9.87E-01	2.53E-01
Joint	[0.80, 0.81)	1.13E-01	1.59E+00	-1.87E-01
Joint	[0.81, 0.82)	1.42E-01	1.16E+00	1.38E-01
Joint	[0.82, 0.84)	1.29E-01	1.53E+00	-1.84E-01
Joint	[0.84, 0.85)	1.42E-01	1.25E+00	6.46E-02
Joint	[0.85, 0.86)	1.44E-01	1.26E+00	4.35E-02
Joint	[0.86, 0.88)	1.49E-01	1.20E+00	1.54E-01
Joint	[0.88, 0.89)	1.11E-01	7.62E-01	-1.09E-01
Joint	[0.89, 0.91)	1.47E-01	1.09E+00	7.87E-02
Joint	[0.91, 0.92)	1.47E-01	1.17E+00	-2.99E-03
Joint	[0.92, 0.93)	1.52E-01	1.25E+00	9.66E-03
Joint	[0.93, 0.95)	1.52E-01	1.28E+00	-8.69E-02
Joint	[0.95, 0.96)	1.37E-01	1.13E+00	-2.30E-01
Joint	[0.96, 0.97)	1.66E-01	1.47E+00	-1.78E-01
Joint	[0.97, 0.99)	1.93E-01	1.74E+00	-3.26E-01
Joint	[0.99, 1.00]	2.07E-01	1.88E+00	-3.62E-01
Joint	(1.00, 1.02]	2.76E-01	2.49E+00	-4.84E-01
Joint	(1.02, 1.03]	3.08E-01	3.19E+00	-5.16E-01
Joint	(1.03, 1.04]	4.46E-01	3.73E+00	-6.40E-01
Joint	(1.04, 1.06]	1.02E+00	6.44E+00	2.50E-01
Joint	(1.06, 1.07]	8.72E-01	6.24E+00	-7.86E-01
Joint	(1.07, 1.08]	4.22E-01	2.85E+00	-7.82E-01
Joint	(1.08, 1.10]	8.09E-01	6.34E+00	-7.64E-01
Joint	(1.10, 1.15]	7.37E-02	1.10E+00	-5.10E-01

Joint	(1.15, 1.21]	1.08E+00	4.62E+00	-9.41E-01
Wing	(1.21,+∞)	1.05E-01	3.48E-01	-9.98E-01
Wing	(-∞, 0.22)	4.62E-01	3.87E-01	1.72E-01
Joint	[0.22, 0.25)	4.09E-01	4.13E-01	2.48E-01
Joint	[0.25, 0.28)	4.68E-01	2.62E-01	1.39E-01
Joint	[0.28, 0.30)	4.25E-01	4.22E-01	2.24E-01
Joint	[0.30, 0.33)	3.58E-01	4.76E-01	3.54E-01
Joint	[0.33, 0.36)	3.37E-01	4.81E-01	3.89E-01
Joint	[0.36, 0.39)	3.29E-01	4.81E-01	3.52E-01
Joint	[0.39, 0.41)	3.77E-01	3.36E-01	3.28E-01
Joint	[0.41, 0.43)	2.98E-01	5.32E-01	3.94E-01
Joint	[0.43, 0.44)	2.93E-01	5.23E-01	3.72E-01
Joint	[0.44, 0.46)	2.78E-01	5.39E-01	3.98E-01
Joint	[0.46, 0.47)	3.11E-01	3.38E-01	3.14E-01
Joint	[0.47, 0.48)	2.51E-01	5.70E-01	3.57E-01
Joint	[0.48, 0.50)	2.48E-01	5.62E-01	3.66E-01
Joint	[0.50, 0.51)	2.24E-01	5.91E-01	3.84E-01
Joint	[0.51, 0.52)	2.08E-01	6.08E-01	3.32E-01
Joint	[0.52, 0.54)	2.27E-01	5.89E-01	3.87E-01
Joint	[0.54, 0.55)	1.65E-01	6.66E-01	2.51E-01
Joint	[0.55, 0.57)	1.42E-01	6.55E-01	1.08E-01
Joint	[0.57, 0.58)	2.09E-01	6.05E-01	3.69E-01
Joint	[0.58, 0.59)	1.84E-01	6.37E-01	3.10E-01
Joint	[0.59, 0.61)	1.54E-01	6.67E-01	2.00E-01
Joint	[0.61, 0.62)	1.69E-01	6.69E-01	3.18E-01
Joint	[0.62, 0.63)	1.68E-01	6.69E-01	3.14E-01
Joint	[0.63, 0.65)	1.69E-01	6.69E-01	3.28E-01

Joint	[0.65, 0.66)	1.61E-01	6.82E-01	2.96E-01
Joint	[0.66, 0.68)	1.48E-01	7.15E-01	2.70E-01
Joint	[0.68, 0.69)	1.48E-01	7.15E-01	2.65E-01
Joint	[0.69, 0.70)	1.10E-01	8.56E-01	9.37E-02
Joint	[0.70, 0.72)	8.85E-02	1.07E+00	-1.66E-01
Joint	[0.72, 0.73)	9.67E-02	9.60E-01	-1.31E-01
Joint	[0.73, 0.75)	9.67E-02	1.04E+00	-1.60E-01
Joint	[0.75, 0.76)	9.66E-02	1.37E+00	-3.58E-01
Joint	[0.76, 0.77)	1.01E-01	1.11E+00	-1.71E-01
Joint	[0.77, 0.79)	1.03E-01	1.19E+00	-1.99E-01
Joint	[0.79, 0.80)	1.10E-01	1.32E+00	-3.14E-01
Joint	[0.80, 0.81)	1.22E-01	1.52E+00	-4.23E-01
Joint	[0.81, 0.83)	1.15E-01	1.19E+00	-1.99E-01
Joint	[0.83, 0.84)	1.17E-01	1.08E+00	-1.30E-01
Joint	[0.84, 0.86)	1.21E-01	1.19E+00	-2.80E-01
Joint	[0.86, 0.87)	1.24E-01	1.23E+00	-2.00E-01
Joint	[0.87, 0.88)	1.26E-01	1.21E+00	-1.88E-01
Joint	[0.88, 0.90)	1.29E-01	1.21E+00	-1.91E-01
Joint	[0.90, 0.91)	3.04E-01	1.99E+00	-4.06E-01
Joint	[0.91, 0.92)	1.24E-01	1.03E+00	-1.54E-01
Joint	[0.92, 0.94)	1.26E-01	1.06E+00	-1.60E-01
Joint	[0.94, 0.95)	1.32E-01	1.15E+00	-2.25E-01
Joint	[0.95, 0.97)	1.36E-01	1.21E+00	-2.31E-01
Joint	[0.97, 0.98)	1.46E-01	1.30E+00	-3.20E-01
Joint	[0.98, 0.99)	1.59E-01	1.45E+00	-3.51E-01
Joint	[0.99, 1.01]	1.04E-01	6.76E-01	-7.09E-01
Joint	(1.01, 1.02]	8.65E-01	6.55E+00	-2.82E-01

Joint	(1.02, 1.03]	1.10E-01	9.22E-01	-3.07E-01
Joint	(1.03, 1.05]	1.20E+00	6.53E+00	1.15E-01
Joint	(1.05, 1.06]	7.28E-01	6.19E+00	-7.27E-01
Joint	(1.06, 1.08]	4.43E-01	4.02E+00	-5.73E-01
Joint	(1.08, 1.09]	3.41E-01	2.79E+00	1.88E-01
Joint	(1.09, 1.10]	5.60E-01	5.74E+00	-2.97E-01
Joint	(1.10, 1.12]	2.85E-01	2.76E+00	2.62E-01
Joint	(1.12, 1.13]	7.60E-01	6.52E+00	-7.50E-01
Joint	(1.13, 1.15]	4.34E-01	5.73E+00	-2.95E-01
Joint	(1.15, 1.16]	7.63E-01	6.23E+00	-7.92E-01
Joint	(1.16, 1.19]	9.74E-01	5.30E+00	-9.03E-01
Joint	(1.19, 1.21]	1.02E+00	5.17E+00	-9.18E-01
Joint	(1.21, 1.24]	9.76E-01	4.65E+00	-9.27E-01
Joint	(1.24, 1.27]	1.09E+00	4.99E+00	-9.33E-01
Joint	(1.27, 1.32]	1.14E+00	4.71E+00	-9.45E-01
Joint	(1.32, 1.38]	1.35E+00	5.59E+00	-9.46E-01
Wing	(1.38,+∞)	8.99E-02	1.98E+00	1.83E-01
Wing	(-∞, 0.11)	4.08E-01	3.18E-01	-2.44E-01
Joint	[0.11, 0.17)	3.43E-01	3.13E-01	-1.02E-01
Joint	[0.17, 0.22)	3.03E-01	3.24E-01	8.36E-02
Joint	[0.22, 0.28)	3.11E-01	3.18E-01	1.06E-01
Joint	[0.28, 0.31)	2.80E-01	3.69E-01	2.42E-01
Joint	[0.31, 0.33)	2.56E-01	3.94E-01	2.01E-01
Joint	[0.33, 0.36)	2.34E-01	4.11E-01	2.58E-01
Joint	[0.36, 0.39)	2.28E-01	4.12E-01	2.43E-01
Joint	[0.39, 0.42)	2.00E-01	4.50E-01	2.25E-01
Joint	[0.42, 0.44)	1.37E-01	5.39E-01	7.33E-02

Joint	[0.44, 0.47)	1.67E-01	4.90E-01	2.12E-01
Joint	[0.47, 0.50)	1.63E-01	4.97E-01	2.34E-01
Joint	[0.50, 0.53)	1.56E-01	5.08E-01	2.44E-01
Joint	[0.53, 0.54)	1.36E-01	5.43E-01	1.38E-01
Joint	[0.54, 0.55)	1.28E-01	5.66E-01	1.38E-01
Joint	[0.55, 0.57)	1.25E-01	5.72E-01	1.23E-01
Joint	[0.57, 0.58)	1.33E-01	5.48E-01	1.52E-01
Joint	[0.58, 0.60)	1.18E-01	5.94E-01	8.32E-02
Joint	[0.60, 0.61)	1.22E-01	5.84E-01	1.22E-01
Joint	[0.61, 0.62)	1.18E-01	5.99E-01	1.10E-01
Joint	[0.62, 0.64)	1.18E-01	5.97E-01	1.06E-01
Joint	[0.64, 0.65)	1.15E-01	6.13E-01	1.01E-01
Joint	[0.65, 0.67)	1.04E-01	6.19E-01	-3.69E-02
Joint	[0.67, 0.68)	7.96E-02	1.03E+00	-3.35E-01
Joint	[0.68, 0.69)	8.29E-02	9.78E-01	-2.83E-01
Joint	[0.69, 0.71)	8.61E-02	1.10E+00	-3.88E-01
Joint	[0.71, 0.72)	8.95E-02	9.09E-01	-2.12E-01
Joint	[0.72, 0.74)	9.00E-02	1.05E+00	-2.97E-01
Joint	[0.74, 0.75)	1.01E-01	1.32E+00	-4.84E-01
Joint	[0.75, 0.76)	9.42E-02	1.02E+00	-2.68E-01
Joint	[0.76, 0.78)	1.01E-01	1.10E+00	-3.46E-01
Joint	[0.78, 0.79)	1.13E-01	1.37E+00	-4.58E-01
Joint	[0.79, 0.80)	1.03E-01	9.77E-01	-2.98E-01
Joint	[0.80, 0.82)	1.95E-01	2.55E+00	-6.96E-01
Joint	[0.82, 0.83)	1.09E-01	1.07E+00	-4.44E-01
Joint	[0.83, 0.85)	1.14E-01	1.08E+00	-4.22E-01
Joint	[0.85, 0.86)	5.74E-01	4.30E+00	-4.09E-01

Joint	[0.86, 0.87)	5.86E-01	4.30E+00	-3.92E-01
Joint	[0.87, 0.89)	5.58E-01	4.63E+00	-4.45E-01
Joint	[0.89, 0.90)	3.70E-01	2.79E+00	-3.13E-01
Joint	[0.90, 0.92)	2.10E-01	1.48E+00	-3.68E-01
Joint	[0.92, 0.93)	5.93E-02	2.43E-01	-7.69E-01
Joint	[0.93, 0.94)	5.86E-01	4.85E+00	-3.83E-01
Joint	[0.94, 0.96)	5.93E-02	2.43E-01	-7.69E-01
Joint	[0.96, 0.97)	1.09E-01	9.11E-01	-2.26E-01
Joint	[0.97, 0.98)	1.15E-01	9.13E-01	-4.35E-01
Joint	[0.98, 1.00)	8.06E-02	4.97E-01	-5.68E-01
Joint	[1.00, 1.01]	1.01E-01	6.50E-01	-7.22E-01
Joint	(1.01, 1.03]	5.66E-01	3.52E+00	-1.50E-01
Joint	(1.03, 1.04]	7.43E-02	4.49E-01	-5.35E-01
Joint	(1.04, 1.05]	5.93E-02	2.43E-01	-7.69E-01
Joint	(1.05, 1.07]	7.42E-01	6.07E+00	-2.97E-01
Joint	(1.07, 1.08]	3.25E-01	2.34E+00	-7.91E-01
Joint	(1.08, 1.10]	3.11E-01	3.21E+00	-4.05E-01
Joint	(1.10, 1.11]	8.14E-01	6.62E+00	-1.64E-01
Joint	(1.11, 1.12]	8.02E-01	6.73E+00	-1.11E-01
Joint	(1.12, 1.14]	6.55E-01	5.66E+00	-8.78E-02
Joint	(1.14, 1.17]	6.95E-01	6.51E+00	-6.99E-01
Joint	(1.17, 1.22]	1.02E+00	6.38E+00	-8.77E-01
Joint	(1.22, 1.25]	1.09E+00	6.18E+00	-9.01E-01
Joint	(1.25, 1.39]	1.23E+00	6.31E+00	-9.22E-01
Joint	(1.39, 1.66]	2.57E-01	1.29E+00	-8.79E-01
Wing	(1.66,+∞)	1.00E-05	4.92E-01	-1.85E-01
Wing	(-∞, 0.06)	2.74E-01	3.33E-01	-5.84E-01

Joint	[0.06, 0.11)	2.32E-01	3.16E-01	-5.10E-01
Joint	[0.11, 0.17)	2.11E-01	2.83E-01	-2.21E-01
Joint	[0.17, 0.22)	1.98E-01	2.88E-01	-1.18E-01
Joint	[0.22, 0.28)	1.34E-01	3.79E-01	-1.24E-01
Joint	[0.28, 0.31)	1.36E-01	3.73E-01	-8.26E-02
Joint	[0.31, 0.34)	1.48E-01	3.46E-01	1.93E-02
Joint	[0.34, 0.36)	1.47E-01	3.47E-01	5.20E-02
Joint	[0.36, 0.39)	1.58E-01	3.13E-01	7.67E-02
Joint	[0.39, 0.42)	8.37E-02	5.30E-01	-1.67E-01
Joint	[0.42, 0.45)	1.13E-01	4.28E-01	-1.98E-02
Joint	[0.45, 0.47)	1.12E-01	4.25E-01	2.72E-02
Joint	[0.47, 0.50)	1.07E-01	4.44E-01	-8.16E-03
Joint	[0.50, 0.52)	1.04E-01	4.54E-01	-2.75E-02
Joint	[0.52, 0.53)	1.00E-01	4.71E-01	-3.35E-02
Joint	[0.53, 0.54)	1.21E-01	3.84E-01	1.04E-01
Joint	[0.54, 0.56)	7.93E-02	6.43E-01	-2.64E-01
Joint	[0.56, 0.57)	7.30E-02	8.06E-01	-3.87E-01
Joint	[0.57, 0.59)	7.56E-02	7.76E-01	-3.79E-01
Joint	[0.59, 0.60)	7.55E-02	8.67E-01	-4.31E-01
Joint	[0.60, 0.61)	7.96E-02	1.00E+00	-5.12E-01
Joint	[0.61, 0.63)	7.88E-02	9.10E-01	-4.19E-01
Joint	[0.63, 0.64)	8.51E-02	1.01E+00	-5.08E-01
Joint	[0.64, 0.66)	8.49E-02	7.16E-01	-3.37E-01
Joint	[0.66, 0.67)	8.61E-02	7.07E-01	-3.29E-01
Joint	[0.67, 0.68)	5.09E-01	1.50E+00	3.86E-01
Joint	[0.68, 0.70)	9.68E-02	1.20E+00	-5.18E-01
Joint	[0.70, 0.71)	8.92E-02	7.80E-01	-4.20E-01

Joint	[0.71, 0.73)	9.12E-02	7.88E-01	-4.13E-01
Joint	[0.73, 0.74)	9.54E-02	1.03E+00	-4.44E-01
Joint	[0.74, 0.75)	1.10E-01	1.18E+00	-5.72E-01
Joint	[0.75, 0.77)	4.06E-01	4.06E+00	-5.32E-01
Joint	[0.77, 0.78)	2.40E-01	1.78E+00	-4.63E-01
Joint	[0.78, 0.80)	2.41E-01	1.06E+00	-3.16E-01
Joint	[0.80, 0.81)	1.97E-01	1.01E+00	-3.14E-01
Joint	[0.81, 0.82)	1.82E-01	1.05E+00	-3.35E-01
Joint	[0.82, 0.84)	1.89E-01	1.04E+00	-3.41E-01
Joint	[0.84, 0.85)	2.03E-01	9.62E-01	-2.88E-01
Joint	[0.85, 0.87)	2.23E-01	9.49E-01	-2.49E-01
Joint	[0.87, 0.88)	5.51E-01	3.11E+00	-3.44E-01
Joint	[0.88, 0.89)	2.05E-01	1.06E+00	-2.73E-01
Joint	[0.89, 0.91)	4.60E-01	2.54E+00	1.43E-01
Joint	[0.91, 0.92)	4.20E-01	3.34E+00	-5.98E-01
Joint	[0.92, 0.94)	1.69E+00	6.72E+00	-3.60E-01
Joint	[0.94, 0.95)	7.95E-01	4.46E+00	-3.61E-01
Joint	[0.95, 0.96)	2.09E+00	6.39E+00	-3.25E-01
Joint	[0.96, 0.98)	6.95E-01	7.05E+00	-4.17E-01
Joint	[0.98, 0.99)	1.44E-01	1.16E+00	-6.55E-01
Joint	[0.99, 1.01]	6.69E-01	6.34E+00	-7.56E-01
Joint	(1.01, 1.02]	2.59E-01	1.93E+00	-3.93E-01
Joint	(1.02, 1.03]	9.70E-01	6.74E+00	-3.91E-01
Joint	(1.03, 1.05]	1.98E-01	1.63E+00	-6.31E-01
Joint	(1.05, 1.06]	2.76E+00	7.42E+00	-2.86E-01
Joint	(1.06, 1.07]	2.60E+00	7.97E+00	-2.03E-01
Joint	(1.07, 1.09]	3.21E-01	2.16E+00	-3.73E-01

Joint	(1.09, 1.10]	9.36E-01	6.70E+00	-3.54E-01
Joint	(1.10, 1.12]	1.05E+00	7.07E+00	-2.72E-01
Joint	(1.12, 1.13]	8.42E-01	4.81E+00	-4.44E-01
Joint	(1.13, 1.14]	5.71E-01	4.49E+00	-3.77E-01
Joint	(1.14, 1.17]	1.26E+00	9.99E+00	-3.35E-01
Joint	(1.17, 1.20]	5.66E-01	4.35E+00	-7.80E-01
Joint	(1.20, 1.23]	6.24E-01	5.12E+00	-7.62E-01
Joint	(1.23, 1.26]	8.54E-01	6.33E+00	-8.14E-01
Joint	(1.26, 1.28]	8.90E-01	6.44E+00	-8.27E-01
Joint	(1.28, 1.34]	1.35E-01	7.51E-01	-7.93E-01
Joint	(1.34, 1.40]	1.16E-01	6.04E-01	-7.96E-01
Joint	(1.40, 1.45]	7.96E-01	6.10E+00	-8.45E-01
Joint	(1.45, 1.56]	8.58E-01	5.82E+00	-8.70E-01
Joint	(1.56, 1.68]	1.08E+00	5.45E+00	-9.10E-01
Wing	(1.68,+∞)	5.93E-02	2.43E-01	-7.69E-01

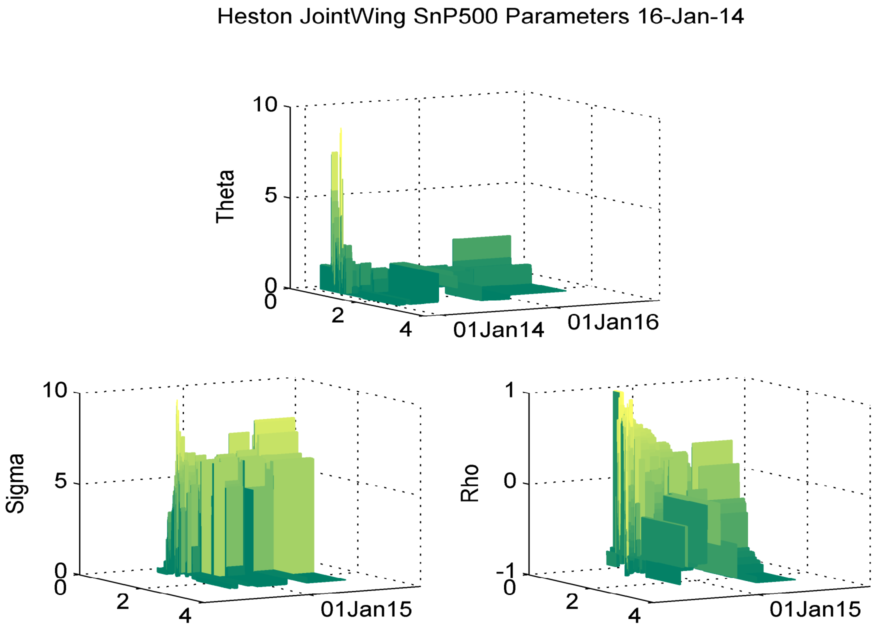


Figure B.25: Heston JointWing SnP500 Parameters

Table B.20: Elapsed Seconds SnP500 Calibration (02-Jan-14∼31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
7.675E+0	1.866E+1	2.335E+2	5.749E+2	1.295E+3	1.886E+4

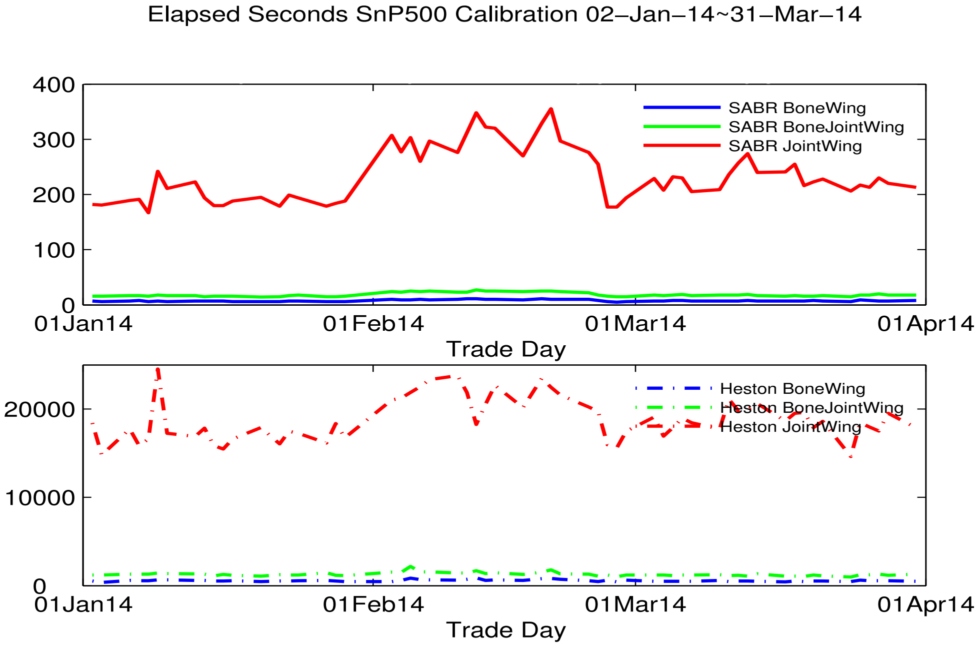


Figure B.26: Elapsed Seconds SnP500 Calibration (02-Jan-14∼31-Mar-14)

Table B.21: SABR BoneWing Nikkei225 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	($-\infty$, 0.67)		9.65E+00	-6.47E-01	4.76E+00
Bone	[0.67, 1.19]	(0.00, 0.08]	2.21E+01	5.91E-01	1.00E+01
Wing	(1.19,$+\infty$)		7.52E+00	3.29E-01	1.87E+00
Wing	($-\infty$, 0.57)		5.64E+00	-6.79E-02	6.83E+00
Bone	[0.57, 1.26]	(0.08, 0.16]	1.73E+01	-4.86E-01	9.94E+00
Wing	(1.26,$+\infty$)		7.57E+00	3.35E-01	1.95E+00
Wing	($-\infty$, 0.61)		5.16E+00	-2.08E-02	4.12E+00
Bone	[0.61, 1.25]	(0.16, 0.23]	1.99E+01	5.48E-01	6.60E+00
Wing	(1.25,$+\infty$)		7.45E+00	3.25E-01	1.69E+00
Wing	($-\infty$, 0.93)		7.22E+00	-1.72E-01	2.63E+00
Bone	[0.93, 1.15]	(0.23, 0.31]	5.18E+00	1.27E-01	1.00E+01
Wing	(1.15,$+\infty$)		4.14E+00	4.53E-01	5.34E+00
Wing	($-\infty$, 0.57)		6.12E+00	-2.44E-01	8.00E+00
Bone	[0.57, 1.28]	(0.31, 0.41]	9.56E+00	-3.15E-02	7.78E+00
Wing	(1.28,$+\infty$)		6.80E+00	2.07E-01	2.44E+00
Wing	($-\infty$, 0.77)		5.36E+00	-4.46E-02	3.44E+00
Bone	[0.77, 1.04]	(0.41, 0.65]	1.14E+01	2.34E-01	4.88E+00
Wing	(1.04,$+\infty$)		3.98E+00	4.15E-02	9.08E-01
Wing	($-\infty$, 0.64)		5.12E+00	-1.50E-02	2.84E+00
Bone	[0.64, 1.12]	(0.65, 0.90]	1.71E+01	-4.01E-03	2.65E+00
Wing	(1.12,$+\infty$)		5.66E+00	3.63E-01	1.27E+00
Wing	($-\infty$, 0.26)		4.78E+00	4.78E-02	2.46E+00
Bone	[0.26, 1.13]	(0.90, 1.40]	1.48E+01	3.35E-01	2.26E+00

Wing	(1.13,+∞)		7.21E+00	2.15E-01	7.96E-01
Wing	(-∞, 0.36)		4.45E+00	7.56E-02	1.15E+00
Bone	[0.36, 1.13]	(1.40, 1.90]	2.20E+01	3.15E-01	6.76E-01
Wing	(1.13,+∞)		9.42E+00	5.13E-01	5.65E-01
Wing	(-∞, 1.05]	(1.90, 2.90]	4.31E+01	7.02E-01	3.00E-01
Wing	(1.05,+∞)		8.05E+00	1.71E-01	9.94E-01
Wing	(-∞, 0.50)		4.80E+00	3.03E-02	1.47E+00
Bone	[0.50, 1.06]	(2.90,+∞]	4.57E+01	-1.88E-01	3.48E-01
Wing	(1.06,+∞)		5.69E+00	2.52E-02	1.05E+00

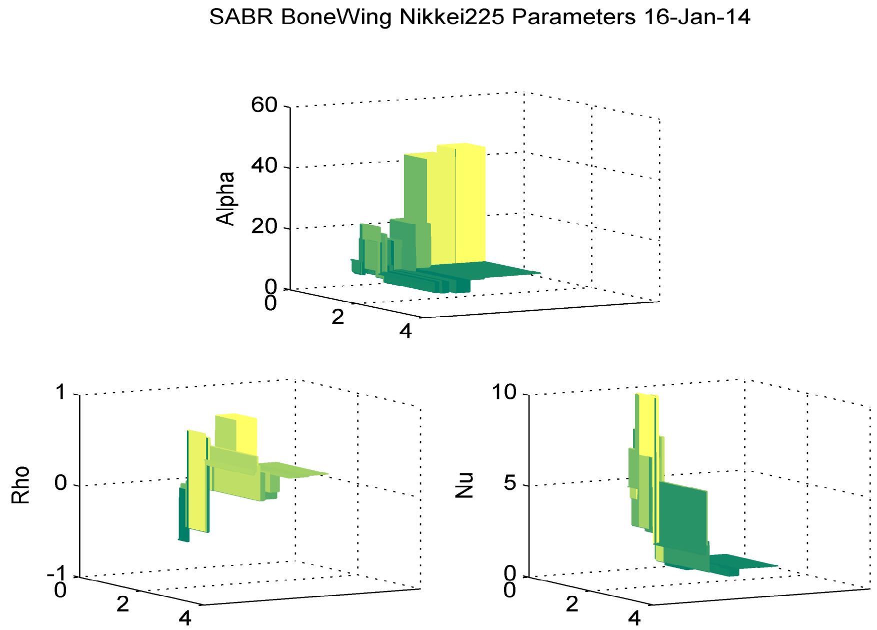


Figure B.27: SABR BoneWing Nikkei225 Parameters

Table B.22: Heston BoneWing Nikkei225 Parameters (κ = 2.08E+00, v0 = 1.02E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.67)		1.00E-05	9.74E-01	-9.99E-01
Bone	[0.67, 1.19]	(0.00, 0.08]	2.11E-01	3.29E-01	-2.16E-01
Wing	(1.19,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.57)		3.82E-01	1.95E-03	5.01E-01
Bone	[0.57, 1.26]	(0.08, 0.16]	2.11E-01	3.29E-01	-2.16E-01
Wing	(1.26,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.61)		4.51E-01	3.01E-03	-4.60E-01
Bone	[0.61, 1.25]	(0.16, 0.23]	2.11E-01	3.29E-01	-2.16E-01
Wing	(1.25,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.93)		1.74E-01	1.81E-03	9.95E-01
Bone	[0.93, 1.15]	(0.23, 0.31]	2.11E-01	3.29E-01	-2.16E-01
Wing	(1.15,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.57)		2.11E-01	3.29E-01	-2.16E-01
Bone	[0.57, 1.28]	(0.31, 0.41]	2.11E-01	3.29E-01	-2.16E-01
Wing	(1.28,+∞)		1.00E+01	5.82E+00	9.99E-01
Wing	(-∞, 0.77)		5.83E-01	4.30E+00	-6.83E-01
Bone	[0.77, 1.04]	(0.41, 0.65]	6.11E-01	5.47E+00	-4.97E-01
Wing	(1.04,+∞)		7.77E-01	2.72E-01	-9.51E-01
Wing	(-∞, 0.64)		1.57E-01	6.78E+00	-6.03E-01
Bone	[0.64, 1.12]	(0.65, 0.90]	2.11E-01	3.29E-01	-2.16E-01
Wing	(1.12,+∞)		3.28E-01	2.30E+00	9.93E-01
Wing	(-∞, 0.26)		2.11E-01	3.29E-01	-2.16E-01
Bone	[0.26, 1.13]	(0.90, 1.40]	2.11E-01	3.29E-01	-2.16E-01

Wing	(1.13,+∞)		7.61E-02	1.10E+00	-2.76E-01
Wing	(-∞, 0.36)		2.11E-01	3.29E-01	-2.16E-01
Bone	[0.36, 1.13]	(1.40, 1.90]	2.11E-01	3.29E-01	-2.16E-01
Wing	(1.13,+∞)		1.00E-05	1.27E+00	-2.65E-01
Wing	(-∞, 1.05]	(1.90, 2.90]	1.52E-01	5.81E-01	9.43E-02
Wing	(1.05,+∞)		5.62E+00	1.15E+00	8.58E-01
Wing	(-∞, 0.50)		1.07E+00	1.92E+00	-8.18E-01
Bone	[0.50, 1.06]	(2.90,+∞]	3.65E-01	2.55E+00	-4.65E-01
Wing	(1.06,+∞)		6.21E-04	5.49E+00	-6.50E-01

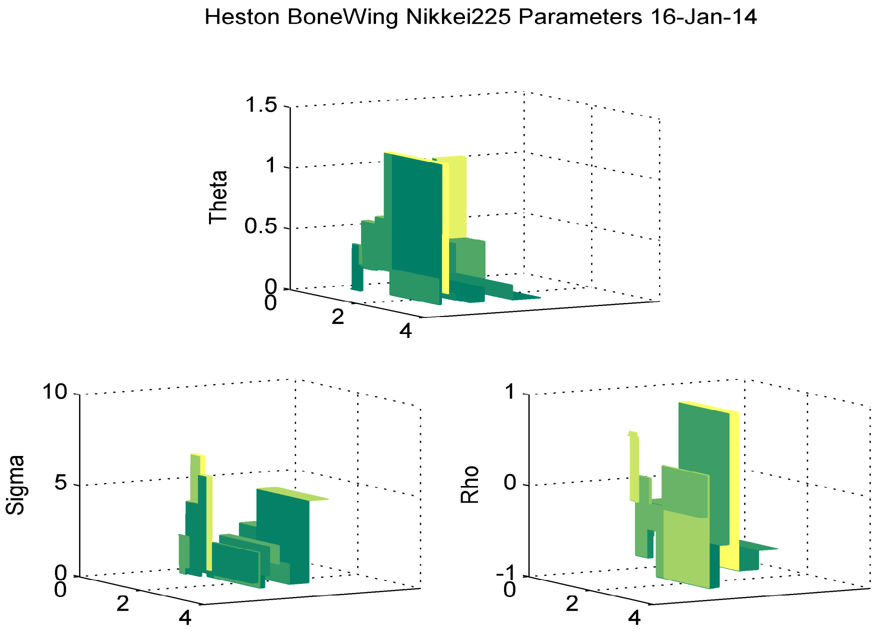


Figure B.28: Heston BoneWing Nikkei225 Parameters

Table B.23: SABR BoneJointWing Nikkei225 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.67)		5.17E+00	-2.04E-02	4.23E+00
Bone	[0.67, 0.95]		9.75E+00	-6.17E-01	5.75E+00
Joint	(0.95, 0.95)		8.49E+00	-3.29E-01	7.60E+00
Bone	[0.95, 1.05]	(0.00, 0.08]	2.21E+01	5.91E-01	1.00E+01
Joint	(1.05, 1.06)		8.92E+00	2.20E-01	9.30E+00
Bone	[1.06, 1.19]		9.50E+00	5.92E-01	4.36E+00
Wing	(1.19,+∞)		7.36E+00	2.64E-01	9.01E-01
Wing	(-∞, 0.57)		5.07E+00	-8.65E-03	4.03E+00
Bone	[0.57, 0.95]		9.72E+00	-5.73E-01	5.19E+00
Joint	(0.95, 0.95)		8.85E+00	-2.97E-01	8.76E+00
Bone	[0.95, 1.05]	(0.08, 0.16]	1.66E+01	-4.54E-01	1.00E+01
Joint	(1.05, 1.06)		8.70E+00	1.25E-01	8.48E+00
Bone	[1.06, 1.26]		9.73E+00	3.58E-01	7.69E+00
Wing	(1.26,+∞)		7.45E+00	3.25E-01	1.65E+00
Wing	(-∞, 0.61)		4.91E+00	1.43E-02	2.91E+00
Bone	[0.61, 0.94]		7.13E+00	-1.72E-01	3.35E+00
Joint	(0.94, 0.96)		9.27E+00	-1.23E-01	6.68E+00
Bone	[0.96, 1.04]	(0.16, 0.23]	1.07E+01	2.58E-01	1.00E+01
Joint	(1.04, 1.05)		9.43E+00	2.29E-01	8.82E+00
Bone	[1.05, 1.25]		9.77E+00	3.16E-01	7.34E+00
Wing	(1.25,+∞)		7.48E+00	3.28E-01	1.75E+00
Wing	(-∞, 0.93)		3.78E+00	1.37E-01	5.02E-01
Joint	[0.93, 1.09)	(0.23, 0.31]	7.62E+00	5.54E-01	9.96E+00

Bone	[1.09, 1.15]		1.00E+01	1.14E-01	1.00E+01
Wing	(1.15,+∞)		9.90E+00	2.94E-01	4.90E+00
Wing	(-∞, 0.57)	(0.31, 0.41]	6.12E+00	-1.90E-01	7.95E+00
Bone	[0.57, 0.93]	(0.31, 0.41]	5.50E+00	-7.25E-02	8.22E+00
Joint	(0.93, 0.96)	(0.31, 0.41]	1.07E+00	-4.10E-01	9.67E+00
Bone	[0.96, 1.02]	(0.31, 0.41]	2.83E+01	5.59E-01	1.07E+00
Joint	(1.02, 1.05)	(0.31, 0.41]	9.70E+00	1.83E-01	4.98E+00
Bone	[1.05, 1.28]	(0.31, 0.41]	6.13E+00	1.05E-01	4.88E+00
Wing	(1.28,+∞)	(0.31, 0.41]	7.39E+00	2.99E-01	1.22E+00
Wing	(-∞, 0.77)	(0.41, 0.65]	5.37E+00	-4.03E-02	3.47E+00
Bone	[0.77, 0.93]	(0.41, 0.65]	5.48E+00	-4.94E-02	5.15E+00
Joint	(0.93, 0.96)	(0.41, 0.65]	5.89E-01	2.09E-01	6.95E+00
Bone	[0.96, 1.04]	(0.41, 0.65]	2.12E+01	4.20E-01	2.05E+00
Wing	(1.04,+∞)	(0.41, 0.65]	9.97E+00	7.59E-01	5.11E-01
Wing	(-∞, 0.64)	(0.65, 0.90]	5.14E+00	8.17E-03	2.91E+00
Bone	[0.64, 0.95]	(0.65, 0.90]	5.10E+00	1.20E-03	3.49E+00
Joint	(0.95, 0.96)	(0.65, 0.90]	5.67E+00	-3.26E-02	3.93E+00
Bone	[0.96, 0.99]	(0.65, 0.90]	9.85E+00	3.54E-01	4.50E+00
Joint	(0.99, 1.12]	(0.65, 0.90]	1.24E+01	-4.25E-01	4.54E+00
Wing	(1.12,+∞)	(0.65, 0.90]	4.66E+00	-4.85E-02	1.03E+00
Wing	(-∞, 0.26)	(0.90, 1.40]	3.85E+00	4.43E-02	5.38E+00
Joint	[0.26, 0.97)	(0.90, 1.40]	4.90E+00	1.94E-02	2.39E+00
Bone	[0.97, 1.03]	(0.90, 1.40]	1.63E+01	7.75E-01	4.97E+00
Joint	(1.03, 1.13]	(0.90, 1.40]	7.28E+00	1.02E-01	2.97E+00
Wing	(1.13,+∞)	(0.90, 1.40]	4.55E+00	-5.98E-02	5.50E-01
Wing	(-∞, 0.36)	(1.40, 1.90]	4.45E+00	7.44E-02	1.16E+00
Bone	[0.36, 0.42]	(1.40, 1.90]	4.38E+00	7.93E-02	1.17E+00

Joint	(0.42, 1.13]		2.30E+01	-6.72E-02	4.22E-01
Wing	(1.13,+∞)		4.55E+00	-6.02E-02	5.19E-01
Wing	(-∞, 1.05]	(1.90, 2.90]	4.31E+01	7.02E-01	3.00E-01
Wing	(1.05,+∞)		8.05E+00	1.71E-01	9.94E-01
Wing	(-∞, 0.50)	(2.90,+∞]	4.85E+00	2.19E-02	1.63E+00
Bone	[0.50, 0.89]		5.46E+00	-2.99E-02	1.84E+00
Joint	(0.89, 1.06]		3.40E+01	7.52E-01	1.94E+00
Wing	(1.06,+∞)		4.68E+00	-5.02E-02	7.57E-01

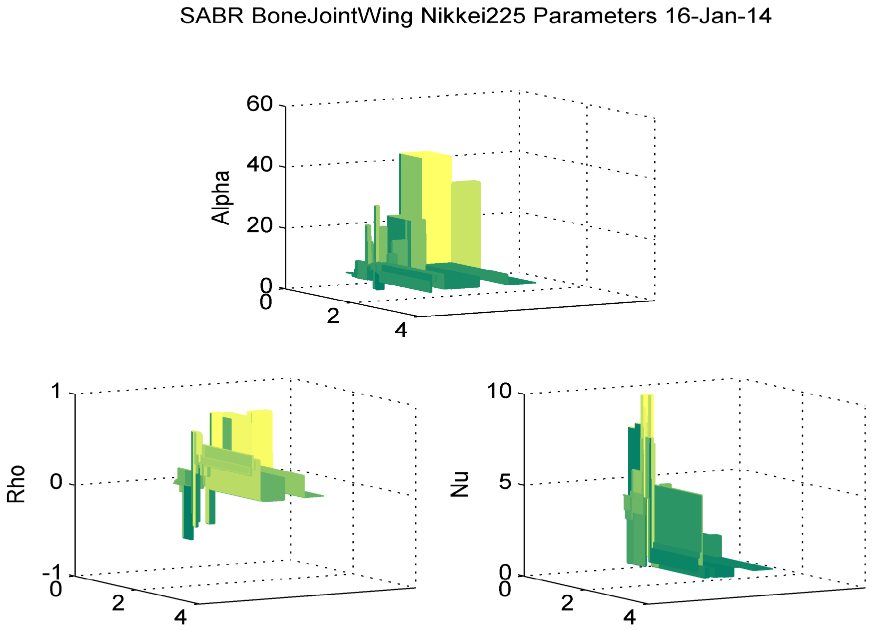


Figure B.29: SABR BoneJointWing Nikkei225 Parameters

Table B.24: Heston BoneJointWing Nikkei225 Parameters (κ = 2.08E+00, v0 = 1.02E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.67)		1.00E-05	9.74E-01	-9.99E-01
Bone	[0.67, 0.95]		2.11E-01	3.29E-01	-2.16E-01
Joint	(0.95, 0.95)		2.11E-01	3.29E-01	-2.16E-01
Bone	[0.95, 1.05]	(0.00, 0.08]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.05, 1.06)		2.11E-01	3.29E-01	-2.16E-01
Bone	[1.06, 1.19]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.19,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.57)		3.82E-01	1.95E-03	5.01E-01
Bone	[0.57, 0.95]		2.11E-01	3.29E-01	-2.16E-01
Joint	(0.95, 0.95)		2.11E-01	3.29E-01	-2.16E-01
Bone	[0.95, 1.05]	(0.08, 0.16]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.05, 1.06)		2.11E-01	3.29E-01	-2.16E-01
Bone	[1.06, 1.26]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.26,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.61)		4.51E-01	3.01E-03	-4.60E-01
Bone	[0.61, 0.94]		2.36E-01	1.46E+00	-3.86E-01
Joint	(0.94, 0.96)		4.71E-01	5.16E+00	2.08E-01
Bone	[0.96, 1.04]	(0.16, 0.23]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.04, 1.05)		2.11E-01	3.29E-01	-2.16E-01
Bone	[1.05, 1.25]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.25,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.93)		1.74E-01	1.81E-03	9.95E-01
Joint	[0.93, 1.09)	(0.23, 0.31]	2.11E-01	3.29E-01	-2.16E-01

Bone	[1.09, 1.15]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.15,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.57)		2.11E-01	3.29E-01	-2.16E-01
Bone	[0.57, 0.93]		6.19E-02	1.22E+00	9.99E-01
Joint	(0.93, 0.96)		1.43E-01	1.78E+00	9.97E-01
Bone	[0.96, 1.02]	(0.31, 0.41]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.02, 1.05)		2.11E-01	3.29E-01	-2.16E-01
Bone	[1.05, 1.28]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.28,+∞)		1.00E+01	5.82E+00	9.99E-01
Wing	(-∞, 0.77)		5.83E-01	4.30E+00	-6.83E-01
Bone	[0.77, 0.93]		6.11E-01	5.47E+00	-4.97E-01
Joint	(0.93, 0.96)	(0.41, 0.65]	4.25E-01	5.74E+00	-5.19E-01
Bone	[0.96, 1.04]		1.31E-01	1.43E+00	6.50E-01
Wing	(1.04,+∞)		7.77E-01	2.72E-01	-9.51E-01
Wing	(-∞, 0.64)		3.02E-01	4.89E+00	-4.95E-01
Bone	[0.64, 0.95]		2.90E-01	1.93E+00	-1.44E-01
Joint	(0.95, 0.96)		3.91E-01	5.11E+00	-5.25E-01
Bone	[0.96, 0.99]	(0.65, 0.90]	3.34E-01	5.81E+00	-5.61E-01
Joint	(0.99, 1.12]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.12,+∞)		3.28E-01	2.30E+00	9.93E-01
Wing	(-∞, 0.26)		2.11E-01	3.29E-01	-2.16E-01
Joint	[0.26, 0.97)		5.61E-01	1.69E+00	-2.64E-01
Bone	[0.97, 1.03]	(0.90, 1.40]	3.26E-01	5.96E+00	-6.20E-01
Joint	(1.03, 1.13]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.13,+∞)		7.61E-02	1.10E+00	-2.76E-01
Wing	(-∞, 0.36)		1.44E-01	7.81E-01	7.43E-02
Bone	[0.36, 0.42]	(1.40, 1.90]	2.50E-02	1.19E+00	3.81E-02

Joint	(0.42, 1.13]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.13,+∞)		1.00E-05	1.27E+00	-2.65E-01
Wing	(-∞, 1.05]	(1.90, 2.90]	1.52E-01	5.81E-01	9.43E-02
Wing	(1.05,+∞)		5.62E+00	1.15E+00	8.58E-01
Wing	(-∞, 0.50)	(2.90,+∞]	3.98E-01	5.43E-01	-2.38E-01
Bone	[0.50, 0.89]		4.46E-01	5.11E-01	-1.53E-01
Joint	(0.89, 1.06]		3.88E-01	3.66E+00	-5.62E-01
Wing	(1.06,+∞)		8.02E-04	5.49E+00	-6.53E-01

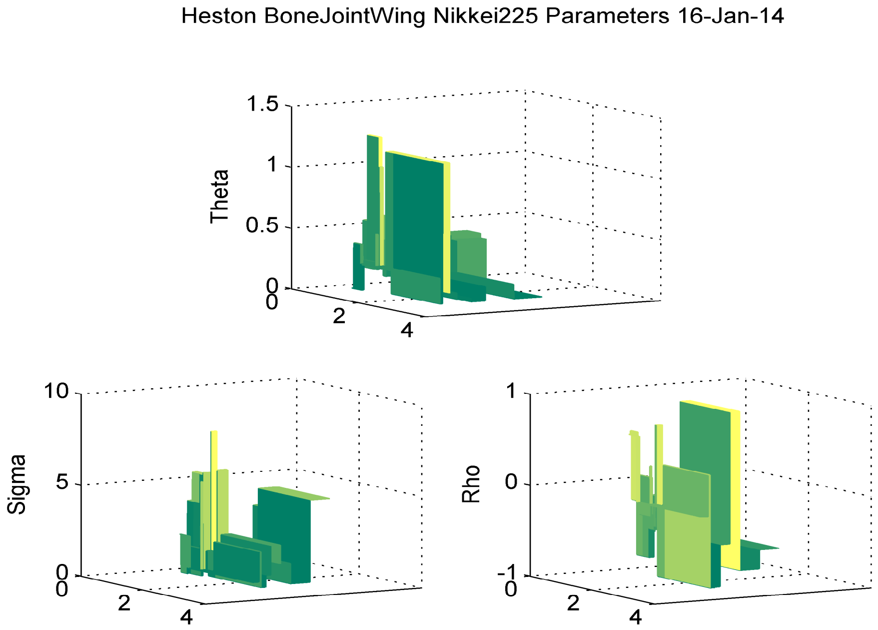


Figure B.30: Heston BoneJointWing Nikkei225 Parameters

Table B.25: SABR JointWing Nikkei225 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.67)		5.03E+00	-1.58E-03	3.56E+00
Joint	[0.67, 0.68)		3.82E+00	1.72E-01	5.24E+00
Joint	[0.68, 0.70)		3.81E+00	1.88E-01	5.23E+00
Joint	[0.70, 0.72)		3.81E+00	1.99E-01	5.21E+00
Joint	[0.72, 0.73)		3.81E+00	2.16E-01	5.21E+00
Joint	[0.73, 0.75)		5.18E+00	2.99E-02	4.74E+00
Joint	[0.75, 0.76)		3.83E+00	1.18E-01	5.30E+00
Joint	[0.76, 0.77)		3.83E+00	1.23E-01	5.28E+00
Joint	[0.77, 0.78)		3.83E+00	1.39E-01	5.28E+00
Joint	[0.78, 0.79)		5.28E+00	-1.35E-03	4.82E+00
Joint	[0.79, 0.80)		3.85E+00	6.94E-02	5.36E+00
Joint	[0.80, 0.80)		5.64E+00	-6.36E-02	4.80E+00
Joint	[0.80, 0.81)		5.44E+00	-1.01E-01	4.94E+00
Joint	[0.81, 0.82)		5.57E+00	-1.35E-01	4.97E+00
Joint	[0.82, 0.83)		5.72E+00	-1.63E-01	4.99E+00
Joint	[0.83, 0.83)		5.48E+00	-2.24E-01	5.13E+00
Joint	[0.83, 0.84)		5.69E+00	-2.63E-01	5.19E+00
Joint	[0.84, 0.85)		5.61E+00	-3.27E-01	5.30E+00
Joint	[0.85, 0.86)		5.14E+00	-4.67E-01	5.59E+00
Joint	[0.86, 0.87)		5.71E+00	-3.73E-01	5.54E+00
Joint	[0.87, 0.87)		6.11E+00	-3.39E-01	5.60E+00
Joint	[0.87, 0.88)		6.66E+00	-2.96E-01	5.68E+00
Joint	[0.88, 0.89)		6.83E+00	-2.29E-01	5.87E+00

Joint	[0.89, 0.89)	6.01E+00	-4.53E-02	8.06E+00
Joint	[0.89, 0.90)	5.16E+00	1.40E-02	8.54E+00
Joint	[0.90, 0.91)	7.06E+00	-3.01E-01	6.18E+00
Joint	[0.91, 0.91)	7.48E+00	-2.75E-01	6.27E+00
Joint	[0.91, 0.92)	7.67E+00	-2.66E-01	6.53E+00
Joint	[0.92, 0.92)	7.30E+00	-1.90E-01	9.03E+00
Joint	[0.92, 0.93)	6.76E+00	-1.26E-01	9.44E+00
Joint	[0.93, 0.94)	8.08E+00	-3.14E-01	7.17E+00
Joint	[0.94, 0.95)	8.33E+00	-2.88E-01	7.60E+00
Joint	[0.95, 0.95)	8.60E+00	-3.02E-01	8.12E+00
Joint	[0.95, 0.95)	6.12E+00	-1.97E-01	7.65E+00
Joint	[0.95, 0.96)	8.58E+00	-3.38E-01	6.84E+00
Joint	[0.96, 0.97)	9.22E+00	-2.29E-01	9.28E+00
Joint	[0.97, 0.97)	1.00E+01	-2.41E-01	1.00E+01
Joint	[0.97, 0.98)	1.00E+01	-2.41E-01	1.00E+01
Joint	[0.98, 0.99)	1.00E+01	-2.46E-01	1.00E+01
Joint	[0.99, 0.99)	1.00E+01	-2.46E-01	1.00E+01
Joint	[0.99, 0.99)	1.00E+01	-2.03E-01	1.00E+01
Joint	[0.99, 1.00]	2.92E+01	-6.54E-01	1.80E+00
Joint	(1.00, 1.00]	1.00E+01	9.32E-02	1.00E+01
Joint	(1.00, 1.01]	1.00E+01	2.41E-01	1.00E+01
Joint	(1.01, 1.01]	8.93E+00	1.95E-01	9.12E+00
Joint	(1.01, 1.02]	6.64E+00	2.24E-01	9.67E+00
Joint	(1.02, 1.02]	1.00E+01	2.55E-01	1.00E+01
Joint	(1.02, 1.03]	1.00E+01	2.55E-01	1.00E+01
Joint	(1.03, 1.03]	9.35E+00	2.62E-01	9.35E+00
Joint	(1.03, 1.03]	1.00E+01	2.45E-01	1.00E+01

Joint	(1.03, 1.04]	1.00E+01	2.45E-01	1.00E+01
Joint	(1.04, 1.05]	8.64E+00	2.50E-01	7.94E+00
Joint	(1.05, 1.05]	6.41E+00	2.36E-01	7.92E+00
Joint	(1.05, 1.06]	3.82E+00	1.14E-01	8.75E+00
Joint	(1.06, 1.07]	7.99E+00	3.62E-01	6.82E+00
Joint	(1.07, 1.07]	5.71E+00	2.36E-01	7.43E+00
Joint	(1.07, 1.07]	7.72E+00	3.25E-01	6.33E+00
Joint	(1.07, 1.08]	7.36E+00	3.39E-01	5.91E+00
Joint	(1.08, 1.08]	5.70E+00	2.37E-01	7.19E+00
Joint	(1.08, 1.09]	5.12E+00	3.43E-01	7.19E+00
Joint	(1.09, 1.10]	6.79E+00	3.56E-01	5.37E+00
Joint	(1.10, 1.11]	6.43E+00	3.64E-01	5.16E+00
Joint	(1.11, 1.11]	5.22E+00	4.37E-01	5.27E+00
Joint	(1.11, 1.12]	6.31E+00	2.34E-01	4.78E+00
Joint	(1.12, 1.13]	6.26E+00	1.97E-01	4.68E+00
Joint	(1.13, 1.14]	6.19E+00	1.45E-01	4.53E+00
Joint	(1.14, 1.14]	6.02E+00	9.41E-02	4.44E+00
Joint	(1.14, 1.15]	3.80E+00	-1.14E-01	5.22E+00
Joint	(1.15, 1.16]	5.58E+00	8.61E-03	4.37E+00
Joint	(1.16, 1.18]	6.35E+00	-4.51E-02	4.04E+00
Joint	(1.18, 1.19]	3.50E+00	-2.71E-01	5.04E+00
Wing	(1.19,+∞)	7.30E+00	2.56E-01	7.70E-01
Wing	(-∞, 0.57)	5.63E+00	-7.61E-02	2.95E+00
Joint	[0.57, 0.59)	4.74E+00	3.47E-02	3.57E+00
Joint	[0.59, 0.62)	4.79E+00	2.85E-02	3.59E+00
Joint	[0.62, 0.64)	4.74E+00	3.41E-02	3.64E+00
Joint	[0.64, 0.65)	4.74E+00	3.51E-02	3.62E+00

Joint	[0.65, 0.67)	4.77E+00	3.05E-02	3.64E+00
Joint	[0.67, 0.68)	4.85E+00	1.84E-02	3.69E+00
Joint	[0.68, 0.70)	4.87E+00	1.66E-02	3.77E+00
Joint	[0.70, 0.72)	4.89E+00	1.69E-02	3.85E+00
Joint	[0.72, 0.73)	4.91E+00	2.04E-02	3.91E+00
Joint	[0.73, 0.75)	4.92E+00	1.30E-02	3.99E+00
Joint	[0.75, 0.76)	4.94E+00	1.11E-02	4.09E+00
Joint	[0.76, 0.76)	4.40E+00	-3.91E-01	6.15E+00
Joint	[0.76, 0.78)	2.98E+00	-4.45E-01	6.75E+00
Joint	[0.78, 0.80)	6.07E+00	2.07E-01	4.35E+00
Joint	[0.80, 0.81)	6.29E+00	1.12E-01	4.31E+00
Joint	[0.81, 0.83)	6.68E+00	2.86E-02	4.31E+00
Joint	[0.83, 0.83)	5.41E+00	6.97E-02	4.72E+00
Joint	[0.83, 0.84)	3.82E+00	1.55E-01	5.27E+00
Joint	[0.84, 0.85)	3.83E+00	1.46E-01	5.27E+00
Joint	[0.85, 0.86)	3.93E+00	7.76E-02	5.31E+00
Joint	[0.86, 0.86)	4.62E+00	-3.31E-01	7.90E+00
Joint	[0.86, 0.87)	4.06E+00	-2.73E-01	8.09E+00
Joint	[0.87, 0.87)	6.31E+00	-1.64E-01	4.91E+00
Joint	[0.87, 0.88)	6.66E+00	-2.52E-01	4.92E+00
Joint	[0.88, 0.89)	6.33E+00	-3.36E-01	5.19E+00
Joint	[0.89, 0.90)	6.52E+00	-4.49E-01	5.40E+00
Joint	[0.90, 0.91)	6.77E+00	-4.13E-01	5.49E+00
Joint	[0.91, 0.91)	4.58E+00	-3.15E-01	6.16E+00
Joint	[0.91, 0.91)	7.28E+00	-3.22E-01	5.51E+00
Joint	[0.91, 0.92)	7.63E+00	-2.36E-01	5.70E+00
Joint	[0.92, 0.93)	7.61E+00	1.39E-01	6.51E+00

Joint	[0.93, 0.94)	7.89E+00	2.44E-02	6.53E+00
Joint	[0.94, 0.95)	8.30E+00	-5.15E-02	6.65E+00
Joint	[0.95, 0.95)	8.54E+00	-1.91E-03	7.07E+00
Joint	[0.95, 0.95)	1.00E+01	-1.67E-01	1.00E+01
Joint	[0.95, 0.96)	1.00E+01	-1.67E-01	1.00E+01
Joint	[0.96, 0.97)	9.03E+00	-2.11E-01	7.77E+00
Joint	[0.97, 0.98)	9.30E+00	-2.12E-01	8.42E+00
Joint	[0.98, 0.99)	9.60E+00	-1.64E-01	9.33E+00
Joint	[0.99, 0.99)	1.00E+01	-1.82E-01	1.00E+01
Joint	[0.99, 0.99)	1.00E+01	-1.82E-01	1.00E+01
Joint	[0.99, 1.00]	3.04E+01	8.10E-01	6.16E+00
Joint	(1.00, 1.00]	1.00E+01	7.04E-02	1.00E+01
Joint	(1.00, 1.01]	1.00E+01	7.59E-02	1.00E+01
Joint	(1.01, 1.02]	9.90E+00	1.73E-01	9.93E+00
Joint	(1.02, 1.02]	8.99E+00	2.03E-01	9.53E+00
Joint	(1.02, 1.03]	9.43E+00	1.96E-01	8.79E+00
Joint	(1.03, 1.03]	9.15E+00	2.31E-01	8.03E+00
Joint	(1.03, 1.03]	6.81E+00	1.55E-01	8.20E+00
Joint	(1.03, 1.04]	8.83E+00	1.93E-01	7.24E+00
Joint	(1.04, 1.05]	8.57E+00	2.50E-01	6.90E+00
Joint	(1.05, 1.06]	4.83E+00	1.82E-01	7.58E+00
Joint	(1.06, 1.07]	8.06E+00	2.53E-01	6.24E+00
Joint	(1.07, 1.07]	7.81E+00	2.43E-01	5.90E+00
Joint	(1.07, 1.08]	7.44E+00	3.38E-01	5.58E+00
Joint	(1.08, 1.08]	9.47E+00	1.55E-01	9.89E+00
Joint	(1.08, 1.09]	9.27E+00	1.75E-01	9.78E+00
Joint	(1.09, 1.10]	5.81E+00	5.07E-01	5.59E+00

Joint	(1.10, 1.11]	6.75E+00	2.69E-01	4.94E+00
Joint	(1.11, 1.11]	6.64E+00	2.06E-01	4.80E+00
Joint	(1.11, 1.11]	3.81E+00	5.61E-01	6.34E+00
Joint	(1.11, 1.12]	3.99E+00	2.98E-01	5.82E+00
Joint	(1.12, 1.13]	6.29E+00	9.94E-02	4.59E+00
Joint	(1.13, 1.14]	5.94E+00	2.05E-02	4.54E+00
Joint	(1.14, 1.15]	5.81E+00	-9.95E-03	4.42E+00
Joint	(1.15, 1.15]	3.78E+00	-1.58E-01	5.10E+00
Joint	(1.15, 1.16]	5.68E+00	-7.40E-02	4.38E+00
Joint	(1.16, 1.18]	5.84E+00	-1.49E-01	4.12E+00
Joint	(1.18, 1.19]	5.84E+00	-1.89E-01	4.00E+00
Joint	(1.19, 1.21]	5.83E+00	-2.45E-01	3.90E+00
Joint	(1.21, 1.22]	5.43E+00	-2.91E-01	3.90E+00
Joint	(1.22, 1.24]	4.94E+00	-3.54E-01	3.99E+00
Joint	(1.24, 1.26]	5.12E+00	-3.88E-01	3.86E+00
Wing	(1.26,+∞)	4.89E+00	-2.07E-03	8.81E-01
Wing	(-∞, 0.61)	4.85E+00	2.21E-02	2.65E+00
Joint	[0.61, 0.64)	4.73E+00	3.68E-02	3.17E+00
Joint	[0.64, 0.66)	4.73E+00	3.71E-02	3.26E+00
Joint	[0.66, 0.70)	4.77E+00	3.16E-02	3.33E+00
Joint	[0.70, 0.72)	4.83E+00	2.83E-02	3.42E+00
Joint	[0.72, 0.77)	5.07E+00	-3.29E-03	3.47E+00
Joint	[0.77, 0.80)	5.02E+00	2.76E-03	3.70E+00
Joint	[0.80, 0.81)	4.94E+00	1.40E-02	3.89E+00
Joint	[0.81, 0.85)	5.16E+00	-1.33E-02	4.11E+00
Joint	[0.85, 0.85)	5.29E+00	1.81E-01	4.57E+00
Joint	[0.85, 0.86)	5.76E+00	1.39E-01	4.53E+00

Joint	[0.86, 0.89)	8.87E+00	-2.99E-01	4.21E+00
Joint	[0.89, 0.90)	3.04E+00	-5.23E-02	5.89E+00
Joint	[0.90, 0.92)	8.41E+00	-3.43E-01	4.61E+00
Joint	[0.92, 0.93)	7.42E+00	-4.27E-01	5.19E+00
Joint	[0.93, 0.93)	4.22E+00	-3.99E-01	6.27E+00
Joint	[0.93, 0.94)	8.04E+00	-3.67E-01	5.47E+00
Joint	[0.94, 0.96)	1.06E+00	-1.69E-01	7.65E+00
Joint	[0.96, 1.01]	2.84E+01	8.16E-01	6.44E+00
Joint	(1.01, 1.04]	1.00E+01	1.57E-01	1.00E+01
Joint	(1.04, 1.05]	1.00E+01	1.57E-01	1.00E+01
Joint	(1.05, 1.06]	8.07E+00	2.90E-01	5.76E+00
Joint	(1.06, 1.07]	8.84E+00	1.47E-01	9.85E+00
Joint	(1.07, 1.08]	9.03E+00	2.63E-01	9.59E+00
Joint	(1.08, 1.09]	5.63E+00	5.67E-01	6.02E+00
Joint	(1.09, 1.10]	8.25E+00	2.39E-01	4.33E+00
Joint	(1.10, 1.12]	7.89E+00	1.75E-01	4.07E+00
Joint	(1.12, 1.12]	4.67E+00	-9.40E-02	4.82E+00
Joint	(1.12, 1.13]	7.43E+00	1.78E-02	4.01E+00
Joint	(1.13, 1.13]	4.50E+00	-1.83E-01	4.69E+00
Joint	(1.13, 1.14]	3.74E+00	-2.65E-01	4.95E+00
Joint	(1.14, 1.15]	5.70E+00	-1.26E-01	4.26E+00
Joint	(1.15, 1.17]	6.76E+00	-1.03E-01	3.92E+00
Joint	(1.17, 1.17]	4.41E+00	-2.98E-01	4.47E+00
Joint	(1.17, 1.18]	3.33E+00	-3.75E-01	4.80E+00
Joint	(1.18, 1.20]	5.76E+00	-2.52E-01	3.97E+00
Joint	(1.20, 1.21]	5.68E+00	-3.42E-01	3.95E+00
Joint	(1.21, 1.25]	6.53E+00	-3.13E-01	3.49E+00

Wing	(1.25,+∞)		7.28E+00	2.54E-01	7.56E-01
Wing	(-∞, 0.93)		9.37E+00	-2.62E-01	4.29E-01
Joint	[0.93, 1.09]	(0.23, 0.31]	1.82E+01	2.51E-01	2.87E-01
Joint	(1.09, 1.15]		1.00E+01	1.14E-01	1.00E+01
Wing	(1.15,+∞)		4.03E+00	2.60E-01	6.14E+00
Wing	(-∞, 0.57)		6.19E+00	-1.11E-01	8.49E+00
Joint	[0.57, 0.64)		7.84E+00	-7.51E-02	9.66E+00
Joint	[0.64, 0.70)		5.50E+00	-7.25E-02	8.22E+00
Joint	[0.70, 0.72)		4.67E+00	4.51E-02	2.86E+00
Joint	[0.72, 0.80)		5.25E+00	-2.05E-02	2.92E+00
Joint	[0.80, 0.81)		4.86E+00	2.57E-02	3.31E+00
Joint	[0.81, 0.83)		4.90E+00	2.16E-02	3.47E+00
Joint	[0.83, 0.86)		5.13E+00	-7.34E-03	3.61E+00
Joint	[0.86, 0.89)		2.17E+00	6.72E-01	4.62E+00
Joint	[0.89, 0.93)	(0.31, 0.41]	9.26E+00	-4.18E-01	5.26E+00
Joint	[0.93, 0.96)		1.17E+00	-3.69E-01	9.75E+00
Joint	[0.96, 0.99)		9.83E+00	1.50E-02	5.64E+00
Joint	[0.99, 1.02]		2.90E+01	2.78E-01	1.64E-01
Joint	(1.02, 1.05]		9.70E+00	1.94E-01	5.12E+00
Joint	(1.05, 1.07]		8.65E+00	2.56E-01	4.62E+00
Joint	(1.07, 1.09]		8.65E+00	1.94E-01	4.32E+00
Joint	(1.09, 1.15]		5.02E+00	7.85E-03	6.42E+00
Joint	(1.15, 1.25]		5.24E+00	2.63E-02	5.22E+00
Joint	(1.25, 1.28]		4.95E+00	-5.44E-02	2.81E+00
Wing	(1.28,+∞)		6.93E+00	2.10E-01	7.43E-01
Wing	(-∞, 0.77)		5.30E+00	-3.61E-02	3.20E+00
Joint	[0.77, 0.89)		5.48E+00	-4.94E-02	5.15E+00

(0.41, 0.65]

Joint	[0.89, 0.93)		4.86E+00	1.11E-02	5.38E+00
Joint	[0.93, 0.96)		6.00E+00	-6.02E-02	3.96E+00
Joint	[0.96, 0.99)		9.83E+00	9.96E-02	4.61E+00
Joint	[0.99, 1.02]		2.35E+01	7.97E-01	3.26E+00
Joint	(1.02, 1.04]		9.55E+00	2.71E-01	4.75E+00
Wing	(1.04,+∞)		9.49E+00	3.89E-01	5.63E-01
Wing	(-∞, 0.64)		5.12E+00	-9.16E-03	2.86E+00
Joint	[0.64, 0.71)		4.66E+00	5.01E-02	2.95E+00
Joint	[0.71, 0.77)		5.03E+00	1.27E-02	2.19E+00
Joint	[0.77, 0.83)		5.01E+00	9.27E-03	3.73E+00
Joint	[0.83, 0.87)		4.77E+00	2.93E-02	4.19E+00
Joint	[0.87, 0.93)	(0.65, 0.90]	5.23E+00	-1.85E-02	4.28E+00
Joint	[0.93, 0.95)		4.87E+00	8.58E-03	5.21E+00
Joint	[0.95, 0.96)		5.71E+00	-2.34E-02	4.01E+00
Joint	[0.96, 0.99)		9.85E+00	3.54E-01	4.50E+00
Joint	[0.99, 1.12]		1.24E+01	-4.25E-01	4.54E+00
Wing	(1.12,+∞)		7.67E+00	2.69E-01	1.45E+00
Wing	(-∞, 0.26)		4.76E+00	3.92E-02	2.32E+00
Joint	[0.26, 0.97)		7.40E+00	-8.87E-02	2.23E+00
Joint	[0.97, 1.03]	(0.90, 1.40]	1.63E+01	7.75E-01	4.97E+00
Joint	(1.03, 1.13]		7.42E+00	-1.01E-02	3.19E+00
Wing	(1.13,+∞)		7.58E+00	2.21E-01	6.45E-01
Wing	(-∞, 0.32)		4.46E+00	7.40E-02	1.15E+00
Joint	[0.32, 0.36)		4.49E+00	7.04E-02	1.16E+00
Joint	[0.36, 0.42)	(1.40, 1.90]	4.38E+00	7.93E-02	1.17E+00
Joint	[0.42, 1.13]		2.30E+01	-4.59E-02	4.02E-01
Wing	(1.13,+∞)		7.56E+00	2.13E-01	6.14E-01

Wing	(-∞, 1.05]	(1.90, 2.90]	4.31E+01	7.02E-01	3.00E-01
Wing	(1.05,+∞)		8.05E+00	1.71E-01	9.94E-01
Wing	(-∞, 0.50)	(2.90,+∞]	4.78E+00	3.30E-02	1.43E+00
Joint	[0.50, 0.89)		5.46E+00	-2.99E-02	1.84E+00
Joint	[0.89, 1.06]		2.95E+01	8.01E-01	4.16E+00
Wing	(1.06,+∞)		8.08E+00	1.97E-01	9.80E-01

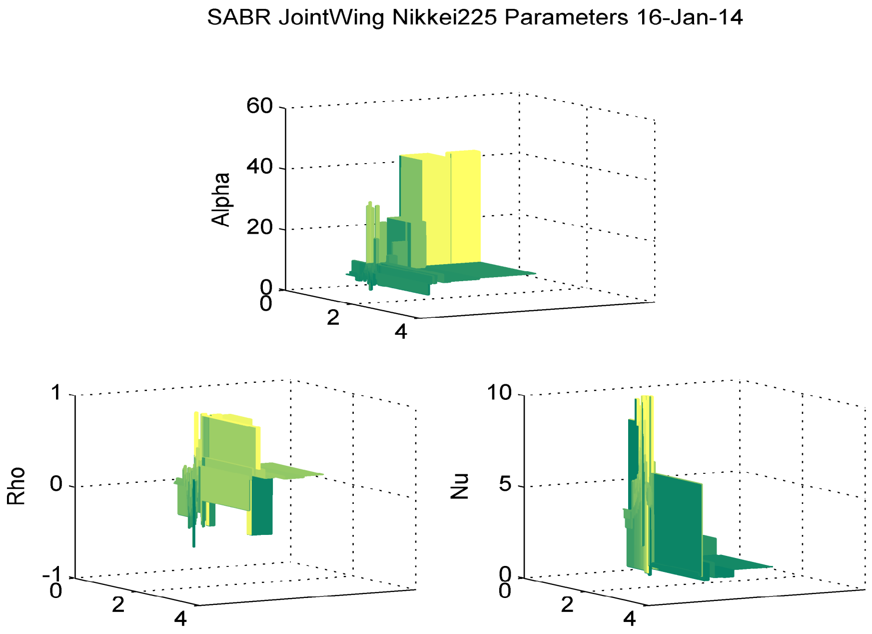


Figure B.31: SABR JointWing Nikkei225 Parameters

Table B.26: Heston JointWing Nikkei225 Parameters (κ = 2.08E+00, v0 = 1.02E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	($-\infty$, 0.67)		1.00E-05	9.74E-01	-9.99E-01
Joint	[0.67, 0.68)		8.46E-02	2.87E+00	-5.03E-01
Joint	[0.68, 0.70)		2.11E-01	3.29E-01	-2.16E-01
Joint	[0.70, 0.72)		4.91E-02	2.32E+00	-5.99E-01
Joint	[0.72, 0.73)		8.95E-02	2.37E+00	-3.78E-01
Joint	[0.73, 0.75)		2.62E+00	6.71E-01	9.98E-01
Joint	[0.75, 0.76)		1.80E-02	2.95E+00	-4.02E-01
Joint	[0.76, 0.77)		1.18E-01	1.82E+00	-7.29E-01
Joint	[0.77, 0.78)		1.47E-01	2.03E+00	-4.44E-01
Joint	[0.78, 0.79)		1.97E+00	6.21E-01	9.61E-01
Joint	[0.79, 0.80)		1.76E-01	2.68E+00	-1.79E-01
Joint	[0.80, 0.80)		1.24E+00	6.05E-01	1.36E-01
Joint	[0.80, 0.81)		7.46E-01	1.09E+00	-3.40E-01
Joint	[0.81, 0.82)		4.65E-01	1.66E+00	-3.47E-01
Joint	[0.82, 0.83)		2.69E-01	2.05E+00	-4.18E-01
Joint	[0.83, 0.83)		8.54E-01	7.05E-01	-3.61E-01
Joint	[0.83, 0.84)		5.92E-01	1.40E+00	-3.30E-01
Joint	[0.84, 0.85)		3.41E-01	1.69E+00	-5.82E-01
Joint	[0.85, 0.86)		5.84E-01	1.14E+00	-4.79E-01
Joint	[0.86, 0.87)		4.12E-01	1.69E+00	-4.75E-01
Joint	[0.87, 0.87)		7.27E-01	7.61E-01	-3.96E-01
Joint	[0.87, 0.88)		7.05E-01	8.66E-01	-3.76E-01
Joint	[0.88, 0.89)		1.42E+00	4.49E+00	5.91E-01

Joint	[0.89, 0.89)	2.11E-01	3.29E-01	-2.16E-01
Joint	[0.89, 0.90)	2.11E-01	3.29E-01	-2.16E-01
Joint	[0.90, 0.91)	8.37E-01	5.13E+00	2.87E-01
Joint	[0.91, 0.91)	1.34E+00	4.80E+00	5.74E-01
Joint	[0.91, 0.92)	8.07E-01	1.67E+00	1.83E-01
Joint	[0.92, 0.92)	2.11E-01	3.29E-01	-2.16E-01
Joint	[0.92, 0.93)	2.11E-01	3.29E-01	-2.16E-01
Joint	[0.93, 0.94)	8.16E-01	2.02E+00	1.93E-01
Joint	[0.94, 0.95)	8.37E-01	2.21E+00	2.15E-01
Joint	[0.95, 0.95)	1.38E+00	4.68E+00	5.68E-01
Joint	[0.95, 0.95)	2.11E-01	3.29E-01	-2.16E-01
Joint	[0.95, 0.96)	2.11E-01	3.29E-01	-2.16E-01
Joint	[0.96, 0.97)	1.22E+00	5.65E+00	3.51E-01
Joint	[0.97, 0.97)	5.14E+00	4.96E+00	2.82E-04
Joint	[0.97, 0.98)	5.54E+00	5.11E+00	-1.91E-01
Joint	[0.98, 0.99)	1.20E+00	5.82E+00	2.90E-01
Joint	[0.99, 0.99)	1.02E+00	5.99E+00	8.73E-02
Joint	[0.99, 0.99)	1.06E+00	5.85E+00	3.41E-01
Joint	[0.99, 1.00]	1.18E+00	6.03E+00	-1.86E-01
Joint	(1.00, 1.00]	2.00E-01	3.40E-01	-2.15E-01
Joint	(1.00, 1.01]	8.38E-01	8.11E+00	9.52E-01
Joint	(1.01, 1.01]	1.69E-02	4.14E-01	-2.37E-01
Joint	(1.01, 1.02]	2.74E-01	7.47E+00	9.89E-01
Joint	(1.02, 1.02]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.02, 1.03]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.03, 1.03]	1.12E+00	5.86E+00	-3.19E-01
Joint	(1.03, 1.03]	2.11E-01	3.29E-01	-2.16E-01

Joint	(1.03, 1.04]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.04, 1.05]	9.87E-01	3.48E+00	-5.05E-01
Joint	(1.05, 1.05]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.05, 1.06]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.06, 1.07]	1.15E+00	5.54E+00	-5.50E-01
Joint	(1.07, 1.07]	9.31E-01	5.30E+00	-4.37E-01
Joint	(1.07, 1.07]	1.05E+00	5.52E+00	-6.01E-01
Joint	(1.07, 1.08]	4.66E-01	5.83E-01	-1.37E-01
Joint	(1.08, 1.08]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.08, 1.09]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.09, 1.10]	4.50E-01	5.83E-01	-1.39E-01
Joint	(1.10, 1.11]	4.40E-01	5.84E-01	-1.41E-01
Joint	(1.11, 1.11]	4.32E-01	5.85E-01	-1.43E-01
Joint	(1.11, 1.12]	4.42E-01	1.00E+00	-2.42E-01
Joint	(1.12, 1.13]	6.27E-01	2.09E+00	-5.43E-01
Joint	(1.13, 1.14]	4.29E-01	7.20E-01	-1.83E-01
Joint	(1.14, 1.14]	1.14E+00	3.57E+00	-7.91E-01
Joint	(1.14, 1.15]	6.65E-01	1.41E+00	-5.80E-01
Joint	(1.15, 1.16]	6.40E-01	1.24E+00	-5.70E-01
Joint	(1.16, 1.18]	7.45E-01	1.36E+00	-6.72E-01
Joint	(1.18, 1.19]	2.98E-02	2.69E+00	-4.71E-01
Wing	(1.19,+∞)	2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.57)	3.82E-01	1.95E-03	5.01E-01
Joint	[0.57, 0.59)	2.34E-01	3.47E-01	-2.14E-01
Joint	[0.59, 0.62)	1.83E-01	1.92E+00	-6.25E-01
Joint	[0.62, 0.64)	1.72E-01	2.41E+00	-3.10E-01
Joint	[0.64, 0.65)	1.02E-01	1.83E+00	-7.07E-01

Joint	[0.65, 0.67)	4.42E-01	1.14E+00	-5.40E-01
Joint	[0.67, 0.68)	1.98E-01	1.77E+00	-4.92E-01
Joint	[0.68, 0.70)	6.56E-01	8.78E-01	-3.25E-01
Joint	[0.70, 0.72)	1.14E-01	2.08E+00	-5.96E-01
Joint	[0.72, 0.73)	1.99E-01	1.69E+00	-5.60E-01
Joint	[0.73, 0.75)	3.45E-01	1.40E+00	-4.03E-01
Joint	[0.75, 0.76)	2.00E-01	1.93E+00	-4.52E-01
Joint	[0.76, 0.76)	2.11E-01	3.29E-01	-2.16E-01
Joint	[0.76, 0.78)	2.11E-01	3.29E-01	-2.16E-01
Joint	[0.78, 0.80)	6.71E-01	3.80E+00	4.97E-01
Joint	[0.80, 0.81)	1.05E+00	3.71E+00	6.66E-01
Joint	[0.81, 0.83)	9.62E-01	4.41E+00	6.46E-01
Joint	[0.83, 0.83)	8.91E-01	4.70E+00	6.21E-01
Joint	[0.83, 0.84)	5.65E-01	4.64E+00	4.16E-01
Joint	[0.84, 0.85)	6.54E-01	5.00E+00	5.25E-01
Joint	[0.85, 0.86)	4.47E-01	8.40E-01	-4.01E-02
Joint	[0.86, 0.86)	2.11E-01	3.29E-01	-2.16E-01
Joint	[0.86, 0.87)	2.11E-01	3.29E-01	-2.16E-01
Joint	[0.87, 0.87)	8.57E-01	4.80E+00	5.72E-01
Joint	[0.87, 0.88)	7.94E-01	4.98E+00	5.25E-01
Joint	[0.88, 0.89)	4.45E-01	8.39E-01	-5.87E-02
Joint	[0.89, 0.90)	5.61E-01	5.54E+00	1.61E-01
Joint	[0.90, 0.91)	4.37E-01	1.17E+00	2.98E-02
Joint	[0.91, 0.91)	6.75E-01	5.62E+00	2.98E-01
Joint	[0.91, 0.91)	7.25E-01	5.17E+00	3.93E-01
Joint	[0.91, 0.92)	7.61E-01	5.23E+00	4.06E-01
Joint	[0.92, 0.93)	8.03E-01	5.57E+00	3.92E-01

Joint	[0.93, 0.94)	7.97E-01	5.66E+00	3.49E-01
Joint	[0.94, 0.95)	7.76E-01	5.41E+00	3.32E-01
Joint	[0.95, 0.95)	8.01E-01	5.49E+00	3.66E-01
Joint	[0.95, 0.95)	2.16E+00	5.63E+00	4.54E-02
Joint	[0.95, 0.96)	2.23E+00	5.55E+00	-6.58E-02
Joint	[0.96, 0.97)	7.47E-01	6.14E+00	-5.30E-03
Joint	[0.97, 0.98)	8.04E-01	5.82E+00	3.43E-01
Joint	[0.98, 0.99)	7.79E-01	6.03E+00	7.45E-02
Joint	[0.99, 0.99)	6.03E+00	7.55E+00	4.05E-01
Joint	[0.99, 0.99)	5.17E+00	7.48E+00	4.44E-01
Joint	[0.99, 1.00]	4.92E-01	2.50E+00	6.20E-01
Joint	(1.00, 1.00]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.00, 1.01]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.01, 1.02]	8.65E-01	6.06E+00	-3.34E-01
Joint	(1.02, 1.02]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.02, 1.03]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.03, 1.03]	8.72E-01	5.88E+00	-4.41E-01
Joint	(1.03, 1.03]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.03, 1.04]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.04, 1.05]	7.15E-01	4.85E+00	-3.62E-01
Joint	(1.05, 1.06]	6.61E-01	6.13E+00	-1.29E-01
Joint	(1.06, 1.07]	8.25E-01	5.90E+00	-4.27E-01
Joint	(1.07, 1.07]	8.19E-01	5.85E+00	-4.70E-01
Joint	(1.07, 1.08]	3.68E-01	1.40E+00	-2.01E-01
Joint	(1.08, 1.08]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.08, 1.09]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.09, 1.10]	3.47E-01	1.18E+00	-2.08E-01

Joint	(1.10, 1.11]	8.13E-01	5.55E+00	-6.10E-01
Joint	(1.11, 1.11]	8.72E-01	5.60E+00	-6.56E-01
Joint	(1.11, 1.11]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.11, 1.12]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.12, 1.13]	3.35E-01	1.01E+00	-2.52E-01
Joint	(1.13, 1.14]	3.04E-01	1.03E+00	-2.05E-01
Joint	(1.14, 1.15]	3.01E-01	9.67E-01	-2.12E-01
Joint	(1.15, 1.15]	3.26E-01	4.53E+00	-3.80E-01
Joint	(1.15, 1.16]	3.14E-01	1.07E+00	-2.36E-01
Joint	(1.16, 1.18]	2.85E-01	8.46E-01	-2.32E-01
Joint	(1.18, 1.19]	4.46E-01	1.86E+00	-5.80E-01
Joint	(1.19, 1.21]	5.27E-01	1.81E+00	-6.60E-01
Joint	(1.21, 1.22]	9.34E-01	4.22E+00	-8.22E-01
Joint	(1.22, 1.24]	5.39E-01	1.54E+00	-6.65E-01
Joint	(1.24, 1.26]	4.48E-01	1.04E+00	-5.86E-01
Wing	(1.26,+∞)	2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.61)	4.51E-01	3.01E-03	-4.60E-01
Joint	[0.61, 0.64)	4.03E-01	1.13E+00	-3.43E-01
Joint	[0.64, 0.66)	1.26E-01	2.26E+00	-3.74E-01
Joint	[0.66, 0.70)	8.11E-01	3.40E+00	6.86E-01
Joint	[0.70, 0.72)	7.03E-01	3.51E+00	6.64E-01
Joint	[0.72, 0.77)	6.16E-01	3.60E+00	6.41E-01
Joint	[0.77, 0.80)	5.68E-01	3.59E+00	6.03E-01
Joint	[0.80, 0.81)	4.69E-01	3.65E+00	5.14E-01
Joint	[0.81, 0.85)	3.19E-01	5.47E-01	-1.47E-01
Joint	[0.85, 0.85)	3.53E-01	4.94E+00	2.55E-01
Joint	[0.85, 0.86)	2.98E-01	6.40E-01	-1.22E-01

Joint	[0.86, 0.89)	3.29E-01	6.04E-01	-1.42E-01
Joint	[0.89, 0.90)	4.87E-01	5.14E+00	1.84E-01
Joint	[0.90, 0.92)	2.75E-01	9.31E-01	-4.50E-02
Joint	[0.92, 0.93)	4.49E-01	5.84E+00	1.15E-01
Joint	[0.93, 0.93)	2.87E-01	1.52E+00	6.24E-02
Joint	[0.93, 0.94)	4.73E-01	5.02E+00	2.20E-01
Joint	[0.94, 0.96)	2.93E-01	5.79E+00	5.64E-02
Joint	[0.96, 1.01]	3.53E-01	3.36E+00	9.98E-01
Joint	(1.01, 1.04]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.04, 1.05]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.05, 1.06]	5.56E-01	6.10E+00	-1.35E-01
Joint	(1.06, 1.07]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.07, 1.08]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.08, 1.09]	6.95E-01	5.96E+00	-4.68E-01
Joint	(1.09, 1.10]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.10, 1.12]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.12, 1.12]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.12, 1.13]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.13, 1.13]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.13, 1.14]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.14, 1.15]	7.58E-01	5.66E+00	-7.05E-01
Joint	(1.15, 1.17]	4.80E-01	5.63E+00	-5.44E-01
Joint	(1.17, 1.17]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.17, 1.18]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.18, 1.20]	2.54E-01	7.93E-01	-2.48E-01
Joint	(1.20, 1.21]	2.37E-01	8.86E-01	-2.41E-01
Joint	(1.21, 1.25]	2.32E-01	3.35E-01	-2.00E-01

Wing	(1.25,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.93)		2.11E-01	3.29E-01	-2.16E-01
Joint	[0.93, 1.09]	(0.23, 0.31]	2.11E-01	3.29E-01	-2.16E-01
Joint	(1.09, 1.15]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.15,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.57)		2.11E-01	3.29E-01	-2.16E-01
Joint	[0.57, 0.64)		9.99E+00	5.06E-01	6.83E-01
Joint	[0.64, 0.70)		1.24E+00	6.20E-01	-3.96E-01
Joint	[0.70, 0.72)		4.01E-01	2.06E+00	5.65E-01
Joint	[0.72, 0.80)		2.02E-01	2.73E+00	2.63E-01
Joint	[0.80, 0.81)		1.91E-01	3.45E-01	-1.98E-01
Joint	[0.81, 0.83)		1.88E-01	3.73E+00	-2.76E-02
Joint	[0.83, 0.86)		2.61E-01	2.73E+00	3.47E-01
Joint	[0.86, 0.89)		6.19E-02	1.22E+00	9.99E-01
Joint	[0.89, 0.93)	(0.31, 0.41]	6.19E-02	1.22E+00	9.99E-01
Joint	[0.93, 0.96)		1.27E+00	5.48E+00	-3.44E-01
Joint	[0.96, 0.99)		1.61E-01	6.40E-01	-6.71E-01
Joint	[0.99, 1.02]		2.11E-01	3.29E-01	-2.16E-01
Joint	(1.02, 1.05]		2.11E-01	3.29E-01	-2.16E-01
Joint	(1.05, 1.07]		2.11E-01	3.29E-01	-2.16E-01
Joint	(1.07, 1.09]		4.90E-01	6.15E+00	-2.41E-01
Joint	(1.09, 1.15]		2.11E-01	3.29E-01	-2.16E-01
Joint	(1.15, 1.25]		2.11E-01	3.29E-01	-2.16E-01
Joint	(1.25, 1.28]		7.73E-01	5.15E+00	-8.32E-01
Wing	(1.28,+∞)		1.00E+01	5.82E+00	9.99E-01
Wing	(-∞, 0.77)		1.13E+00	8.42E-01	-4.23E-01
Joint	[0.77, 0.89)		1.44E+00	1.46E+00	-7.49E-03

(0.41, 0.65]

Joint	[0.89, 0.93)		6.20E-01	5.56E+00	-5.05E-01
Joint	[0.93, 0.96)		1.18E-01	1.89E+00	4.26E-01
Joint	[0.96, 0.99)		3.02E-01	6.38E+00	-5.34E-01
Joint	[0.99, 1.02]		1.30E-01	1.40E+00	8.83E-01
Joint	(1.02, 1.04]		3.84E-01	5.90E+00	-5.02E-01
Wing	(1.04,+∞)		7.77E-01	2.72E-01	-9.51E-01
Wing	(-∞, 0.64)		1.08E+00	1.41E+00	-2.20E-01
Joint	[0.64, 0.71)		5.08E-01	4.90E+00	-5.24E-01
Joint	[0.71, 0.77)		1.18E-01	1.58E+00	1.75E-01
Joint	[0.77, 0.83)		2.60E-01	3.64E-01	-2.10E-01
Joint	[0.83, 0.87)		5.34E-01	5.15E+00	-5.33E-01
Joint	[0.87, 0.93)	(0.65, 0.90]	2.51E-01	1.82E+00	1.78E-01
Joint	[0.93, 0.95)		8.36E-01	5.50E+00	-5.72E-01
Joint	[0.95, 0.96)		1.56E-01	1.14E+00	7.07E-01
Joint	[0.96, 0.99)		3.34E-01	5.81E+00	-5.61E-01
Joint	[0.99, 1.12]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.12,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.26)		7.09E-01	1.38E+00	-2.54E-01
Joint	[0.26, 0.97)		1.03E+00	1.13E+00	-3.59E-01
Joint	[0.97, 1.03]	(0.90, 1.40]	3.26E-01	5.96E+00	-6.20E-01
Joint	(1.03, 1.13]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.13,+∞)		2.11E-01	3.29E-01	-2.16E-01
Wing	(-∞, 0.32)		1.52E-01	7.51E-01	6.43E-02
Joint	[0.32, 0.36)		1.39E-01	8.06E-01	8.59E-02
Joint	[0.36, 0.42)	(1.40, 1.90]	2.50E-02	1.19E+00	3.81E-02
Joint	[0.42, 1.13]		2.11E-01	3.29E-01	-2.16E-01
Wing	(1.13,+∞)		2.11E-01	3.29E-01	-2.16E-01

Wing	(-∞, 1.05]	(1.90, 2.90]	1.52E-01	5.81E-01	9.43E-02
Wing	(1.05,+∞)		5.62E+00	1.15E+00	8.58E-01
Wing	(-∞, 0.50)	(2.90,+∞]	5.76E-01	2.87E+00	-7.04E-01
Joint	[0.50, 0.89)		4.46E-01	5.11E-01	-1.53E-01
Joint	[0.89, 1.06]		3.74E-01	3.31E+00	-5.35E-01
Wing	(1.06,+∞)		6.21E-04	5.49E+00	-6.50E-01

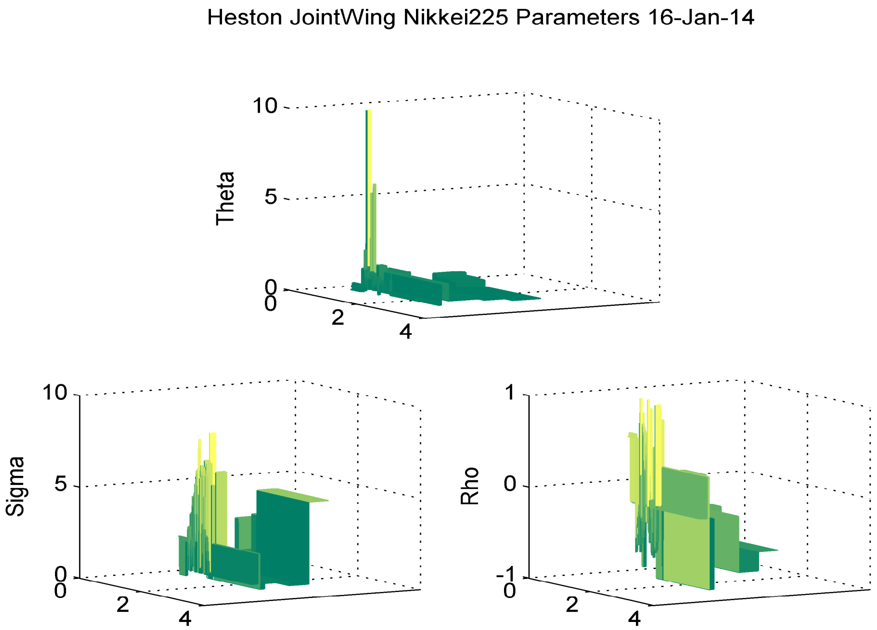


Figure B.32: Heston JointWing Nikkei225 Parameters

Table B.27: Elapsed Seconds Nikkei225 Calibration (06-Jan-14∼31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
7.453E+0	1.309E+1	5.041E+1	3.419E+2	6.362E+2	3.230E+3

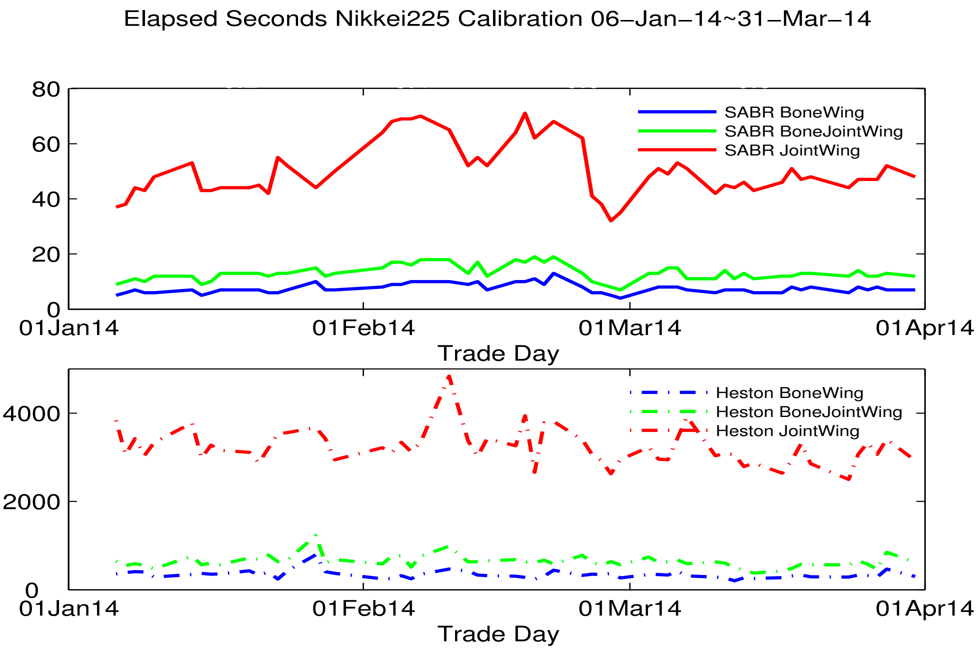


Figure B.33: Elapsed Seconds Nikkei225 Calibration (02-Jan-14∼31-Mar-14)

Table B.28: SABR BoneWing EuroStoxx50 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.69)		8.18E+00	-4.38E-01	8.12E+00
Bone	[0.69, 1.05]	(0.00, 0.01]	4.22E+00	-8.16E-01	1.00E+01
Wing	(1.05,+∞)		5.99E+00	-5.45E-02	4.82E-01
Wing	(-∞, 0.71)		9.85E+00	-7.25E-01	2.97E+00
Bone	[0.71, 1.09]	(0.01, 0.10]	1.92E+00	-5.85E-01	9.63E+00
Wing	(1.09,+∞)		9.88E+00	7.51E-01	2.30E-01
Wing	(-∞, 0.47)		8.78E+00	6.13E-01	3.89E+00
Bone	[0.47, 1.20]	(0.10, 0.18]	2.93E+00	-7.49E-01	5.31E+00
Wing	(1.20,+∞)		9.85E+00	7.66E-01	3.42E-01
Wing	(-∞, 0.48)		8.86E+00	6.25E-01	2.54E+00
Bone	[0.48, 1.19]	(0.18, 0.43]	4.16E+00	-7.63E-01	3.00E+00
Wing	(1.19,+∞)		7.78E+00	-1.80E-01	3.83E-01
Wing	(-∞, 0.42)		7.52E+00	6.35E-01	2.16E+00
Bone	[0.42, 1.26]	(0.43, 0.68]	5.29E+00	-7.48E-01	2.00E+00
Wing	(1.26,+∞)		7.78E+00	-3.23E-01	5.05E-01
Wing	(-∞, 0.45)		8.47E+00	6.42E-01	1.93E+00
Bone	[0.45, 1.49]	(0.68, 0.93]	5.86E+00	-6.72E-01	1.72E+00
Wing	(1.49,+∞)		7.14E+00	-2.47E-03	8.26E-01
Wing	(-∞, 0.37)		5.14E+00	6.03E-01	1.19E+00
Bone	[0.37, 1.50]	(0.93, 1.42]	1.44E+01	7.17E-01	1.60E-02
Wing	(1.50,+∞)		9.92E+00	7.70E-01	2.78E-01
Wing	(-∞, 0.37)		4.20E+00	-6.06E-01	1.36E+00
Bone	[0.37, 1.66]	(1.42, 1.92]	1.27E+01	-1.37E-02	8.07E-01

Wing	(1.66,+∞)		3.09E+00	5.11E-01	1.53E+00
Wing	(-∞, 0.37)		6.83E+00	-2.23E-01	7.05E-01
Bone	[0.37, 1.70]	(1.92,+∞]	1.32E+01	-4.87E-01	3.77E-01
Wing	(1.70,+∞)		9.27E+00	5.25E-01	2.66E-01

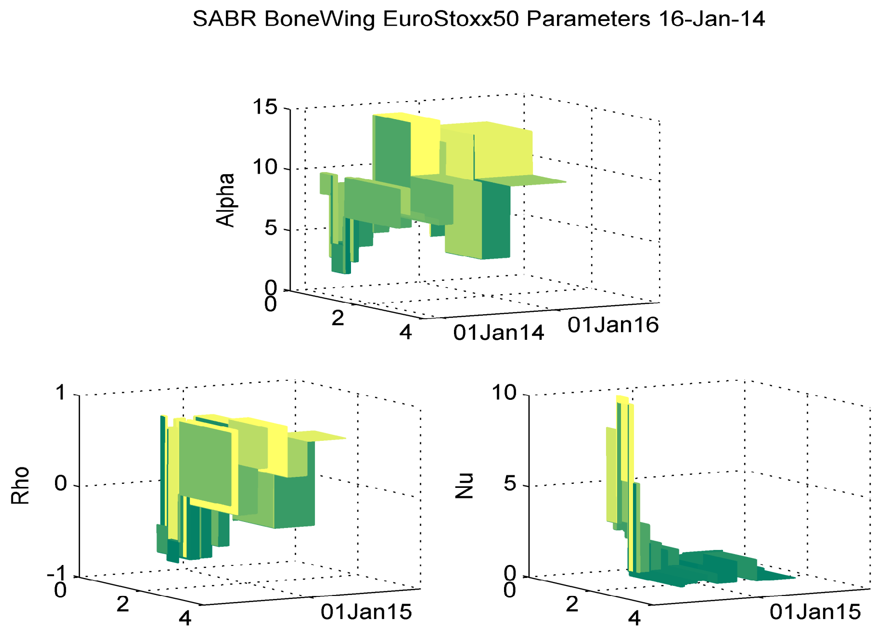


Figure B.34: SABR BoneWing EuroStoxx50 Parameters

Table B.29: Heston BoneWing EuroStoxx50 Parameters (κ = 2.00E+00, v0 = 1.00E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.69)		1.25E-05	1.00E+01	-4.44E-01
Bone	[0.69, 1.05]	(0.00, 0.01]	1.00E+01	2.90E+00	-9.99E-01
Wing	(1.05,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.71)		2.07E-03	7.63E-01	-5.98E-01
Bone	[0.71, 1.09]	(0.01, 0.10]	1.00E-01	3.50E-01	-2.50E-01
Wing	(1.09,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.47)		1.00E-01	3.50E-01	-2.50E-01
Bone	[0.47, 1.20]	(0.10, 0.18]	1.00E-01	3.50E-01	-2.50E-01
Wing	(1.20,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.48)		8.29E-02	1.49E+00	-7.38E-01
Bone	[0.48, 1.19]	(0.18, 0.43]	1.32E-01	1.04E+00	-9.54E-01
Wing	(1.19,+∞)		4.20E-04	1.46E+00	-5.92E-01
Wing	(-∞, 0.42)		5.49E-02	1.65E+00	-6.02E-01
Bone	[0.42, 1.26]	(0.43, 0.68]	1.03E-01	9.22E-01	-9.18E-01
Wing	(1.26,+∞)		4.40E-02	6.83E-04	5.61E-01
Wing	(-∞, 0.45)		1.10E-01	1.70E+00	-5.83E-02
Bone	[0.45, 1.49]	(0.68, 0.93]	9.12E-02	9.02E-01	-8.52E-01
Wing	(1.49,+∞)		3.24E+00	1.00E+01	9.15E-01
Wing	(-∞, 0.37)		1.90E-01	8.16E-01	1.64E-01
Bone	[0.37, 1.50]	(0.93, 1.42]	7.31E-02	7.89E-01	-8.21E-01
Wing	(1.50,+∞)		4.11E-01	3.60E-01	-9.95E-01
Wing	(-∞, 0.37)		1.72E-01	9.28E-01	-6.69E-02
Bone	[0.37, 1.66]	(1.42, 1.92]	2.28E-02	1.90E+00	-2.68E-01

Wing	(1.66,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.37)		1.05E-01	8.39E-01	-1.07E-01
Bone	[0.37, 1.70]	(1.92,+∞]	6.73E-02	1.03E+00	-6.88E-01
Wing	(1.70,+∞)		1.00E-02	9.81E-01	-2.59E-01

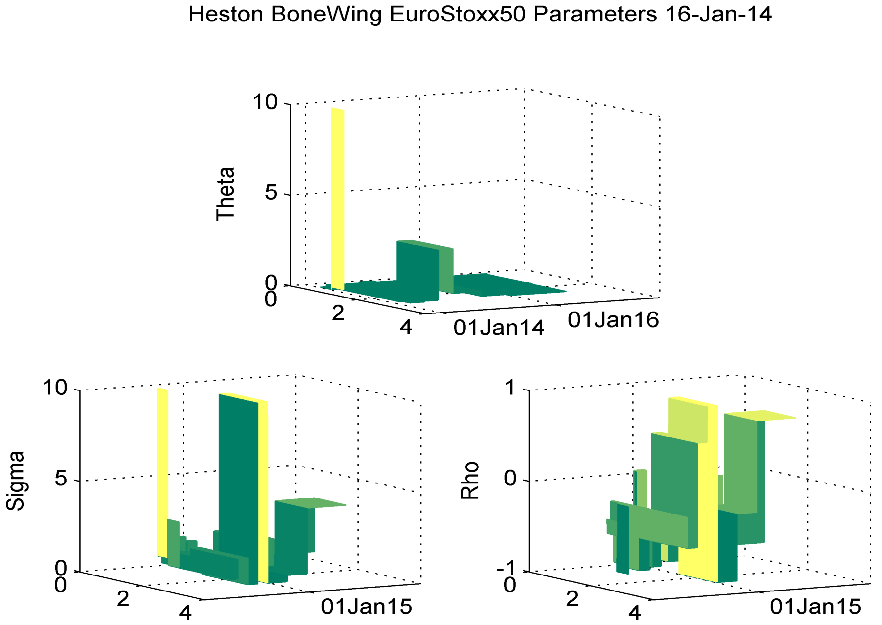


Figure B.35: Heston BoneWing EuroStoxx50 Parameters

Table B.30: SABR BoneJointWing EuroStoxx50 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.69)		8.52E+00	-4.94E-01	8.85E+00
Bone	[0.69, 0.95]		1.00E+01	-8.15E-01	1.00E+01
Joint	(0.95, 0.96)	(0.00, 0.01]	7.60E+00	-6.06E-01	9.79E+00
Bone	[0.96, 1.05]		4.22E+00	-8.16E-01	1.00E+01
Wing	(1.05,+∞)		5.99E+00	-5.45E-02	4.82E-01
Wing	(-∞, 0.71)		9.85E+00	-7.56E-01	2.14E+00
Bone	[0.71, 0.95]		8.43E+00	-2.67E-01	4.65E+00
Joint	(0.95, 0.96)		5.42E+00	-5.72E-02	6.68E+00
Bone	[0.96, 1.04]	(0.01, 0.10]	1.74E+00	-5.01E-01	1.00E+01
Joint	(1.04, 1.05)		7.72E+00	-4.06E-01	1.53E+00
Bone	[1.05, 1.09]		3.81E+00	8.36E-02	2.61E+00
Wing	(1.09,+∞)		7.14E+00	6.25E-02	2.23E-01
Wing	(-∞, 0.47)		8.49E+00	6.97E-01	3.67E+00
Bone	[0.47, 0.95]		9.93E+00	-5.79E-01	2.34E+00
Joint	(0.95, 0.96)		6.15E+00	-6.32E-02	5.82E+00
Bone	[0.96, 1.05]	(0.10, 0.18]	1.46E+00	-3.98E-01	1.00E+01
Joint	(1.05, 1.06)		9.03E+00	5.40E-02	1.53E+00
Bone	[1.06, 1.20]		5.59E+00	-5.41E-01	2.38E+00
Wing	(1.20,+∞)		9.01E+00	-1.15E-01	4.18E-01
Wing	(-∞, 0.48)		9.32E+00	3.03E-01	1.67E+00
Bone	[0.48, 0.94]		9.93E+00	-5.98E-01	1.31E+00
Joint	(0.94, 0.95)		6.33E+00	-2.58E-02	4.49E+00
Bone	[0.95, 1.05]	(0.18, 0.43]	2.02E+00	-6.72E-01	9.96E+00

Joint	(1.05, 1.07)		9.42E+00	-4.19E-01	1.61E+00
Bone	[1.07, 1.19]		6.00E+00	-4.69E-01	1.33E+00
Wing	(1.19,+∞)		7.37E+00	-3.16E-01	4.07E-01
Wing	(-∞, 0.42)		7.72E+00	6.71E-01	1.99E+00
Bone	[0.42, 0.94]		9.93E+00	-3.54E-01	1.06E+00
Joint	(0.94, 0.95)		6.28E+00	7.97E-03	3.84E+00
Bone	[0.95, 1.05]	(0.43, 0.68]	2.85E+00	-7.39E-01	9.60E+00
Joint	(1.05, 1.07)		9.31E+00	-5.10E-01	1.62E+00
Bone	[1.07, 1.26]		5.95E+00	1.86E-01	4.68E-01
Wing	(1.26,+∞)		7.27E+00	-3.15E-01	5.12E-01
Wing	(-∞, 0.45)		8.60E+00	6.20E-01	1.68E+00
Bone	[0.45, 0.94]		9.93E+00	-1.44E-01	1.03E+00
Joint	(0.94, 0.96)		5.07E+00	2.15E-02	3.92E+00
Bone	[0.96, 1.04]	(0.68, 0.93]	2.50E+00	-6.78E-01	7.95E+00
Joint	(1.04, 1.05)		8.99E+00	-5.93E-01	2.17E+00
Bone	[1.05, 1.49]		6.47E+00	2.16E-01	3.76E-01
Wing	(1.49,+∞)		5.55E+00	-6.18E-01	1.07E+00
Wing	(-∞, 0.37)		5.46E+00	7.06E-01	1.78E+00
Bone	[0.37, 0.95]		9.96E+00	3.96E-01	1.04E+00
Joint	(0.95, 0.96)		4.99E+00	4.21E-02	3.40E+00
Bone	[0.96, 1.05]	(0.93, 1.42]	1.17E+01	-7.72E-01	5.66E+00
Joint	(1.05, 1.06)		8.36E+00	-5.20E-01	1.68E+00
Bone	[1.06, 1.50]		7.04E+00	2.51E-01	2.29E-01
Wing	(1.50,+∞)		6.65E+00	1.62E-01	3.13E-01
Wing	(-∞, 0.37)		5.08E+00	-2.84E-01	9.42E-01
Bone	[0.37, 0.95]		9.94E+00	1.47E-01	7.53E-01
Joint	(0.95, 0.97)		6.41E+00	1.91E-01	2.73E+00
		(1.42, 1.92]			

Bone	[0.97, 1.05]		1.16E+01	-7.75E-01	5.08E+00
Joint	(1.05, 1.07)		7.58E+00	-6.06E-01	1.52E+00
Bone	[1.07, 1.66]		2.11E+00	-1.47E-01	1.77E+00
Wing	(1.66,+∞)		4.07E+00	2.36E-01	1.41E+00
Wing	(-∞, 0.37)		5.37E+00	-1.02E-01	7.60E-01
Bone	[0.37, 0.93]		9.97E+00	5.24E-04	5.59E-01
Joint	(0.93, 0.95)		7.70E+00	7.37E-01	4.61E+00
Bone	[0.95, 1.02]	(1.92,+∞]	1.08E+01	-7.41E-01	3.04E+00
Joint	(1.02, 1.05)		7.47E+00	-5.00E-01	1.82E+00
Bone	[1.05, 1.70]		8.06E+00	2.18E-01	1.40E-01
Wing	(1.70,+∞)		5.58E+00	5.10E-01	3.15E-01

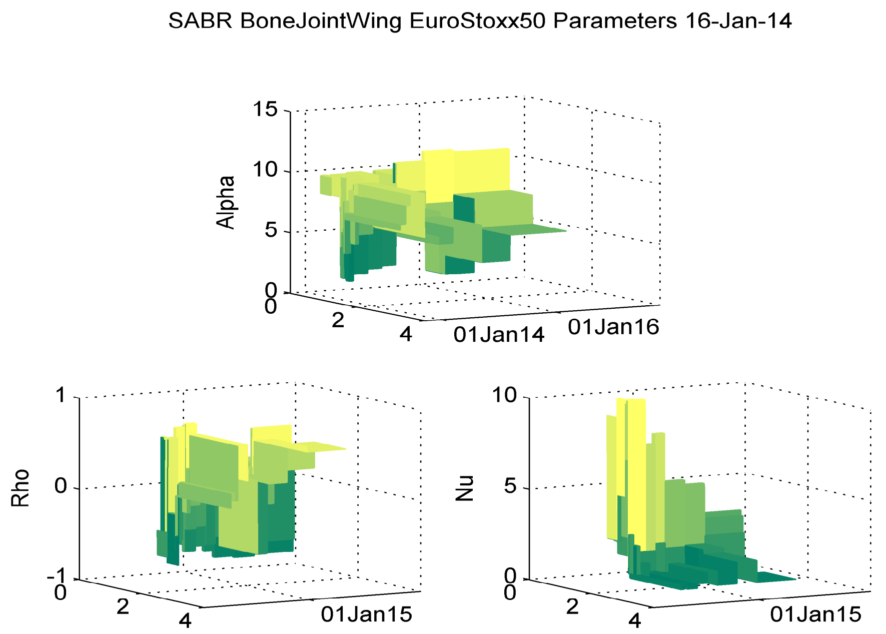


Figure B.36: SABR BoneJointWing EuroStoxx50 Parameters

Table B.31: Heston BoneJointWing EuroStoxx50 Parameters (κ = 2.00E+00, v0 = 1.00E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.69)		1.25E-05	1.00E+01	-4.44E-01
Bone	[0.69, 0.95]		1.00E-01	3.50E-01	-2.50E-01
Joint	(0.95, 0.96)	(0.00, 0.01]	1.00E-01	3.50E-01	-2.50E-01
Bone	[0.96, 1.05]		1.00E+01	2.90E+00	-9.99E-01
Wing	(1.05,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.71)	(0.01, 0.10]	2.07E-03	7.63E-01	-5.98E-01
Bone	[0.71, 0.95]		1.00E-01	3.50E-01	-2.50E-01

Joint	(0.95, 0.96)		2.28E-01	5.75E-01	-2.26E-01
Bone	[0.96, 1.04]		8.95E-01	5.95E+00	-7.74E-01
Joint	(1.04, 1.05)		4.98E-01	2.26E+00	-9.35E-01
Bone	[1.05, 1.09]		5.40E-02	1.66E+00	-5.44E-01
Wing	(1.09,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.47)		1.00E-01	3.50E-01	-2.50E-01
Bone	[0.47, 0.95]		1.00E-01	3.50E-01	-2.50E-01
Joint	(0.95, 0.96)		5.24E-01	5.10E+00	3.71E-01
Bone	[0.96, 1.05]	(0.10, 0.18]	1.22E+00	9.02E+00	-9.89E-01
Joint	(1.05, 1.06)		4.62E-02	7.84E+00	-3.92E-01
Bone	[1.06, 1.20]		2.84E-02	1.05E+00	-4.91E-01
Wing	(1.20,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.48)		1.36E-02	2.04E+00	-7.99E-01
Bone	[0.48, 0.94]		2.16E-01	1.22E+00	-2.35E-01
Joint	(0.94, 0.95)		4.02E-01	5.46E+00	-3.43E-01
Bone	[0.95, 1.05]	(0.18, 0.43]	8.28E-01	9.97E+00	-9.97E-01
Joint	(1.05, 1.07)		1.70E-01	2.09E+00	-9.37E-01
Bone	[1.07, 1.19]		5.84E-02	1.03E+00	-6.79E-01
Wing	(1.19,+∞)		4.20E-04	1.46E+00	-5.92E-01
Wing	(-∞, 0.42)		2.17E-02	3.24E+00	-4.38E-01
Bone	[0.42, 0.94]		2.05E-01	1.55E+00	2.13E-01
Joint	(0.94, 0.95)		3.59E-01	5.88E+00	-5.29E-01
Bone	[0.95, 1.05]	(0.43, 0.68]	5.77E-01	8.68E+00	-9.59E-01
Joint	(1.05, 1.07)		2.71E-01	5.07E+00	-8.47E-01
Bone	[1.07, 1.26]		4.69E-02	7.34E-01	-6.34E-01
Wing	(1.26,+∞)		4.40E-02	6.83E-04	5.61E-01
Wing	(-∞, 0.45)		1.74E-01	1.30E+00	2.16E-01

(0.68, 0.93]

Bone	[0.45, 0.94]		1.64E-01	1.34E+00	1.92E-01
Joint	(0.94, 0.96)		2.28E-01	2.56E+00	-1.16E-01
Bone	[0.96, 1.04]		4.24E-01	6.57E+00	-9.99E-01
Joint	(1.04, 1.05)		3.62E-01	7.23E+00	-8.79E-01
Bone	[1.05, 1.49]		5.15E-02	7.72E-01	-6.68E-01
Wing	(1.49,+∞)		3.24E+00	1.00E+01	9.15E-01
Wing	(-∞, 0.37)		1.86E-01	7.97E-01	1.88E-01
Bone	[0.37, 0.95]		1.14E-01	1.08E+00	-3.08E-04
Joint	(0.95, 0.96)		3.20E-01	7.20E+00	-6.62E-01
Bone	[0.96, 1.05]	(0.93, 1.42]	2.10E-01	5.41E+00	-6.05E-01
Joint	(1.05, 1.06)		2.29E-02	2.54E-01	-7.04E-01
Bone	[1.06, 1.50]		3.26E-02	1.71E+00	-6.06E-01
Wing	(1.50,+∞)		4.11E-01	3.60E-01	-9.95E-01
Wing	(-∞, 0.37)		1.39E-01	7.74E-01	8.43E-02
Bone	[0.37, 0.95]		9.96E-02	1.01E+00	-6.88E-02
Joint	(0.95, 0.97)		1.42E-01	3.29E+00	-4.94E-01
Bone	[0.97, 1.05]	(1.42, 1.92]	3.10E-01	6.89E+00	-8.94E-01
Joint	(1.05, 1.07)		2.04E-02	4.40E-01	-2.98E-01
Bone	[1.07, 1.66]		9.17E-02	8.12E+00	2.04E-01
Wing	(1.66,+∞)		1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.37)		7.92E-02	1.08E+00	-1.91E-01
Bone	[0.37, 0.93]		8.41E-02	1.02E+00	-2.00E-01
Joint	(0.93, 0.95)		1.01E-01	2.52E+00	-4.43E-01
Bone	[0.95, 1.02]	(1.92,+∞]	2.31E-01	6.95E+00	-6.89E-01
Joint	(1.02, 1.05)		6.92E-02	2.36E+00	-4.14E-01
Bone	[1.05, 1.70]		3.23E-02	4.16E-01	-4.17E-01
Wing	(1.70,+∞)		1.00E-02	9.81E-01	-2.59E-01

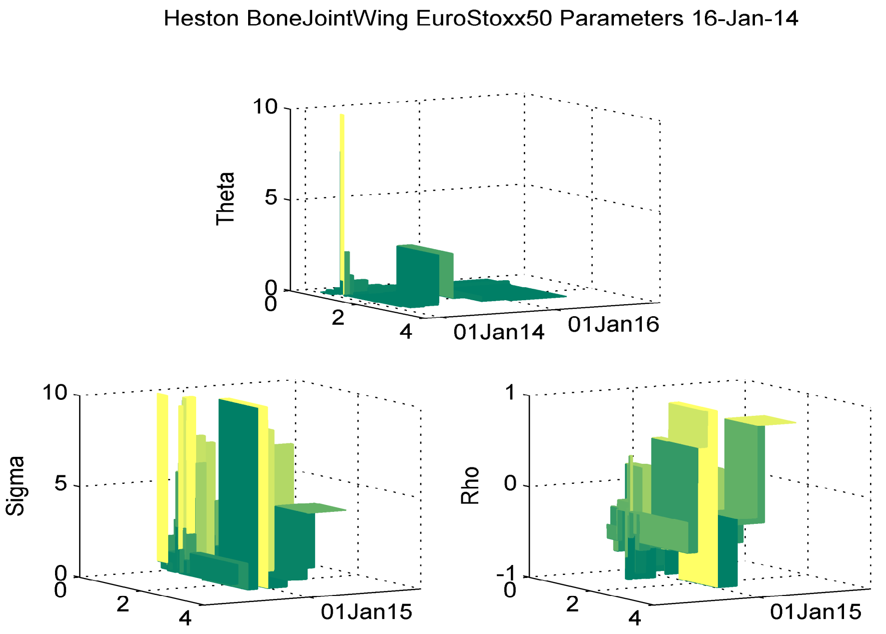


Figure B.37: Heston BoneJointWing EuroStoxx50 Parameters

Table B.32: SABR JointWing EuroStoxx50 Parameters (β = 5.00E-01, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	α	ρ	ν
Wing	(-∞, 0.69)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.69, 0.70)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.70, 0.70)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.70, 0.71)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.71, 0.72)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.72, 0.73)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.73, 0.73)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.73, 0.74)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.74, 0.75)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.75, 0.76)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.76, 0.77)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.77, 0.77)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.77, 0.78)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.78, 0.79)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.79, 0.80)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.80, 0.81)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.81, 0.81)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.81, 0.82)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.82, 0.83)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.83, 0.84)		1.00E+01	-8.14E-01	1.00E+01
Joint	[0.84, 0.85)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.85, 0.85)		1.00E+01	-8.15E-01	1.00E+01
Joint	[0.85, 0.86)		1.00E+01	-8.15E-01	1.00E+01

(0.00, 0.01]

Joint	[0.86, 0.87)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.87, 0.88)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.88, 0.88)	1.00E+01	-8.16E-01	1.00E+01
Joint	[0.88, 0.89)	1.00E+01	-8.16E-01	1.00E+01
Joint	[0.89, 0.90)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.90, 0.91)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.91, 0.92)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.92, 0.92)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.92, 0.93)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.93, 0.94)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.94, 0.95)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.95, 0.96)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.96, 0.96)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.96, 0.97)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.97, 0.98)	1.00E+01	-8.16E-01	1.00E+01
Joint	[0.98, 0.99)	1.00E+01	-8.15E-01	1.00E+01
Joint	[0.99, 1.00)	1.00E+01	-7.00E-01	9.98E+00
Joint	[1.00, 1.00]	4.22E+00	-8.16E-01	1.00E+01
Joint	(1.00, 1.01]	4.43E+00	6.71E-01	9.32E+00
Joint	(1.01, 1.02]	5.91E+00	6.06E-01	9.28E+00
Joint	(1.02, 1.03]	6.92E+00	5.65E-01	9.26E+00
Joint	(1.03, 1.03]	7.61E+00	5.43E-01	9.26E+00
Joint	(1.03, 1.04]	8.08E+00	5.34E-01	9.28E+00
Joint	(1.04, 1.05]	8.47E+00	5.45E-01	9.32E+00
Wing	(1.05,+∞)	9.99E+00	7.94E-01	4.87E-01
Wing	(-∞, 0.71)	9.85E+00	-7.57E-01	1.63E+00
Joint	[0.71, 0.72)	6.54E+00	6.94E-01	6.04E+00

Joint	[0.72, 0.73)	6.34E+00	6.70E-01	5.96E+00
Joint	[0.73, 0.74)	6.61E+00	6.59E-01	5.88E+00
Joint	[0.74, 0.74)	6.07E+00	6.98E-01	6.40E+00
Joint	[0.74, 0.75)	6.28E+00	6.88E-01	6.34E+00
Joint	[0.75, 0.76)	6.54E+00	6.88E-01	6.36E+00
Joint	[0.76, 0.77)	6.42E+00	6.53E-01	6.19E+00
Joint	[0.77, 0.77)	6.46E+00	6.62E-01	6.35E+00
Joint	[0.77, 0.78)	6.71E+00	6.88E-01	6.63E+00
Joint	[0.78, 0.79)	6.45E+00	6.53E-01	6.50E+00
Joint	[0.79, 0.80)	6.43E+00	6.57E-01	6.67E+00
Joint	[0.80, 0.81)	6.52E+00	7.05E-01	7.28E+00
Joint	[0.81, 0.81)	5.11E+00	-3.80E-03	4.86E+00
Joint	[0.81, 0.82)	5.14E+00	-1.05E-02	4.94E+00
Joint	[0.82, 0.83)	5.18E+00	-1.63E-02	5.04E+00
Joint	[0.83, 0.84)	5.10E+00	-1.08E-02	5.21E+00
Joint	[0.84, 0.85)	5.27E+00	-2.79E-02	5.27E+00
Joint	[0.85, 0.85)	5.33E+00	-3.33E-02	5.41E+00
Joint	[0.85, 0.86)	5.40E+00	-4.05E-02	5.56E+00
Joint	[0.86, 0.87)	5.48E+00	-4.66E-02	5.73E+00
Joint	[0.87, 0.88)	5.57E+00	-5.38E-02	5.92E+00
Joint	[0.88, 0.89)	4.94E+00	-5.15E-02	6.42E+00
Joint	[0.89, 0.89)	5.80E+00	-7.33E-02	6.35E+00
Joint	[0.89, 0.90)	5.96E+00	-7.28E-02	6.60E+00
Joint	[0.90, 0.91)	6.13E+00	-9.18E-02	6.87E+00
Joint	[0.91, 0.92)	6.35E+00	-1.10E-01	7.17E+00
Joint	[0.92, 0.92)	6.60E+00	-1.26E-01	7.49E+00
Joint	[0.92, 0.93)	6.92E+00	-1.42E-01	7.85E+00

Joint	[0.93, 0.94)	7.33E+00	-1.59E-01	8.24E+00
Joint	[0.94, 0.95)	8.37E+00	-1.94E-01	8.41E+00
Joint	[0.95, 0.96)	8.79E+00	-2.03E-01	8.95E+00
Joint	[0.96, 0.96)	9.33E+00	-2.05E-01	9.53E+00
Joint	[0.96, 0.97)	1.00E+01	-2.11E-01	1.00E+01
Joint	[0.97, 0.98)	1.00E+01	-1.97E-01	1.00E+01
Joint	[0.98, 0.99)	1.00E+01	-1.60E-01	1.00E+01
Joint	[0.99, 1.00)	1.00E+01	-6.69E-02	1.00E+01
Joint	[1.00, 1.00]	1.74E+00	-5.01E-01	1.00E+01
Joint	(1.00, 1.01]	2.04E+00	4.30E-01	4.90E+00
Joint	(1.01, 1.02]	2.53E+00	-1.16E-01	4.76E+00
Joint	(1.02, 1.03]	2.87E+00	3.18E-03	3.78E+00
Joint	(1.03, 1.04]	3.15E+00	-2.41E-02	3.47E+00
Joint	(1.04, 1.04]	3.43E+00	-1.37E-01	3.44E+00
Joint	(1.04, 1.05]	3.67E+00	-6.04E-02	3.01E+00
Joint	(1.05, 1.06]	3.74E+00	1.77E-02	2.78E+00
Joint	(1.06, 1.07]	3.19E+00	2.98E-01	2.79E+00
Joint	(1.07, 1.08]	4.03E+00	4.71E-01	1.95E+00
Joint	(1.08, 1.09]	3.44E+00	1.42E-01	2.75E+00
Wing	(1.09,+∞)	7.18E+00	3.40E-03	2.27E-01
Wing	(-∞, 0.47)	8.93E+00	5.32E-01	2.75E+00
Joint	[0.47, 0.49)	6.36E+00	6.80E-01	3.73E+00
Joint	[0.49, 0.54)	7.66E+00	6.24E-01	3.42E+00
Joint	[0.54, 0.55)	6.55E+00	6.97E-01	3.94E+00
Joint	[0.55, 0.57)	6.37E+00	6.23E-01	3.69E+00
Joint	[0.57, 0.58)	6.19E+00	6.66E-01	3.95E+00
Joint	[0.58, 0.60)	6.40E+00	6.52E-01	3.91E+00

Joint	[0.60, 0.62)	6.45E+00	6.56E-01	3.95E+00
Joint	[0.62, 0.63)	6.44E+00	6.60E-01	4.03E+00
Joint	[0.63, 0.65)	6.97E+00	6.46E-01	3.94E+00
Joint	[0.65, 0.66)	6.64E+00	6.28E-01	3.99E+00
Joint	[0.66, 0.68)	6.62E+00	6.70E-01	4.28E+00
Joint	[0.68, 0.70)	6.75E+00	6.40E-01	4.18E+00
Joint	[0.70, 0.71)	7.04E+00	7.00E-01	4.57E+00
Joint	[0.71, 0.73)	6.83E+00	6.79E-01	4.58E+00
Joint	[0.73, 0.74)	6.74E+00	6.66E-01	4.65E+00
Joint	[0.74, 0.76)	6.61E+00	7.08E-01	5.14E+00
Joint	[0.76, 0.77)	6.61E+00	6.76E-01	5.08E+00
Joint	[0.77, 0.79)	6.97E+00	6.70E-01	5.15E+00
Joint	[0.79, 0.81)	7.13E+00	6.74E-01	5.38E+00
Joint	[0.81, 0.82)	7.13E+00	6.77E-01	5.68E+00
Joint	[0.82, 0.84)	7.42E+00	6.79E-01	5.97E+00
Joint	[0.84, 0.85)	6.68E+00	6.61E-01	6.26E+00
Joint	[0.85, 0.85)	5.03E+00	1.05E-02	4.41E+00
Joint	[0.85, 0.86)	5.08E+00	1.08E-02	4.54E+00
Joint	[0.86, 0.87)	5.13E+00	1.50E-03	4.67E+00
Joint	[0.87, 0.88)	5.19E+00	-7.50E-03	4.81E+00
Joint	[0.88, 0.89)	5.25E+00	-1.86E-02	4.97E+00
Joint	[0.89, 0.89)	5.34E+00	-2.53E-02	5.15E+00
Joint	[0.89, 0.90)	5.43E+00	-3.39E-02	5.34E+00
Joint	[0.90, 0.91)	5.55E+00	-2.84E-02	5.57E+00
Joint	[0.91, 0.92)	5.66E+00	-5.26E-02	5.78E+00
Joint	[0.92, 0.92)	5.82E+00	-6.29E-02	6.04E+00
Joint	[0.92, 0.93)	6.00E+00	-7.31E-02	6.32E+00

Joint	[0.93, 0.94)	6.23E+00	-8.49E-02	6.62E+00
Joint	[0.94, 0.95)	6.51E+00	-9.74E-02	6.95E+00
Joint	[0.95, 0.96)	6.85E+00	-1.06E-01	7.31E+00
Joint	[0.96, 0.96)	7.26E+00	-1.19E-01	7.69E+00
Joint	[0.96, 0.97)	7.77E+00	-1.22E-01	8.11E+00
Joint	[0.97, 0.98)	8.34E+00	-1.14E-01	8.58E+00
Joint	[0.98, 0.99)	9.41E+00	4.90E-02	9.01E+00
Joint	[0.99, 1.00)	9.60E+00	9.87E-03	9.50E+00
Joint	[1.00, 1.00]	1.46E+00	-3.98E-01	1.00E+01
Joint	(1.00, 1.01]	2.35E+00	7.00E-02	4.32E+00
Joint	(1.01, 1.02]	2.81E+00	4.71E-02	3.38E+00
Joint	(1.02, 1.03]	3.30E+00	-2.35E-01	3.36E+00
Joint	(1.03, 1.04]	3.49E+00	-2.39E-02	2.49E+00
Joint	(1.04, 1.04]	3.71E+00	-9.48E-02	2.39E+00
Joint	(1.04, 1.05]	3.87E+00	-2.92E-02	2.08E+00
Joint	(1.05, 1.06]	3.92E+00	-1.97E-02	2.02E+00
Joint	(1.06, 1.07]	4.10E+00	-2.14E-01	2.29E+00
Joint	(1.07, 1.09]	4.41E+00	1.75E-02	1.65E+00
Joint	(1.09, 1.14]	3.97E+00	1.01E-01	1.79E+00
Joint	(1.14, 1.20]	3.88E+00	4.46E-02	1.88E+00
Wing	(1.20,+∞)	8.64E+00	3.68E-02	4.05E-01
Wing	(-∞, 0.48)	9.21E+00	4.55E-01	1.69E+00
Joint	[0.48, 0.50)	6.09E+00	6.52E-01	2.43E+00
Joint	[0.50, 0.52)	6.33E+00	6.41E-01	2.41E+00
Joint	[0.52, 0.53)	6.21E+00	6.54E-01	2.51E+00
Joint	[0.53, 0.55)	6.74E+00	-2.86E-01	1.46E+00
Joint	[0.55, 0.57)	4.24E+00	-4.09E-01	1.80E+00

Joint	[0.57, 0.58)	5.22E+00	-4.74E-01	1.65E+00
Joint	[0.58, 0.60)	5.75E+00	-4.11E-01	1.61E+00
Joint	[0.60, 0.61)	7.23E+00	-2.65E-01	1.48E+00
Joint	[0.61, 0.63)	7.19E+00	-3.70E-01	1.47E+00
Joint	[0.63, 0.65)	6.29E+00	-4.08E-01	1.62E+00
Joint	[0.65, 0.66)	7.41E+00	-2.86E-01	1.53E+00
Joint	[0.66, 0.68)	7.15E+00	-3.65E-01	1.56E+00
Joint	[0.68, 0.69)	7.28E+00	-4.00E-01	1.56E+00
Joint	[0.69, 0.71)	7.30E+00	-3.89E-01	1.60E+00
Joint	[0.71, 0.73)	7.86E+00	-2.81E-01	1.60E+00
Joint	[0.73, 0.74)	6.45E+00	6.38E-01	3.28E+00
Joint	[0.74, 0.76)	6.62E+00	6.94E-01	3.68E+00
Joint	[0.76, 0.78)	7.18E+00	7.08E-01	3.84E+00
Joint	[0.78, 0.79)	6.83E+00	6.93E-01	3.98E+00
Joint	[0.79, 0.81)	8.52E+00	7.56E-01	4.48E+00
Joint	[0.81, 0.82)	6.93E+00	7.23E-01	4.70E+00
Joint	[0.82, 0.84)	7.27E+00	7.22E-01	4.92E+00
Joint	[0.84, 0.86)	7.24E+00	7.41E-01	5.55E+00
Joint	[0.86, 0.87)	8.03E+00	7.24E-01	5.55E+00
Joint	[0.87, 0.89)	5.19E+00	1.62E-02	3.55E+00
Joint	[0.89, 0.90)	5.35E+00	-4.95E-03	3.76E+00
Joint	[0.90, 0.92)	5.59E+00	-1.71E-02	4.01E+00
Joint	[0.92, 0.94)	5.91E+00	-2.56E-02	4.30E+00
Joint	[0.94, 0.95)	6.40E+00	-1.59E-02	4.62E+00
Joint	[0.95, 0.97)	7.02E+00	-5.75E-03	4.97E+00
Joint	[0.97, 0.98)	7.74E+00	1.27E-01	5.53E+00
Joint	[0.98, 1.00]	9.77E-01	-1.76E-01	1.00E+01

Joint	(1.00, 1.02]	3.20E+00	-2.11E-02	3.11E+00
Joint	(1.02, 1.03]	4.01E+00	-1.19E-01	2.21E+00
Joint	(1.03, 1.05]	4.59E+00	-2.80E-01	1.98E+00
Joint	(1.05, 1.07]	4.84E+00	-1.88E-01	1.51E+00
Joint	(1.07, 1.08]	5.11E+00	-1.85E-01	1.28E+00
Joint	(1.08, 1.10]	5.15E+00	-1.34E-02	9.61E-01
Joint	(1.10, 1.11]	5.22E+00	9.93E-02	7.89E-01
Joint	(1.11, 1.13]	5.28E+00	-9.44E-03	8.69E-01
Joint	(1.13, 1.15]	5.21E+00	1.80E-02	8.72E-01
Joint	(1.15, 1.16]	5.59E+00	-1.38E-02	7.47E-01
Joint	(1.16, 1.18]	4.92E+00	1.06E-01	8.81E-01
Joint	(1.18, 1.19]	4.62E+00	1.01E-01	9.76E-01
Wing	(1.19,+∞)	9.56E+00	6.20E-01	2.21E-01
Wing	(-∞, 0.42)	7.22E+00	6.63E-01	1.84E+00
Joint	[0.42, 0.45)	6.86E+00	6.25E-01	1.83E+00
Joint	[0.45, 0.49)	6.92E+00	6.24E-01	1.88E+00
Joint	[0.49, 0.50)	6.82E+00	-2.81E-01	1.16E+00
Joint	[0.50, 0.52)	5.88E+00	-3.78E-01	1.26E+00
Joint	[0.52, 0.53)	7.48E+00	-4.79E-01	1.06E+00
Joint	[0.53, 0.55)	6.94E+00	-3.88E-01	1.16E+00
Joint	[0.55, 0.57)	4.41E+00	-5.63E-01	1.50E+00
Joint	[0.57, 0.58)	6.62E+00	-4.52E-01	1.22E+00
Joint	[0.58, 0.60)	7.34E+00	-2.46E-01	1.22E+00
Joint	[0.60, 0.61)	7.09E+00	-3.00E-01	1.25E+00
Joint	[0.61, 0.63)	7.18E+00	-2.90E-01	1.27E+00
Joint	[0.63, 0.65)	4.38E+00	-4.72E-01	1.66E+00
Joint	[0.65, 0.66)	7.37E+00	-2.65E-01	1.30E+00

Joint	[0.66, 0.68)	4.87E+00	-5.10E-01	1.66E+00
Joint	[0.68, 0.70)	7.30E+00	-3.68E-01	1.34E+00
Joint	[0.70, 0.71)	7.74E+00	-2.78E-01	1.34E+00
Joint	[0.71, 0.73)	7.85E+00	-3.39E-01	1.34E+00
Joint	[0.73, 0.74)	7.99E+00	-2.86E-01	1.39E+00
Joint	[0.74, 0.76)	6.81E+00	-5.45E-01	1.57E+00
Joint	[0.76, 0.78)	8.26E+00	-2.97E-01	1.45E+00
Joint	[0.78, 0.79)	7.43E+00	7.06E-01	3.50E+00
Joint	[0.79, 0.81)	6.97E+00	7.24E-01	3.93E+00
Joint	[0.81, 0.82)	7.13E+00	7.50E-01	4.43E+00
Joint	[0.82, 0.84)	6.93E+00	7.66E-01	5.03E+00
Joint	[0.84, 0.86)	7.97E+00	7.09E-01	4.36E+00
Joint	[0.86, 0.87)	7.79E+00	7.57E-01	5.47E+00
Joint	[0.87, 0.89)	8.35E+00	7.91E-01	6.84E+00
Joint	[0.89, 0.91)	5.31E+00	5.21E-02	3.26E+00
Joint	[0.91, 0.92)	4.80E+00	3.19E-02	3.63E+00
Joint	[0.92, 0.94)	5.83E+00	7.38E-04	3.62E+00
Joint	[0.94, 0.95)	6.28E+00	6.41E-03	3.83E+00
Joint	[0.95, 0.97)	6.88E+00	7.31E-02	4.11E+00
Joint	[0.97, 0.99)	7.39E+00	2.37E-01	4.63E+00
Joint	[0.99, 1.00]	1.82E+00	-6.45E-01	9.70E+00
Joint	(1.00, 1.02]	3.91E+00	-8.12E-02	2.64E+00
Joint	(1.02, 1.03]	4.74E+00	-1.80E-01	1.97E+00
Joint	(1.03, 1.05]	5.26E+00	-1.27E-01	1.40E+00
Joint	(1.05, 1.07]	5.55E+00	-1.78E-01	1.23E+00
Joint	(1.07, 1.08]	5.70E+00	-8.31E-02	9.47E-01
Joint	(1.08, 1.10]	5.90E+00	-8.50E-02	7.97E-01

Joint	(1.10, 1.12]	5.98E+00	-6.76E-02	7.14E-01
Joint	(1.12, 1.13]	6.05E+00	-1.23E-02	6.01E-01
Joint	(1.13, 1.15]	6.15E+00	-9.93E-02	6.46E-01
Joint	(1.15, 1.16]	5.64E+00	1.36E-02	7.54E-01
Joint	(1.16, 1.18]	6.96E+00	-2.64E-01	3.59E-01
Joint	(1.18, 1.20]	5.81E+00	1.16E-01	5.56E-01
Joint	(1.20, 1.21]	6.32E+00	2.09E-03	4.54E-01
Joint	(1.21, 1.23]	5.78E+00	9.08E-02	5.63E-01
Joint	(1.23, 1.24]	6.41E+00	2.28E-02	4.20E-01
Joint	(1.24, 1.26]	6.23E+00	4.38E-02	4.58E-01
Wing	(1.26,+∞)	9.39E+00	5.46E-01	3.06E-01
Wing	(-∞, 0.45)	7.97E+00	5.89E-01	1.53E+00
Joint	[0.45, 0.47)	6.17E+00	6.09E-01	1.72E+00
Joint	[0.47, 0.49)	6.42E+00	6.38E-01	1.78E+00
Joint	[0.49, 0.50)	5.98E+00	-3.45E-01	1.10E+00
Joint	[0.50, 0.52)	6.72E+00	-3.10E-01	1.05E+00
Joint	[0.52, 0.54)	7.12E+00	-4.00E-01	9.94E-01
Joint	[0.54, 0.55)	6.73E+00	-4.32E-01	1.04E+00
Joint	[0.55, 0.57)	6.71E+00	-7.77E-02	1.18E+00
Joint	[0.57, 0.58)	6.85E+00	-3.36E-01	1.09E+00
Joint	[0.58, 0.60)	7.07E+00	-2.70E-01	1.10E+00
Joint	[0.60, 0.62)	5.94E+00	-4.23E-01	1.22E+00
Joint	[0.62, 0.63)	7.29E+00	-2.61E-01	1.13E+00
Joint	[0.63, 0.65)	7.12E+00	-3.36E-01	1.15E+00
Joint	[0.65, 0.67)	6.06E+00	-5.20E-01	1.29E+00
Joint	[0.67, 0.68)	5.60E+00	-4.98E-01	1.40E+00
Joint	[0.68, 0.70)	7.40E+00	-3.34E-01	1.21E+00

Joint	[0.70, 0.71)	7.73E+00	-2.57E-01	1.24E+00
Joint	[0.71, 0.73)	5.14E+00	-6.32E-01	1.66E+00
Joint	[0.73, 0.75)	7.89E+00	-2.81E-01	1.29E+00
Joint	[0.75, 0.76)	6.15E+00	-5.33E-01	1.59E+00
Joint	[0.76, 0.78)	6.95E+00	-5.84E-01	1.52E+00
Joint	[0.78, 0.80)	6.96E+00	-6.73E-01	1.63E+00
Joint	[0.80, 0.81)	7.43E+00	-4.67E-01	1.58E+00
Joint	[0.81, 0.83)	7.25E+00	7.34E-01	3.93E+00
Joint	[0.83, 0.84)	6.68E+00	7.92E-01	5.38E+00
Joint	[0.84, 0.86)	7.65E+00	7.60E-01	4.80E+00
Joint	[0.86, 0.88)	8.10E+00	7.56E-01	4.99E+00
Joint	[0.88, 0.89)	8.37E+00	7.65E-01	5.55E+00
Joint	[0.89, 0.91)	5.30E+00	3.91E-02	2.94E+00
Joint	[0.91, 0.92)	5.60E+00	1.08E-01	3.16E+00
Joint	[0.92, 0.94)	5.90E+00	6.93E-02	3.28E+00
Joint	[0.94, 0.96)	6.32E+00	5.37E-02	3.43E+00
Joint	[0.96, 0.97)	6.84E+00	1.19E-01	3.65E+00
Joint	[0.97, 0.99)	7.07E+00	3.37E-01	4.34E+00
Joint	[0.99, 1.01]	2.23E+00	-7.07E-01	9.70E+00
Joint	(1.01, 1.02]	4.54E+00	-1.14E-01	2.24E+00
Joint	(1.02, 1.04]	5.44E+00	-1.26E-01	1.55E+00
Joint	(1.04, 1.05]	5.85E+00	-1.30E-01	1.23E+00
Joint	(1.05, 1.07]	6.12E+00	-1.03E-01	9.83E-01
Joint	(1.07, 1.09]	6.34E+00	-1.09E-01	8.28E-01
Joint	(1.09, 1.10]	6.53E+00	-1.70E-01	7.77E-01
Joint	(1.10, 1.12]	6.26E+00	6.57E-02	6.43E-01
Joint	(1.12, 1.14]	6.65E+00	-9.13E-02	5.77E-01

Joint	(1.14, 1.15]	6.69E+00	-9.04E-02	5.52E-01
Joint	(1.15, 1.17]	6.89E+00	-1.70E-01	5.36E-01
Joint	(1.17, 1.20]	6.69E+00	6.76E-02	3.90E-01
Joint	(1.20, 1.22]	6.75E+00	-3.38E-03	4.21E-01
Joint	(1.22, 1.23]	6.63E+00	5.63E-02	4.18E-01
Joint	(1.23, 1.27]	6.57E+00	5.43E-02	4.38E-01
Joint	(1.27, 1.30]	6.67E+00	6.84E-02	4.03E-01
Joint	(1.30, 1.33]	6.89E+00	2.51E-02	3.75E-01
Joint	(1.33, 1.36]	3.92E+00	3.71E-01	7.45E-01
Joint	(1.36, 1.49]	6.29E+00	1.68E-01	4.23E-01
Wing	(1.49,+∞)	5.63E+00	-6.17E-01	1.07E+00
Wing	(-∞, 0.30)	5.75E+00	-3.55E-01	7.54E-01
Joint	[0.30, 0.33)	8.18E+00	-3.87E-01	5.86E-01
Joint	[0.33, 0.35)	6.15E+00	-4.66E-01	7.37E-01
Joint	[0.35, 0.37)	6.82E+00	-2.17E-01	7.51E-01
Joint	[0.37, 0.38)	5.66E+00	6.62E-01	1.42E+00
Joint	[0.38, 0.40)	5.78E+00	6.79E-01	1.47E+00
Joint	[0.40, 0.42)	6.05E+00	6.56E-01	1.44E+00
Joint	[0.42, 0.43)	5.91E+00	6.61E-01	1.49E+00
Joint	[0.43, 0.45)	5.95E+00	6.40E-01	1.47E+00
Joint	[0.45, 0.47)	6.24E+00	5.79E-01	1.38E+00
Joint	[0.47, 0.48)	6.20E+00	6.10E-01	1.46E+00
Joint	[0.48, 0.50)	6.44E+00	6.03E-01	1.46E+00
Joint	[0.50, 0.52)	6.20E+00	6.16E-01	1.53E+00
Joint	[0.52, 0.53)	6.82E+00	-2.97E-01	8.64E-01
Joint	[0.53, 0.55)	6.63E+00	-3.25E-01	8.92E-01
Joint	[0.55, 0.56)	6.17E+00	-3.77E-01	9.48E-01

Joint	[0.56, 0.58)	7.15E+00	-2.51E-01	9.00E-01
Joint	[0.58, 0.60)	6.71E+00	-4.49E-01	9.20E-01
Joint	[0.60, 0.61)	4.26E+00	-4.36E-01	1.25E+00
Joint	[0.61, 0.63)	7.34E+00	-2.69E-01	9.43E-01
Joint	[0.63, 0.65)	7.39E+00	-3.32E-01	9.43E-01
Joint	[0.65, 0.66)	7.99E+00	-3.95E-03	1.03E+00
Joint	[0.66, 0.68)	7.02E+00	-5.02E-01	1.02E+00
Joint	[0.68, 0.70)	7.18E+00	-5.73E-01	1.03E+00
Joint	[0.70, 0.71)	6.31E+00	-4.73E-01	1.20E+00
Joint	[0.71, 0.73)	7.05E+00	-3.64E-01	1.15E+00
Joint	[0.73, 0.75)	6.35E+00	-6.83E-01	1.34E+00
Joint	[0.75, 0.76)	5.86E+00	-6.22E-01	1.47E+00
Joint	[0.76, 0.78)	8.42E+00	-2.65E-01	1.12E+00
Joint	[0.78, 0.80)	8.52E+00	-3.05E-01	1.15E+00
Joint	[0.80, 0.81)	8.58E+00	-2.64E-01	1.21E+00
Joint	[0.81, 0.83)	8.32E+00	-3.02E-01	1.30E+00
Joint	[0.83, 0.85)	8.09E+00	7.29E-01	3.50E+00
Joint	[0.85, 0.86)	8.23E+00	7.49E-01	3.97E+00
Joint	[0.86, 0.88)	8.42E+00	7.64E-01	4.50E+00
Joint	[0.88, 0.90)	8.64E+00	7.75E-01	5.09E+00
Joint	[0.90, 0.91)	8.97E+00	7.80E-01	5.66E+00
Joint	[0.91, 0.93)	5.64E+00	1.25E-01	2.69E+00
Joint	[0.93, 0.95)	5.94E+00	7.18E-02	2.76E+00
Joint	[0.95, 0.96)	6.37E+00	1.40E-01	2.92E+00
Joint	[0.96, 0.98)	6.67E+00	2.72E-01	3.24E+00
Joint	[0.98, 1.00)	5.70E+00	3.17E-01	3.91E+00
Joint	[1.00, 1.01]	1.04E+01	-7.93E-01	9.77E+00

Joint	(1.01, 1.03]	5.33E+00	-1.51E-01	1.66E+00
Joint	(1.03, 1.05]	5.80E+00	-2.60E-01	1.52E+00
Joint	(1.05, 1.06]	6.50E+00	-1.04E-01	8.83E-01
Joint	(1.06, 1.08]	6.64E+00	-2.22E-01	9.44E-01
Joint	(1.08, 1.10]	6.80E+00	-1.69E-01	7.61E-01
Joint	(1.10, 1.11]	6.95E+00	-9.85E-02	5.74E-01
Joint	(1.11, 1.13]	7.00E+00	-9.48E-02	5.35E-01
Joint	(1.13, 1.15]	7.14E+00	-1.99E-01	5.84E-01
Joint	(1.15, 1.16]	6.62E+00	-9.93E-04	5.98E-01
Joint	(1.16, 1.18]	6.68E+00	-2.40E-02	5.78E-01
Joint	(1.18, 1.20]	7.20E+00	-1.14E-01	4.33E-01
Joint	(1.20, 1.21]	7.31E+00	-1.71E-01	4.47E-01
Joint	(1.21, 1.23]	3.82E+00	5.31E-01	9.32E-01
Joint	(1.23, 1.25]	5.12E+00	3.18E-01	6.77E-01
Joint	(1.25, 1.26]	7.32E+00	-6.09E-02	3.22E-01
Joint	(1.26, 1.28]	7.21E+00	-7.83E-02	3.76E-01
Joint	(1.28, 1.30]	7.12E+00	-6.76E-02	3.89E-01
Joint	(1.30, 1.31]	7.05E+00	-2.72E-02	3.73E-01
Joint	(1.31, 1.33]	6.98E+00	-1.11E-02	3.71E-01
Joint	(1.33, 1.36]	7.43E+00	-3.38E-02	2.62E-01
Joint	(1.36, 1.40]	7.40E+00	-4.40E-02	2.75E-01
Joint	(1.40, 1.46]	7.15E+00	7.48E-02	2.68E-01
Joint	(1.46, 1.50]	6.49E+00	8.09E-02	3.61E-01
Wing	(1.50,+∞)	6.59E+00	1.64E-01	3.13E-01
Wing	(-∞, 0.27)	5.28E+00	-7.30E-02	7.35E-01
Joint	[0.27, 0.30)	8.29E+00	-3.34E-01	4.97E-01
Joint	[0.30, 0.33)	8.26E+00	-3.44E-01	5.16E-01

Joint	[0.33, 0.37)	8.30E+00	-3.26E-01	5.35E-01
Joint	[0.37, 0.40)	8.11E+00	-3.84E-01	5.52E-01
Joint	[0.40, 0.43)	5.39E+00	-5.33E-01	7.73E-01
Joint	[0.43, 0.47)	7.49E+00	5.60E-01	1.13E+00
Joint	[0.47, 0.50)	7.09E+00	6.37E-01	1.32E+00
Joint	[0.50, 0.53)	7.70E+00	5.88E-01	1.25E+00
Joint	[0.53, 0.57)	6.98E+00	6.81E-01	1.55E+00
Joint	[0.57, 0.60)	8.60E+00	-3.00E-01	6.75E-01
Joint	[0.60, 0.63)	8.72E+00	-3.10E-01	6.96E-01
Joint	[0.63, 0.67)	7.46E+00	-7.03E-01	8.32E-01
Joint	[0.67, 0.68)	7.70E+00	-3.07E-01	8.89E-01
Joint	[0.68, 0.70)	7.16E+00	-5.92E-01	9.57E-01
Joint	[0.70, 0.72)	6.21E+00	-4.87E-01	1.12E+00
Joint	[0.72, 0.73)	8.06E+00	-3.04E-01	9.42E-01
Joint	[0.73, 0.75)	6.35E+00	-5.30E-01	1.20E+00
Joint	[0.75, 0.77)	6.30E+00	-5.25E-01	1.26E+00
Joint	[0.77, 0.78)	8.30E+00	-2.43E-01	1.04E+00
Joint	[0.78, 0.80)	8.55E+00	-2.68E-01	1.05E+00
Joint	[0.80, 0.82)	8.06E+00	-3.45E-01	1.16E+00
Joint	[0.82, 0.83)	8.68E+00	-4.47E-01	1.11E+00
Joint	[0.83, 0.85)	8.81E+00	-3.94E-01	1.16E+00
Joint	[0.85, 0.87)	7.34E+00	7.88E-01	4.56E+00
Joint	[0.87, 0.88)	8.00E+00	7.90E-01	4.84E+00
Joint	[0.88, 0.90)	8.70E+00	7.75E-01	4.63E+00
Joint	[0.90, 0.92)	9.14E+00	7.96E-01	5.79E+00
Joint	[0.92, 0.93)	9.46E+00	7.77E-01	5.41E+00
Joint	[0.93, 0.95)	5.95E+00	1.43E-01	2.50E+00

Joint	[0.95, 0.97)	6.27E+00	1.77E-01	2.62E+00
Joint	[0.97, 0.98)	6.34E+00	2.54E-01	2.85E+00
Joint	[0.98, 1.00)	4.92E+00	2.79E-01	3.61E+00
Joint	[1.00, 1.02]	9.06E+00	-6.94E-01	3.84E+00
Joint	(1.02, 1.03]	5.36E+00	-2.12E-01	1.69E+00
Joint	(1.03, 1.05]	6.20E+00	-2.32E-01	1.30E+00
Joint	(1.05, 1.07]	6.71E+00	-1.95E-01	1.00E+00
Joint	(1.07, 1.08]	7.02E+00	-2.78E-01	9.52E-01
Joint	(1.08, 1.10]	7.26E+00	-1.47E-01	6.39E-01
Joint	(1.10, 1.12]	7.46E+00	-2.92E-01	7.29E-01
Joint	(1.12, 1.13]	7.49E+00	-1.74E-01	5.28E-01
Joint	(1.13, 1.15]	7.58E+00	-1.79E-01	4.76E-01
Joint	(1.15, 1.17]	7.69E+00	-2.12E-01	4.50E-01
Joint	(1.17, 1.18]	7.53E+00	-1.29E-01	4.30E-01
Joint	(1.18, 1.20]	7.59E+00	1.05E-01	2.02E-01
Joint	(1.20, 1.23]	7.79E+00	-2.07E-01	3.67E-01
Joint	(1.23, 1.25]	3.40E+00	4.72E-01	1.00E+00
Joint	(1.25, 1.26]	3.11E+00	4.61E-01	1.02E+00
Joint	(1.26, 1.30]	7.72E+00	-1.79E-01	3.50E-01
Joint	(1.30, 1.33]	7.88E+00	-1.79E-01	2.76E-01
Joint	(1.33, 1.36]	7.90E+00	-1.83E-01	2.74E-01
Joint	(1.36, 1.40]	7.47E+00	-9.78E-02	3.25E-01
Joint	(1.40, 1.43]	2.38E+00	6.93E-01	8.73E-01
Joint	(1.43, 1.46]	7.42E+00	-1.07E-01	3.23E-01
Joint	(1.46, 1.50]	2.99E+00	3.42E-01	7.23E-01
Joint	(1.50, 1.53]	6.07E+00	1.77E-01	3.69E-01
Joint	(1.53, 1.56]	2.16E+00	2.79E-01	8.07E-01

Joint	(1.56, 1.60]	3.38E+00	2.10E-02	1.51E+00
Joint	(1.60, 1.63]	3.55E+00	5.38E-01	2.12E+00
Joint	(1.63, 1.66]	3.43E+00	4.33E-01	2.06E+00
Wing	(1.66,+∞)	9.92E+00	7.52E-01	1.43E+00
Wing	(-∞, 0.27)	4.64E+00	-5.75E-02	6.75E-01
Joint	[0.27, 0.31)	8.38E+00	-3.14E-01	4.10E-01
Joint	[0.31, 0.34)	8.38E+00	-2.94E-01	4.32E-01
Joint	[0.34, 0.37)	7.93E+00	-2.82E-01	4.85E-01
Joint	[0.37, 0.41)	8.36E+00	-3.15E-01	4.64E-01
Joint	[0.41, 0.44)	8.47E+00	-3.17E-01	4.73E-01
Joint	[0.44, 0.48)	8.27E+00	-3.47E-01	5.01E-01
Joint	[0.48, 0.51)	8.60E+00	-3.08E-01	5.04E-01
Joint	[0.51, 0.54)	8.66E+00	-3.38E-01	5.16E-01
Joint	[0.54, 0.58)	8.27E+00	5.78E-01	1.11E+00
Joint	[0.58, 0.61)	8.17E+00	6.40E-01	1.29E+00
Joint	[0.61, 0.65)	7.86E+00	6.95E-01	1.54E+00
Joint	[0.65, 0.68)	8.93E+00	-2.37E-01	6.53E-01
Joint	[0.68, 0.71)	8.88E+00	-2.74E-01	6.96E-01
Joint	[0.71, 0.73)	7.68E+00	-2.65E-01	8.76E-01
Joint	[0.73, 0.75)	5.06E+00	-7.00E-01	1.39E+00
Joint	[0.75, 0.76)	5.40E+00	-6.88E-01	1.38E+00
Joint	[0.76, 0.78)	8.17E+00	-2.30E-01	9.27E-01
Joint	[0.78, 0.80)	7.36E+00	-4.31E-01	1.06E+00
Joint	[0.80, 0.82)	7.32E+00	-4.88E-01	1.13E+00
Joint	[0.82, 0.83)	8.61E+00	-7.03E-01	1.12E+00
Joint	[0.83, 0.85)	8.76E+00	-6.48E-01	1.11E+00
Joint	[0.85, 0.87)	8.90E+00	-3.78E-01	1.03E+00

(1.92,+∞]

Joint	[0.87, 0.88)	9.48E+00	8.05E-01	4.42E+00
Joint	[0.88, 0.90)	8.80E+00	7.91E-01	4.46E+00
Joint	[0.90, 0.92)	8.61E+00	8.01E-01	5.44E+00
Joint	[0.92, 0.93)	9.25E+00	7.91E-01	5.20E+00
Joint	[0.93, 0.95)	7.94E+00	7.45E-01	4.53E+00
Joint	[0.95, 0.99)	6.50E+00	4.64E-01	2.59E+00
Joint	[0.99, 1.02]	1.12E+01	-7.93E-01	6.46E+00
Joint	(1.02, 1.05]	6.17E+00	-2.09E-01	1.22E+00
Joint	(1.05, 1.09]	7.14E+00	-2.56E-01	9.04E-01
Joint	(1.09, 1.12]	7.61E+00	-2.72E-01	7.33E-01
Joint	(1.12, 1.16]	7.80E+00	-1.83E-01	5.44E-01
Joint	(1.16, 1.19]	7.78E+00	-9.68E-02	4.56E-01
Joint	(1.19, 1.22]	7.94E+00	-1.39E-01	4.19E-01
Joint	(1.22, 1.26]	8.31E+00	-2.44E-01	3.73E-01
Joint	(1.26, 1.27]	7.18E+00	7.08E-02	4.39E-01
Joint	(1.27, 1.29]	6.66E+00	2.81E-01	4.33E-01
Joint	(1.29, 1.33]	8.22E+00	-2.28E-02	2.00E-01
Joint	(1.33, 1.36]	7.87E+00	2.97E-01	1.66E-01
Joint	(1.36, 1.39]	4.02E+00	4.91E-01	6.82E-01
Joint	(1.39, 1.43]	8.01E+00	7.30E-02	2.01E-01
Joint	(1.43, 1.46]	4.51E+00	4.38E-01	5.59E-01
Joint	(1.46, 1.50]	7.26E+00	1.62E-03	3.43E-01
Joint	(1.50, 1.53]	7.80E+00	8.47E-02	2.22E-01
Joint	(1.53, 1.56]	7.21E+00	-5.85E-02	3.55E-01
Joint	(1.56, 1.63]	8.00E+00	7.12E-02	1.91E-01
Joint	(1.63, 1.67]	2.77E+00	2.77E-01	6.61E-01
Joint	(1.67, 1.70]	3.97E+00	2.15E-01	5.31E-01

Wing	(1.70,+∞)	5.63E+00	5.12E-01	3.14E-01

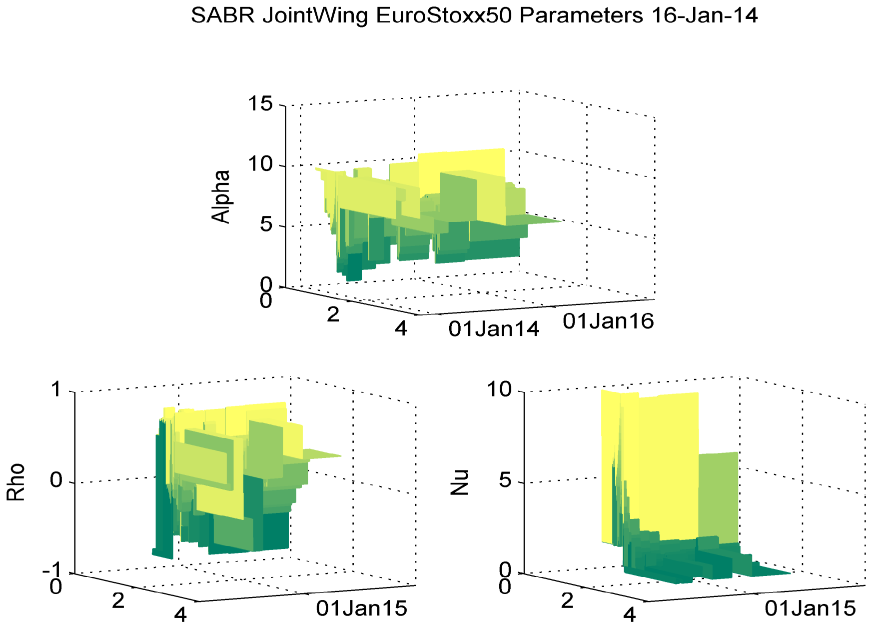


Figure B.38: SABR JointWing EuroStoxx50 Parameters

Table B.33: Heston JointWing EuroStoxx50 Parameters (κ = 2.00E+00, v0 = 1.00E-02, 16-Jan-14)

Mosaic component	Forward moneyness	Time-to-maturity	θ	σ	ρ
Wing	(-∞, 0.69)		1.25E-05	1.00E+01	-4.44E-01
Joint	[0.69, 0.70)		1.05E-01	3.52E-01	-2.49E-01
Joint	[0.70, 0.70)		1.01E+00	8.08E-01	-1.84E-01
Joint	[0.70, 0.71)		1.01E+00	8.08E-01	-1.84E-01
Joint	[0.71, 0.72)		8.97E-01	6.99E-01	-2.00E-01
Joint	[0.72, 0.73)		4.44E+00	2.15E+00	4.62E-01
Joint	[0.73, 0.73)		9.28E-01	7.34E-01	-1.50E-01
Joint	[0.73, 0.74)		9.28E-01	7.34E-01	-1.50E-01
Joint	[0.74, 0.75)		4.00E-01	5.10E-01	-9.92E-01
Joint	[0.75, 0.76)		4.00E-01	5.10E-01	-9.92E-01
Joint	[0.76, 0.77)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.77, 0.77)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.77, 0.78)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.78, 0.79)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.79, 0.80)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.80, 0.81)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.81, 0.81)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.81, 0.82)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.82, 0.83)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.83, 0.84)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.84, 0.85)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.85, 0.85)		1.00E-01	3.50E-01	-2.50E-01
Joint	[0.85, 0.86)		1.00E-01	3.50E-01	-2.50E-01

(0.00, 0.01]

Joint	[0.86, 0.87)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.87, 0.88)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.88, 0.88)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.88, 0.89)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.89, 0.90)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.90, 0.91)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.91, 0.92)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.92, 0.92)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.92, 0.93)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.93, 0.94)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.94, 0.95)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.95, 0.96)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.96, 0.96)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.96, 0.97)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.97, 0.98)	9.99E+00	9.95E+00	-9.98E-01
Joint	[0.98, 0.99)	1.00E+01	7.49E+00	-9.98E-01
Joint	[0.99, 1.00)	1.00E+01	2.90E+00	-9.99E-01
Joint	[1.00, 1.00]	1.00E+01	2.90E+00	-9.99E-01
Joint	(1.00, 1.01]	3.75E-02	9.65E+00	8.16E-01
Joint	(1.01, 1.02]	1.40E-01	9.29E+00	6.56E-01
Joint	(1.02, 1.03]	2.09E-01	9.29E+00	6.13E-01
Joint	(1.03, 1.03]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.03, 1.04]	1.00E-01	3.50E-01	-2.50E-01
Joint	(1.04, 1.05]	1.00E-01	3.50E-01	-2.50E-01
Wing	(1.05,$+\infty$)	1.00E-01	3.50E-01	-2.50E-01
Wing	($-\infty$, 0.71)	2.07E-03	7.63E-01	-5.98E-01
Joint	[0.71, 0.72)	1.00E-01	3.50E-01	-2.50E-01

Joint	[0.72, 0.73)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.73, 0.74)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.74, 0.74)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.74, 0.75)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.75, 0.76)	7.61E-01	1.69E+00	-9.16E-01
Joint	[0.76, 0.77)	6.11E-01	2.22E+00	-7.82E-01
Joint	[0.77, 0.77)	7.29E-01	1.72E+00	-9.13E-01
Joint	[0.77, 0.78)	6.05E-01	2.12E+00	-8.58E-01
Joint	[0.78, 0.79)	7.14E-01	1.97E+00	-7.68E-01
Joint	[0.79, 0.80)	7.80E-01	1.82E+00	-7.48E-01
Joint	[0.80, 0.81)	9.73E-01	2.24E+00	-3.30E-01
Joint	[0.81, 0.81)	1.12E+00	1.80E+00	-3.08E-01
Joint	[0.81, 0.82)	1.05E+00	1.82E+00	-3.85E-01
Joint	[0.82, 0.83)	1.20E+00	1.27E+00	-3.87E-01
Joint	[0.83, 0.84)	1.20E+00	1.18E+00	-4.38E-01
Joint	[0.84, 0.85)	1.19E+00	1.21E+00	-4.45E-01
Joint	[0.85, 0.85)	1.41E+00	8.46E-01	-1.39E-01
Joint	[0.85, 0.86)	1.36E+00	5.30E-01	-4.52E-01
Joint	[0.86, 0.87)	1.57E+00	1.14E+00	2.17E-01
Joint	[0.87, 0.88)	1.62E+00	1.08E+00	3.18E-01
Joint	[0.88, 0.89)	1.73E+00	1.00E+00	5.71E-01
Joint	[0.89, 0.89)	1.77E+00	1.01E+00	6.49E-01
Joint	[0.89, 0.90)	1.85E+00	9.84E-01	8.54E-01
Joint	[0.90, 0.91)	1.67E+00	2.10E+00	8.75E-02
Joint	[0.91, 0.92)	1.64E+00	1.80E+00	-3.56E-02
Joint	[0.92, 0.92)	1.76E+00	2.08E+00	4.10E-02
Joint	[0.92, 0.93)	2.45E+00	5.31E+00	2.87E-01

Joint	[0.93, 0.94)	2.47E+00	5.27E+00	1.93E-01
Joint	[0.94, 0.95)	2.62E+00	5.26E+00	2.20E-01
Joint	[0.95, 0.96)	2.25E+00	2.69E+00	8.50E-02
Joint	[0.96, 0.96)	2.90E+00	5.41E+00	1.48E-01
Joint	[0.96, 0.97)	3.02E+00	4.94E+00	2.58E-01
Joint	[0.97, 0.98)	2.81E+00	3.09E+00	1.84E-01
Joint	[0.98, 0.99)	3.11E+00	3.61E+00	2.86E-01
Joint	[0.99, 1.00)	3.87E+00	6.07E+00	5.83E-01
Joint	[1.00, 1.00]	1.23E+00	6.03E+00	-2.64E-01
Joint	(1.00, 1.01]	7.22E-02	7.34E+00	9.58E-02
Joint	(1.01, 1.02]	6.05E-02	4.08E+00	-3.82E-01
Joint	(1.02, 1.03]	1.70E-01	5.25E+00	-6.84E-01
Joint	(1.03, 1.04]	8.98E-02	2.86E+00	-6.19E-01
Joint	(1.04, 1.04]	1.26E-02	1.44E+00	-3.91E-01
Joint	(1.04, 1.05]	3.47E-02	1.51E+00	-4.83E-01
Joint	(1.05, 1.06]	3.89E-02	1.49E+00	-4.94E-01
Joint	(1.06, 1.07]	2.62E-02	1.65E+00	-4.74E-01
Joint	(1.07, 1.08]	1.19E-02	8.60E-01	-2.68E-01
Joint	(1.08, 1.09]	1.43E-02	1.60E+00	-4.63E-01
Wing	(1.09,+∞)	1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.47)	2.92E+00	6.77E-01	9.80E-01
Joint	[0.47, 0.49)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.49, 0.54)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.54, 0.55)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.55, 0.57)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.57, 0.58)	1.00E-01	3.50E-01	-2.50E-01
Joint	[0.58, 0.60)	1.00E-01	3.50E-01	-2.50E-01

Joint	[0.60, 0.62)	2.20E-01	1.98E+00	-7.89E-01
Joint	[0.62, 0.63)	1.87E-01	2.15E+00	-7.41E-01
Joint	[0.63, 0.65)	2.30E-01	1.91E+00	-7.71E-01
Joint	[0.65, 0.66)	3.83E-01	2.09E+00	-3.50E-01
Joint	[0.66, 0.68)	2.44E-01	2.59E+00	-4.08E-01
Joint	[0.68, 0.70)	2.20E-01	2.03E+00	-7.23E-01
Joint	[0.70, 0.71)	2.97E-01	2.35E+00	-3.80E-01
Joint	[0.71, 0.73)	3.46E-01	1.88E+00	-4.73E-01
Joint	[0.73, 0.74)	3.55E-01	1.69E+00	-5.46E-01
Joint	[0.74, 0.76)	4.03E-01	1.48E+00	-5.48E-01
Joint	[0.76, 0.77)	4.17E-01	1.47E+00	-5.20E-01
Joint	[0.77, 0.79)	4.12E-01	1.28E+00	-6.49E-01
Joint	[0.79, 0.81)	5.03E-01	1.11E+00	-4.76E-01
Joint	[0.81, 0.82)	5.10E-01	9.82E-01	-5.36E-01
Joint	[0.82, 0.84)	6.92E-01	1.72E+00	1.24E-01
Joint	[0.84, 0.85)	7.43E-01	1.82E+00	2.12E-01
Joint	[0.85, 0.85)	6.37E-01	1.15E+00	-6.37E-02
Joint	[0.85, 0.86)	6.79E-01	1.38E+00	6.78E-02
Joint	[0.86, 0.87)	8.32E-01	2.46E+00	2.97E-01
Joint	[0.87, 0.88)	6.73E-01	1.29E+00	-1.89E-02
Joint	[0.88, 0.89)	6.62E-01	1.18E+00	-8.07E-02
Joint	[0.89, 0.89)	6.73E-01	1.20E+00	-8.38E-02
Joint	[0.89, 0.90)	9.84E-01	3.63E+00	3.16E-01
Joint	[0.90, 0.91)	7.02E-01	1.22E+00	-9.69E-02
Joint	[0.91, 0.92)	7.22E-01	1.22E+00	-1.06E-01
Joint	[0.92, 0.92)	7.63E-01	1.41E+00	-3.99E-02
Joint	[0.92, 0.93)	1.27E+00	5.34E+00	3.13E-01

Joint	[0.93, 0.94)	1.28E+00	5.60E+00	1.80E-01
Joint	[0.94, 0.95)	9.18E-01	2.04E+00	2.44E-02
Joint	[0.95, 0.96)	9.16E-01	1.67E+00	-3.11E-02
Joint	[0.96, 0.96)	9.88E-01	1.89E+00	1.75E-02
Joint	[0.96, 0.97)	1.51E+00	5.71E+00	1.32E-01
Joint	[0.97, 0.98)	1.52E+00	5.23E+00	1.85E-01
Joint	[0.98, 0.99)	1.65E+00	5.77E+00	1.39E-01
Joint	[0.99, 1.00)	1.73E+00	5.76E+00	7.80E-02
Joint	[1.00, 1.00]	7.15E-01	6.16E+00	-2.42E-01
Joint	(1.00, 1.01]	7.70E-02	6.45E+00	1.15E-01
Joint	(1.01, 1.02]	1.05E-01	5.85E+00	-4.15E-01
Joint	(1.02, 1.03]	2.29E-01	7.88E+00	-7.11E-01
Joint	(1.03, 1.04]	3.54E-02	1.52E+00	-4.84E-01
Joint	(1.04, 1.04]	4.20E-02	1.47E+00	-5.41E-01
Joint	(1.04, 1.05]	1.07E-02	7.54E-01	-3.39E-01
Joint	(1.05, 1.06]	2.96E-02	1.04E+00	-4.95E-01
Joint	(1.06, 1.07]	2.64E-02	1.12E+00	-4.78E-01
Joint	(1.07, 1.09]	1.65E-02	6.78E-01	-3.43E-01
Joint	(1.09, 1.14]	2.35E-02	1.03E+00	-4.74E-01
Joint	(1.14, 1.20]	2.12E-02	1.10E+00	-4.91E-01
Wing	(1.20,+∞)	1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.48)	5.02E-02	2.09E+00	-4.38E-01
Joint	[0.48, 0.50)	8.71E-02	1.82E+00	-3.51E-01
Joint	[0.50, 0.52)	1.62E-01	1.23E+00	-4.49E-01
Joint	[0.52, 0.53)	1.14E-01	1.38E+00	-5.27E-01
Joint	[0.53, 0.55)	1.01E-01	1.53E+00	-4.69E-01
Joint	[0.55, 0.57)	4.79E-01	1.94E+00	5.43E-01

Joint	[0.57, 0.58)	4.94E-01	1.93E+00	5.61E-01
Joint	[0.58, 0.60)	4.74E-01	1.96E+00	5.54E-01
Joint	[0.60, 0.61)	4.48E-01	2.00E+00	5.43E-01
Joint	[0.61, 0.63)	4.46E-01	1.99E+00	5.33E-01
Joint	[0.63, 0.65)	3.89E-01	2.09E+00	4.83E-01
Joint	[0.65, 0.66)	3.54E-01	2.15E+00	4.44E-01
Joint	[0.66, 0.68)	3.93E-01	2.11E+00	5.04E-01
Joint	[0.68, 0.69)	3.73E-01	2.16E+00	4.86E-01
Joint	[0.69, 0.71)	3.70E-01	2.17E+00	4.84E-01
Joint	[0.71, 0.73)	3.71E-01	2.16E+00	4.81E-01
Joint	[0.73, 0.74)	3.80E-01	2.14E+00	4.95E-01
Joint	[0.74, 0.76)	3.77E-01	2.16E+00	4.91E-01
Joint	[0.76, 0.78)	3.48E-01	2.26E+00	4.33E-01
Joint	[0.78, 0.79)	2.40E-01	1.13E+00	-1.69E-02
Joint	[0.79, 0.81)	3.19E-01	2.53E+00	3.43E-01
Joint	[0.81, 0.82)	2.69E-01	1.61E+00	1.15E-01
Joint	[0.82, 0.84)	3.17E-01	2.79E+00	2.70E-01
Joint	[0.84, 0.86)	2.33E-01	6.83E-01	-1.85E-01
Joint	[0.86, 0.87)	2.44E-01	9.33E-01	-1.07E-01
Joint	[0.87, 0.89)	2.56E-01	1.13E+00	-9.01E-02
Joint	[0.89, 0.90)	2.49E-01	7.96E-01	-1.70E-01
Joint	[0.90, 0.92)	2.74E-01	1.18E+00	-7.65E-02
Joint	[0.92, 0.94)	3.89E-01	2.86E+00	9.48E-02
Joint	[0.94, 0.95)	4.13E-01	3.01E+00	1.31E-01
Joint	[0.95, 0.97)	9.53E-01	3.82E+00	9.69E-02
Joint	[0.97, 0.98)	3.94E-01	2.11E+00	6.43E-02
Joint	[0.98, 1.00]	4.02E-01	6.23E+00	-5.10E-01

Joint	(1.00, 1.02]	4.70E-02	2.96E+00	1.99E-01
Joint	(1.02, 1.03]	1.10E-02	5.69E-01	1.85E-01
Joint	(1.03, 1.05]	1.46E-02	5.79E-01	-5.15E-02
Joint	(1.05, 1.07]	1.48E-02	4.88E-01	-1.14E-01
Joint	(1.07, 1.08]	1.45E-02	3.97E-01	-1.26E-01
Joint	(1.08, 1.10]	2.05E-02	4.50E-01	-2.92E-01
Joint	(1.10, 1.11]	1.53E-02	2.97E-01	-1.34E-01
Joint	(1.11, 1.13]	2.17E-02	4.17E-01	-3.20E-01
Joint	(1.13, 1.15]	2.34E-02	4.56E-01	-3.57E-01
Joint	(1.15, 1.16]	3.17E-02	4.52E-01	-4.61E-01
Joint	(1.16, 1.18]	3.06E-02	6.05E-01	-4.88E-01
Joint	(1.18, 1.19]	2.75E-02	6.75E-01	-4.86E-01
Wing	(1.19,+∞)	4.20E-04	1.46E+00	-5.92E-01
Wing	(-∞, 0.42)	2.20E-02	2.98E+00	-3.38E-01
Joint	[0.42, 0.45)	3.67E-01	1.25E+00	4.20E-01
Joint	[0.45, 0.49)	3.42E-01	1.30E+00	4.35E-01
Joint	[0.49, 0.50)	3.02E-01	1.38E+00	3.79E-01
Joint	[0.50, 0.52)	3.10E-01	1.38E+00	4.44E-01
Joint	[0.52, 0.53)	3.00E-01	1.40E+00	4.43E-01
Joint	[0.53, 0.55)	2.86E-01	1.43E+00	4.37E-01
Joint	[0.55, 0.57)	2.76E-01	1.45E+00	4.37E-01
Joint	[0.57, 0.58)	2.67E-01	1.47E+00	4.18E-01
Joint	[0.58, 0.60)	2.41E-01	1.55E+00	3.82E-01
Joint	[0.60, 0.61)	2.36E-01	1.56E+00	3.73E-01
Joint	[0.61, 0.63)	2.35E-01	1.57E+00	3.82E-01
Joint	[0.63, 0.65)	2.31E-01	1.58E+00	3.70E-01
Joint	[0.65, 0.66)	2.27E-01	1.59E+00	3.60E-01

Joint	[0.66, 0.68)	2.21E-01	1.62E+00	3.47E-01
Joint	[0.68, 0.70)	2.21E-01	1.62E+00	3.49E-01
Joint	[0.70, 0.71)	2.24E-01	1.61E+00	3.60E-01
Joint	[0.71, 0.73)	2.19E-01	1.62E+00	3.35E-01
Joint	[0.73, 0.74)	2.00E-01	1.74E+00	2.54E-01
Joint	[0.74, 0.76)	1.67E-01	1.02E+00	-2.31E-02
Joint	[0.76, 0.78)	1.88E-01	1.97E+00	1.74E-01
Joint	[0.78, 0.79)	1.61E-01	6.57E-01	-1.49E-01
Joint	[0.79, 0.81)	2.00E-01	1.93E+00	1.74E-01
Joint	[0.81, 0.82)	1.97E-01	2.14E+00	1.04E-01
Joint	[0.82, 0.84)	2.08E-01	1.95E+00	1.55E-01
Joint	[0.84, 0.86)	1.74E-01	1.07E+00	-7.84E-02
Joint	[0.86, 0.87)	2.15E-01	2.22E+00	1.68E-02
Joint	[0.87, 0.89)	2.22E-01	2.21E+00	1.36E-01
Joint	[0.89, 0.91)	1.82E-01	1.04E+00	-9.33E-02
Joint	[0.91, 0.92)	1.88E-01	1.07E+00	-9.18E-02
Joint	[0.92, 0.94)	2.40E-01	2.05E+00	8.10E-02
Joint	[0.94, 0.95)	2.55E-01	2.45E+00	3.16E-02
Joint	[0.95, 0.97)	2.70E-01	2.43E+00	-2.09E-02
Joint	[0.97, 0.99)	2.79E-01	2.48E+00	-4.99E-02
Joint	[0.99, 1.00]	3.30E-01	6.87E+00	-5.66E-01
Joint	(1.00, 1.02]	4.36E-02	2.09E+00	1.45E-01
Joint	(1.02, 1.03]	6.96E-02	3.21E+00	-2.50E-01
Joint	(1.03, 1.05]	3.65E-02	1.22E+00	-2.40E-01
Joint	(1.05, 1.07]	2.18E-02	5.23E-01	-1.11E-01
Joint	(1.07, 1.08]	2.20E-02	4.64E-01	-1.59E-01
Joint	(1.08, 1.10]	2.21E-02	3.89E-01	-2.03E-01

Joint	(1.10, 1.12]	2.49E-02	4.25E-01	-2.96E-01
Joint	(1.12, 1.13]	2.71E-02	4.23E-01	-3.60E-01
Joint	(1.13, 1.15]	2.15E-02	2.91E-01	-2.07E-01
Joint	(1.15, 1.16]	1.69E-02	3.27E-01	-5.84E-02
Joint	(1.16, 1.18]	2.06E-02	7.48E-02	-7.87E-02
Joint	(1.18, 1.20]	2.11E-02	1.98E-01	-1.70E-01
Joint	(1.20, 1.21]	2.30E-02	2.27E-01	-2.53E-01
Joint	(1.21, 1.23]	3.37E-02	4.92E-01	-5.07E-01
Joint	(1.23, 1.24]	3.66E-02	3.75E-01	-5.33E-01
Joint	(1.24, 1.26]	3.22E-02	3.49E-01	-4.70E-01
Wing	(1.26,+∞)	4.40E-02	6.83E-04	5.61E-01
Wing	(-∞, 0.45)	6.40E-02	1.92E+00	-1.13E-01
Joint	[0.45, 0.47)	2.29E-01	1.14E+00	3.28E-01
Joint	[0.47, 0.49)	2.26E-01	1.15E+00	3.41E-01
Joint	[0.49, 0.50)	2.13E-01	1.18E+00	3.29E-01
Joint	[0.50, 0.52)	2.01E-01	1.22E+00	3.14E-01
Joint	[0.52, 0.54)	2.06E-01	1.20E+00	3.29E-01
Joint	[0.54, 0.55)	1.88E-01	1.26E+00	3.03E-01
Joint	[0.55, 0.57)	1.85E-01	1.27E+00	3.05E-01
Joint	[0.57, 0.58)	1.71E-01	1.32E+00	2.65E-01
Joint	[0.58, 0.60)	1.58E-01	1.35E+00	1.75E-01
Joint	[0.60, 0.62)	1.68E-01	1.34E+00	2.55E-01
Joint	[0.62, 0.63)	1.72E-01	1.30E+00	2.53E-01
Joint	[0.63, 0.65)	1.73E-01	1.32E+00	2.72E-01
Joint	[0.65, 0.67)	1.68E-01	1.34E+00	2.44E-01
Joint	[0.67, 0.68)	1.64E-01	1.32E+00	2.11E-01
Joint	[0.68, 0.70)	1.75E-01	1.32E+00	2.98E-01

Joint	[0.70, 0.71)	1.54E-01	1.50E+00	1.73E-01
Joint	[0.71, 0.73)	1.42E-01	1.02E+00	-1.89E-02
Joint	[0.73, 0.75)	1.64E-01	1.26E+00	2.02E-01
Joint	[0.75, 0.76)	1.53E-01	1.49E+00	9.69E-02
Joint	[0.76, 0.78)	1.67E-01	1.39E+00	2.22E-01
Joint	[0.78, 0.80)	1.52E-01	1.79E+00	3.52E-02
Joint	[0.80, 0.81)	1.41E-01	1.05E+00	-6.95E-02
Joint	[0.81, 0.83)	1.59E-01	1.77E+00	9.34E-03
Joint	[0.83, 0.84)	1.58E-01	1.55E+00	-7.96E-03
Joint	[0.84, 0.86)	1.66E-01	1.76E+00	-1.55E-03
Joint	[0.86, 0.88)	1.69E-01	1.82E+00	5.60E-02
Joint	[0.88, 0.89)	1.65E-01	1.49E+00	4.44E-04
Joint	[0.89, 0.91)	1.81E-01	2.03E+00	-8.96E-02
Joint	[0.91, 0.92)	1.88E-01	2.17E+00	-7.92E-02
Joint	[0.92, 0.94)	1.87E-01	1.89E+00	-3.25E-02
Joint	[0.94, 0.96)	2.09E-01	2.38E+00	-1.68E-01
Joint	[0.96, 0.97)	1.75E-01	1.33E+00	-4.97E-02
Joint	[0.97, 0.99)	9.31E-01	4.83E+00	5.72E-02
Joint	[0.99, 1.01]	2.84E-01	6.45E+00	-5.99E-01
Joint	(1.01, 1.02]	4.26E-02	1.75E+00	9.61E-02
Joint	(1.02, 1.04]	2.19E-02	4.69E-01	1.69E-01
Joint	(1.04, 1.05]	3.33E-02	9.57E-01	-1.43E-01
Joint	(1.05, 1.07]	2.82E-02	6.47E-01	-1.49E-01
Joint	(1.07, 1.09]	2.83E-02	5.74E-01	-2.16E-01
Joint	(1.09, 1.10]	3.00E-02	5.61E-01	-2.98E-01
Joint	(1.10, 1.12]	2.57E-02	3.90E-01	-2.24E-01
Joint	(1.12, 1.14]	2.08E-02	2.01E-01	-1.13E-02

Joint	(1.14, 1.15]	2.76E-02	3.90E-01	-3.03E-01
Joint	(1.15, 1.17]	2.85E-02	3.57E-01	-3.46E-01
Joint	(1.17, 1.20]	2.52E-02	2.73E-01	-2.51E-01
Joint	(1.20, 1.22]	2.80E-02	3.17E-01	-3.40E-01
Joint	(1.22, 1.23]	1.21E-02	6.32E-01	-4.98E-02
Joint	(1.23, 1.27]	2.64E-02	3.14E-01	-2.97E-01
Joint	(1.27, 1.30]	4.73E-02	5.89E-01	-6.12E-01
Joint	(1.30, 1.33]	3.10E-02	2.95E-01	-3.97E-01
Joint	(1.33, 1.36]	4.05E-03	9.50E-01	-3.07E-01
Joint	(1.36, 1.49]	4.35E-02	4.48E-01	-5.76E-01
Wing	(1.49,+∞)	3.24E+00	1.00E+01	9.15E-01
Wing	(-∞, 0.30)	2.08E-01	7.27E-01	1.56E-01
Joint	[0.30, 0.33)	1.98E-01	7.62E-01	1.72E-01
Joint	[0.33, 0.35)	1.86E-01	8.04E-01	1.98E-01
Joint	[0.35, 0.37)	1.80E-01	8.20E-01	1.99E-01
Joint	[0.37, 0.38)	1.82E-01	8.10E-01	1.95E-01
Joint	[0.38, 0.40)	1.69E-01	8.59E-01	2.05E-01
Joint	[0.40, 0.42)	1.48E-01	9.39E-01	1.20E-01
Joint	[0.42, 0.43)	1.46E-01	9.49E-01	1.56E-01
Joint	[0.43, 0.45)	1.51E-01	9.24E-01	2.14E-01
Joint	[0.45, 0.47)	1.47E-01	9.42E-01	2.01E-01
Joint	[0.47, 0.48)	1.02E-01	1.17E+00	-4.73E-02
Joint	[0.48, 0.50)	1.36E-01	9.95E-01	1.88E-01
Joint	[0.50, 0.52)	1.35E-01	1.00E+00	1.77E-01
Joint	[0.52, 0.53)	1.17E-01	1.11E+00	8.36E-02
Joint	[0.53, 0.55)	1.23E-01	1.06E+00	1.14E-01
Joint	[0.55, 0.56)	1.22E-01	1.08E+00	1.38E-01

Joint	[0.56, 0.58)	1.21E-01	1.08E+00	1.19E-01
Joint	[0.58, 0.60)	1.21E-01	1.10E+00	1.39E-01
Joint	[0.60, 0.61)	9.29E-02	7.49E-01	-3.77E-01
Joint	[0.61, 0.63)	9.30E-02	7.59E-01	-3.70E-01
Joint	[0.63, 0.65)	9.39E-02	7.35E-01	-3.67E-01
Joint	[0.65, 0.66)	9.77E-02	7.42E-01	-2.92E-01
Joint	[0.66, 0.68)	9.86E-02	7.30E-01	-2.79E-01
Joint	[0.68, 0.70)	9.93E-02	7.43E-01	-2.61E-01
Joint	[0.70, 0.71)	9.81E-02	6.39E-01	-3.20E-01
Joint	[0.71, 0.73)	9.82E-02	5.97E-01	-3.32E-01
Joint	[0.73, 0.75)	9.84E-02	5.83E-01	-3.31E-01
Joint	[0.75, 0.76)	9.91E-02	5.67E-01	-3.12E-01
Joint	[0.76, 0.78)	9.93E-02	5.26E-01	-3.09E-01
Joint	[0.78, 0.80)	1.05E-01	6.81E-01	-1.26E-01
Joint	[0.80, 0.81)	1.05E-01	6.46E-01	-1.20E-01
Joint	[0.81, 0.83)	1.02E-01	4.99E-01	-1.88E-01
Joint	[0.83, 0.85)	1.05E-01	6.17E-01	-8.54E-02
Joint	[0.85, 0.86)	1.09E-01	7.08E-01	4.91E-03
Joint	[0.86, 0.88)	1.01E-01	3.48E-01	-1.52E-01
Joint	[0.88, 0.90)	1.19E-01	1.02E+00	1.59E-01
Joint	[0.90, 0.91)	1.01E-01	3.22E-01	1.11E-02
Joint	[0.91, 0.93)	1.03E-01	3.55E-01	2.08E-01
Joint	[0.93, 0.95)	1.20E-01	1.03E+00	2.36E-01
Joint	[0.95, 0.96)	1.48E-01	1.81E+00	-2.46E-01
Joint	[0.96, 0.98)	1.36E-01	1.73E+00	-7.95E-02
Joint	[0.98, 1.00)	3.63E-01	3.59E+00	-1.13E-01
Joint	[1.00, 1.01]	1.38E-01	3.47E+00	-4.71E-01

Joint	(1.01, 1.03]	3.99E-02	1.45E+00	-2.04E-02
Joint	(1.03, 1.05]	4.88E-02	1.90E+00	-1.79E-01
Joint	(1.05, 1.06]	3.00E-02	7.28E-01	-1.08E-01
Joint	(1.06, 1.08]	4.15E-02	1.20E+00	-3.02E-01
Joint	(1.08, 1.10]	2.69E-02	4.88E-01	-1.46E-01
Joint	(1.10, 1.11]	2.59E-02	4.00E-01	-1.68E-01
Joint	(1.11, 1.13]	3.31E-02	6.59E-01	-3.33E-01
Joint	(1.13, 1.15]	2.75E-02	4.15E-01	-2.51E-01
Joint	(1.15, 1.16]	2.64E-02	3.44E-01	-2.41E-01
Joint	(1.16, 1.18]	2.19E-02	1.71E-01	1.34E-02
Joint	(1.18, 1.20]	2.98E-02	4.15E-01	-3.50E-01
Joint	(1.20, 1.21]	2.98E-02	4.06E-01	-3.55E-01
Joint	(1.21, 1.23]	2.97E-02	3.66E-01	-3.70E-01
Joint	(1.23, 1.25]	2.35E-02	1.64E-01	-1.46E-01
Joint	(1.25, 1.26]	3.24E-02	3.92E-01	-4.43E-01
Joint	(1.26, 1.28]	3.24E-02	3.89E-01	-4.43E-01
Joint	(1.28, 1.30]	3.44E-02	4.19E-01	-4.82E-01
Joint	(1.30, 1.31]	3.89E-02	4.69E-01	-5.55E-01
Joint	(1.31, 1.33]	6.40E-01	6.96E+00	9.30E-01
Joint	(1.33, 1.36]	5.02E-02	6.32E-01	-6.59E-01
Joint	(1.36, 1.40]	3.10E-01	8.98E+00	9.25E-01
Joint	(1.40, 1.46]	2.22E-02	1.14E-02	5.21E-01
Joint	(1.46, 1.50]	1.02E-03	1.70E+00	3.74E-01
Wing	(1.50,+∞)	4.11E-01	3.60E-01	-9.95E-01
Wing	(-∞, 0.27)	1.65E-01	6.69E-01	4.45E-02
Joint	[0.27, 0.30)	1.59E-01	6.91E-01	5.57E-02
Joint	[0.30, 0.33)	1.47E-01	7.44E-01	7.38E-02

Joint	[0.33, 0.37)	9.97E-02	9.98E-01	-1.21E-01
Joint	[0.37, 0.40)	1.33E-01	8.00E-01	9.66E-02
Joint	[0.40, 0.43)	1.16E-01	8.97E-01	6.04E-02
Joint	[0.43, 0.47)	1.11E-01	9.29E-01	5.54E-02
Joint	[0.47, 0.50)	1.08E-01	9.52E-01	5.38E-02
Joint	[0.50, 0.53)	1.03E-01	1.00E+00	9.76E-03
Joint	[0.53, 0.57)	8.16E-02	8.28E-01	-4.17E-01
Joint	[0.57, 0.60)	8.39E-02	7.87E-01	-3.95E-01
Joint	[0.60, 0.63)	8.80E-02	8.12E-01	-2.95E-01
Joint	[0.63, 0.67)	8.58E-02	6.88E-01	-3.97E-01
Joint	[0.67, 0.68)	8.72E-02	6.93E-01	-3.56E-01
Joint	[0.68, 0.70)	8.71E-02	6.63E-01	-3.65E-01
Joint	[0.70, 0.72)	8.68E-02	6.42E-01	-3.80E-01
Joint	[0.72, 0.73)	8.75E-02	5.97E-01	-3.63E-01
Joint	[0.73, 0.75)	8.72E-02	6.05E-01	-3.73E-01
Joint	[0.75, 0.77)	8.94E-02	5.98E-01	-2.74E-01
Joint	[0.77, 0.78)	9.00E-02	6.07E-01	-2.48E-01
Joint	[0.78, 0.80)	8.81E-02	4.97E-01	-3.23E-01
Joint	[0.80, 0.82)	8.80E-02	4.48E-01	-3.11E-01
Joint	[0.82, 0.83)	8.78E-02	4.22E-01	-3.14E-01
Joint	[0.83, 0.85)	9.26E-02	6.14E-01	-9.00E-02
Joint	[0.85, 0.87)	9.58E-02	7.40E-01	-1.55E-02
Joint	[0.87, 0.88)	1.01E-01	9.44E-01	9.00E-02
Joint	[0.88, 0.90)	1.01E-01	9.46E-01	1.05E-01
Joint	[0.90, 0.92)	1.00E-01	8.90E-01	1.23E-01
Joint	[0.92, 0.93)	1.01E-01	8.98E-01	1.57E-01
Joint	[0.93, 0.95)	9.64E-02	7.12E-01	1.87E-01

Joint	[0.95, 0.97)	9.97E-02	8.78E-01	2.44E-01
Joint	[0.97, 0.98)	9.35E-02	5.81E-01	3.15E-01
Joint	[0.98, 1.00)	9.97E-02	8.49E-01	2.21E-01
Joint	[1.00, 1.02]	1.22E-01	3.19E+00	-4.86E-01
Joint	(1.02, 1.03]	3.89E-02	1.31E+00	-3.92E-02
Joint	(1.03, 1.05]	4.43E-02	1.60E+00	-1.55E-01
Joint	(1.05, 1.07]	5.05E-02	1.95E+00	-2.58E-01
Joint	(1.07, 1.08]	2.79E-02	4.53E-01	-1.50E-01
Joint	(1.08, 1.10]	3.02E-02	5.80E-01	-1.92E-01
Joint	(1.10, 1.12]	2.86E-02	4.31E-01	-2.32E-01
Joint	(1.12, 1.13]	2.95E-02	4.75E-01	-2.51E-01
Joint	(1.13, 1.15]	2.97E-02	4.53E-01	-2.85E-01
Joint	(1.15, 1.17]	3.02E-02	4.38E-01	-3.28E-01
Joint	(1.17, 1.18]	1.50E-01	1.02E+00	6.49E-01
Joint	(1.18, 1.20]	6.42E-02	2.20E+00	6.78E-01
Joint	(1.20, 1.23]	3.21E-02	4.47E-01	-4.05E-01
Joint	(1.23, 1.25]	3.19E-02	3.90E-01	-4.36E-01
Joint	(1.25, 1.26]	3.34E-02	4.40E-01	-4.57E-01
Joint	(1.26, 1.30]	3.44E-02	4.56E-01	-4.79E-01
Joint	(1.30, 1.33]	3.60E-02	4.63E-01	-5.16E-01
Joint	(1.33, 1.36]	1.20E-01	3.72E+00	9.28E-01
Joint	(1.36, 1.40]	2.39E-02	1.36E-01	-1.98E-01
Joint	(1.40, 1.43]	4.87E-02	6.79E-01	-6.47E-01
Joint	(1.43, 1.46]	4.11E-01	6.76E+00	9.23E-01
Joint	(1.46, 1.50]	4.61E-01	8.08E+00	9.20E-01
Joint	(1.50, 1.53]	2.39E-02	2.87E+00	9.81E-01
Joint	(1.53, 1.56]	3.47E-03	1.71E+00	4.59E-02

Joint	(1.56, 1.60]	3.49E-02	3.76E+00	9.82E-02
Joint	(1.60, 1.63]	3.70E-01	2.30E+00	-3.48E-01
Joint	(1.63, 1.66]	4.37E-01	2.20E+00	-3.63E-01
Wing	(1.66,+∞)	1.00E-01	3.50E-01	-2.50E-01
Wing	(-∞, 0.27)	1.16E-01	7.06E-01	-9.70E-02
Joint	[0.27, 0.31)	1.08E-01	7.70E-01	-1.26E-01
Joint	[0.31, 0.34)	1.07E-01	7.72E-01	-8.14E-02
Joint	[0.34, 0.37)	8.58E-02	1.01E+00	-2.29E-01
Joint	[0.37, 0.41)	9.31E-02	8.91E-01	-9.83E-02
Joint	[0.41, 0.44)	7.20E-02	9.95E-01	-4.50E-01
Joint	[0.44, 0.48)	7.21E-02	1.01E+00	-4.40E-01
Joint	[0.48, 0.51)	7.56E-02	9.93E-01	-3.67E-01
Joint	[0.51, 0.54)	7.41E-02	7.75E-01	-5.12E-01
Joint	[0.54, 0.58)	7.92E-02	8.69E-01	-3.28E-01
Joint	[0.58, 0.61)	7.86E-02	7.18E-01	-3.95E-01
Joint	[0.61, 0.65)	8.17E-02	7.98E-01	-2.70E-01
Joint	[0.65, 0.68)	7.93E-02	6.46E-01	-3.86E-01
Joint	[0.68, 0.71)	7.93E-02	6.17E-01	-3.90E-01
Joint	[0.71, 0.73)	7.90E-02	6.21E-01	-4.11E-01
Joint	[0.73, 0.75)	8.62E-02	8.97E-01	-9.56E-02
Joint	[0.75, 0.76)	8.58E-02	9.59E-01	-1.59E-01
Joint	[0.76, 0.78)	8.63E-02	9.87E-01	-1.44E-01
Joint	[0.78, 0.80)	7.88E-02	5.42E-01	-3.85E-01
Joint	[0.80, 0.82)	8.68E-02	1.01E+00	-1.32E-01
Joint	[0.82, 0.83)	7.90E-02	5.65E-01	-3.62E-01
Joint	[0.83, 0.85)	7.85E-02	5.20E-01	-3.75E-01
Joint	[0.85, 0.87)	8.60E-02	9.52E-01	-1.26E-01

(1.92,+∞]

Joint	[0.87, 0.88)	7.86E-02	5.20E-01	-3.11E-01
Joint	[0.88, 0.90)	8.64E-02	9.78E-01	-9.79E-02
Joint	[0.90, 0.92)	8.59E-02	9.80E-01	-3.64E-02
Joint	[0.92, 0.93)	8.53E-02	9.26E-01	-4.27E-02
Joint	[0.93, 0.95)	8.40E-02	8.86E-01	2.39E-02
Joint	[0.95, 0.99)	8.32E-02	8.64E-01	6.40E-02
Joint	[0.99, 1.02]	9.22E-02	2.68E+00	-4.65E-01
Joint	(1.02, 1.05]	3.84E-02	1.18E+00	-1.07E-01
Joint	(1.05, 1.09]	4.48E-02	1.49E+00	-2.38E-01
Joint	(1.09, 1.12]	3.29E-02	6.70E-01	-2.12E-01
Joint	(1.12, 1.16]	4.18E-02	1.08E+00	-3.48E-01
Joint	(1.16, 1.19]	3.71E-02	7.76E-01	-3.49E-01
Joint	(1.19, 1.22]	9.93E-02	1.02E+00	7.03E-01
Joint	(1.22, 1.26]	6.26E-02	1.43E+00	7.70E-01
Joint	(1.26, 1.27]	3.26E-02	4.63E-01	-3.65E-01
Joint	(1.27, 1.29]	6.45E-02	1.41E+00	8.46E-01
Joint	(1.29, 1.33]	7.84E-02	1.42E+00	9.10E-01
Joint	(1.33, 1.36]	3.80E-02	5.73E-01	-4.91E-01
Joint	(1.36, 1.39]	9.24E-02	1.59E+00	9.93E-01
Joint	(1.39, 1.43]	4.18E-02	6.45E-01	-5.46E-01
Joint	(1.43, 1.46]	1.55E-01	2.71E+00	9.92E-01
Joint	(1.46, 1.50]	5.35E-02	8.99E-01	-6.41E-01
Joint	(1.50, 1.53]	2.37E-01	4.81E+00	-8.55E-01
Joint	(1.53, 1.56]	5.35E-02	8.40E-01	-6.53E-01
Joint	(1.56, 1.63]	2.77E-02	2.41E-01	-3.05E-01
Joint	(1.63, 1.67]	2.09E-02	3.14E+00	-2.70E-01
Joint	(1.67, 1.70]	3.35E-03	1.63E+00	9.52E-01

Wing	(1.70,+∞)	1.00E-02	9.81E-01	-2.59E-01

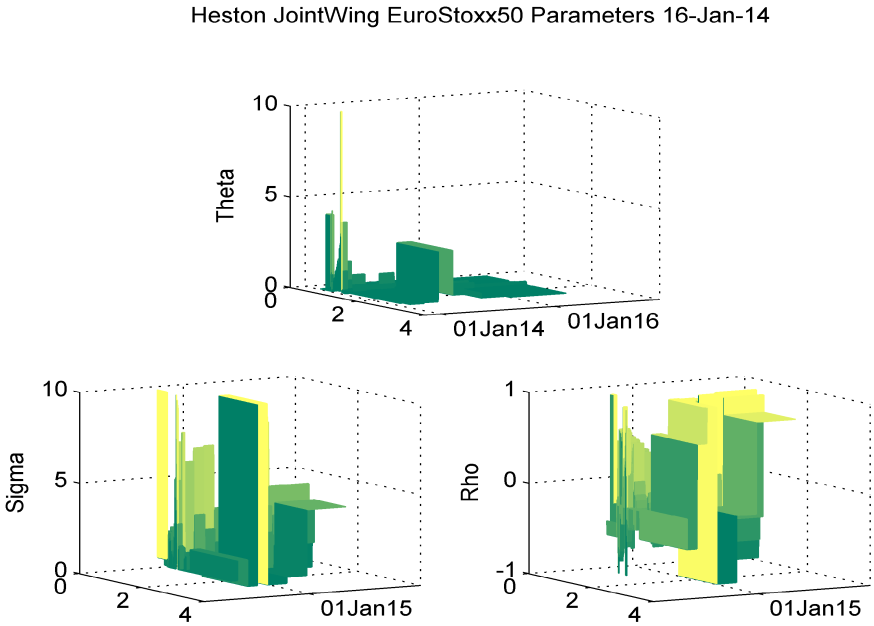


Figure B.39: Heston JointWing EuroStoxx50 Parameters

Table B.34: Elapsed Seconds EuroStoxx50 Calibration (03-Jan-14∼31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
6.480E+0	1.490E+1	1.079E+2	4.570E+2	1.137E+3	9.120E+3

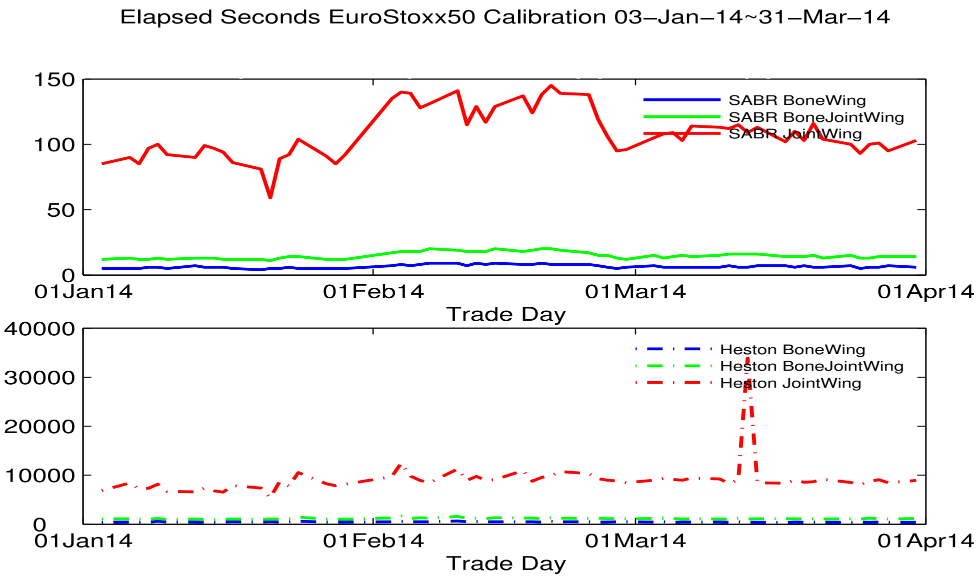
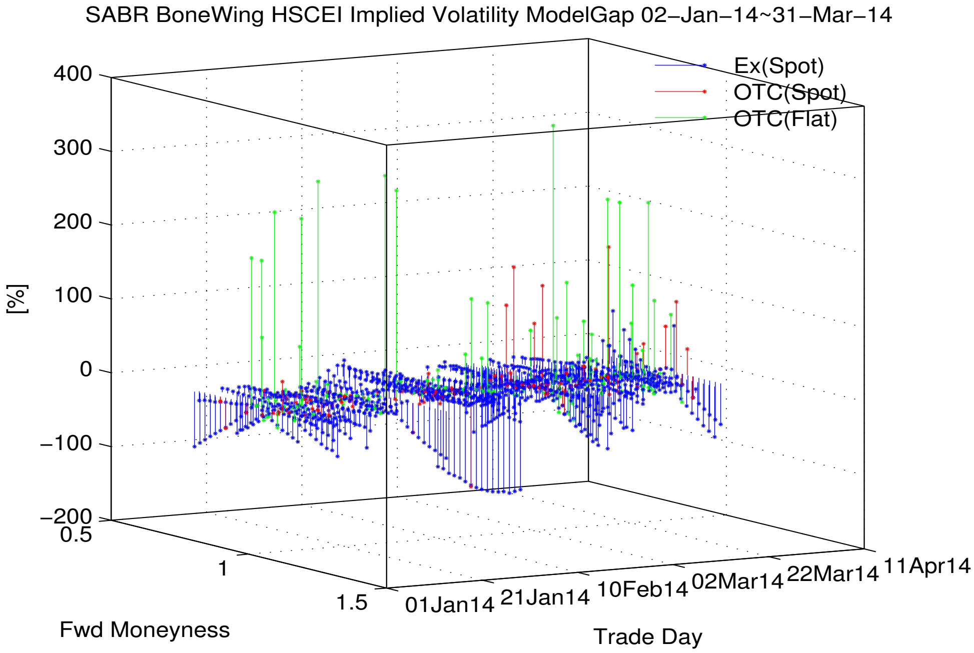


Figure B.40: Elapsed Seconds EuroStoxx50 Calibration (02-Jan-14∼31-Mar-14)



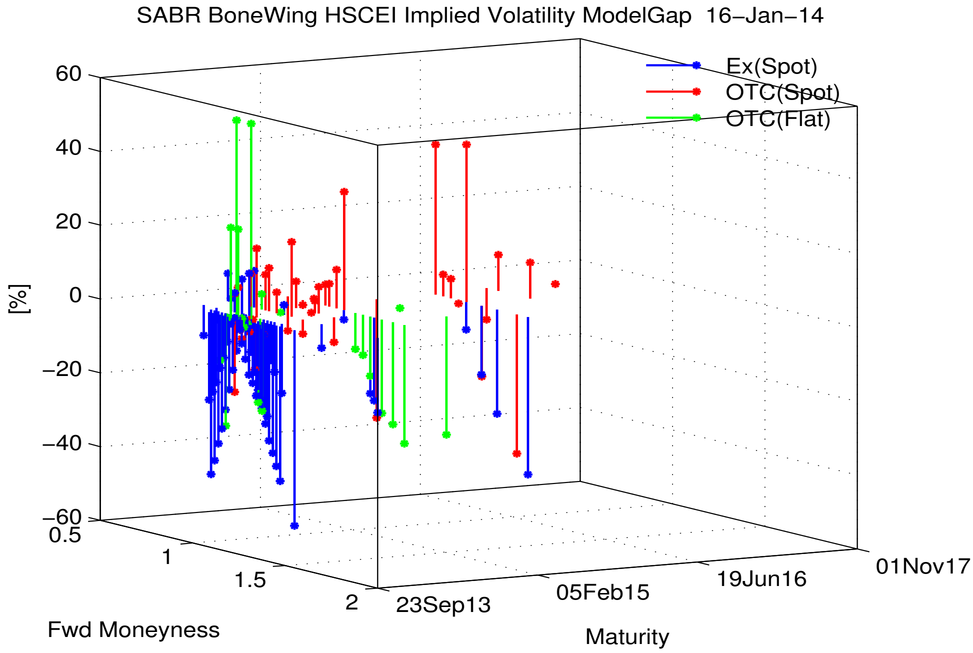


Figure B.41: SABR BoneWing HSCEI Implied Volatility (Nearest) ModelGap (03-Jan-14~31-Mar-14), (16-Jan-14)

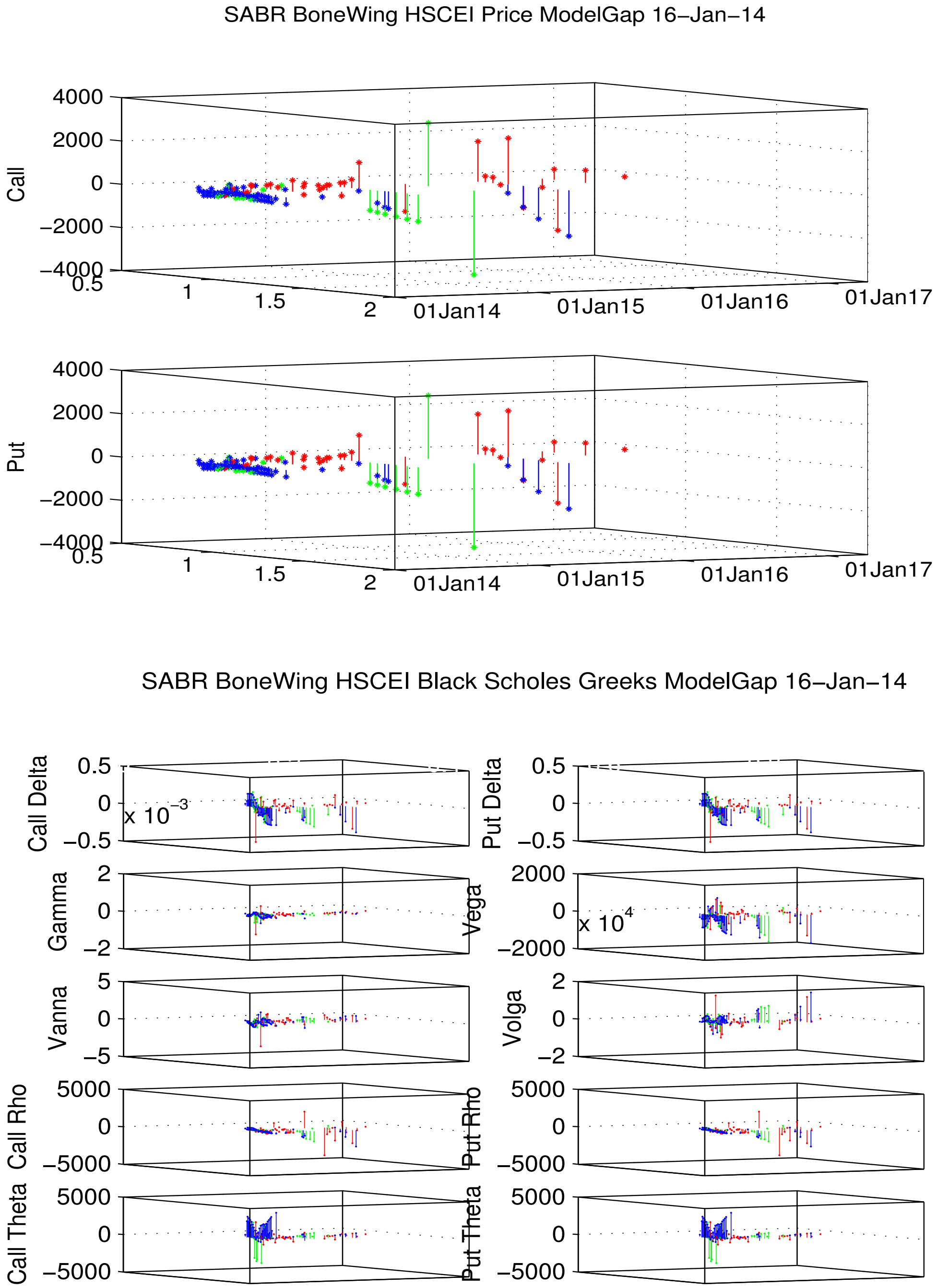
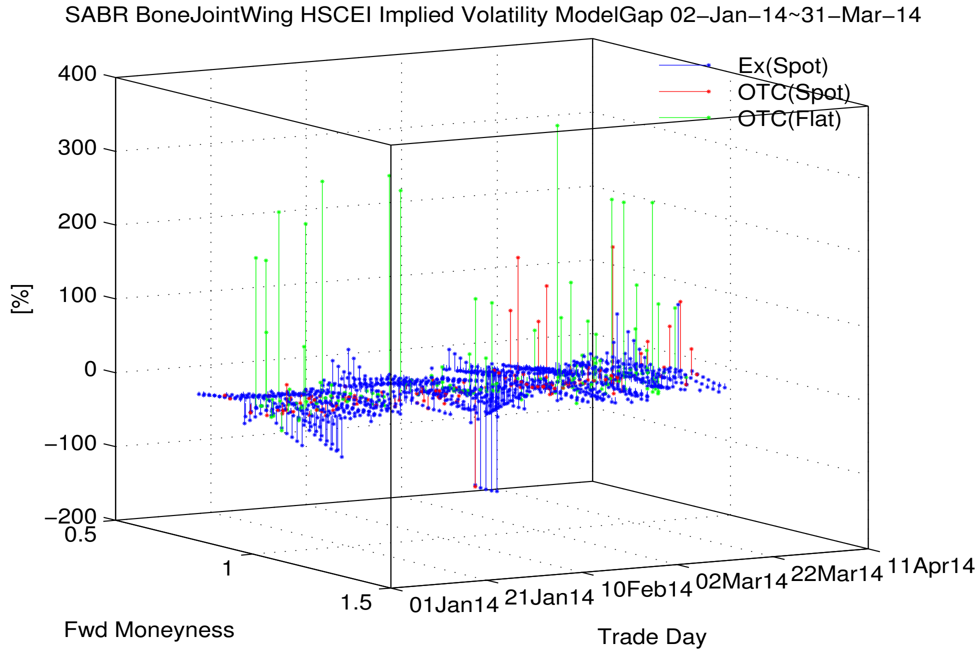


Figure B.42: SABR BoneWing HSCEI Price, Black Scholes Greeks ModelGap (16-Jan-14)



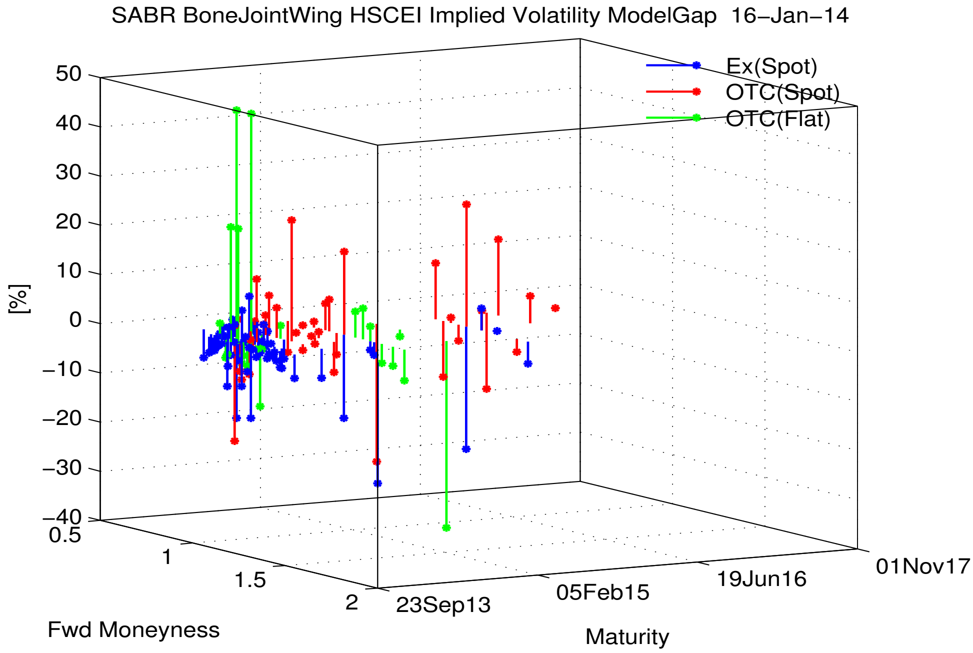
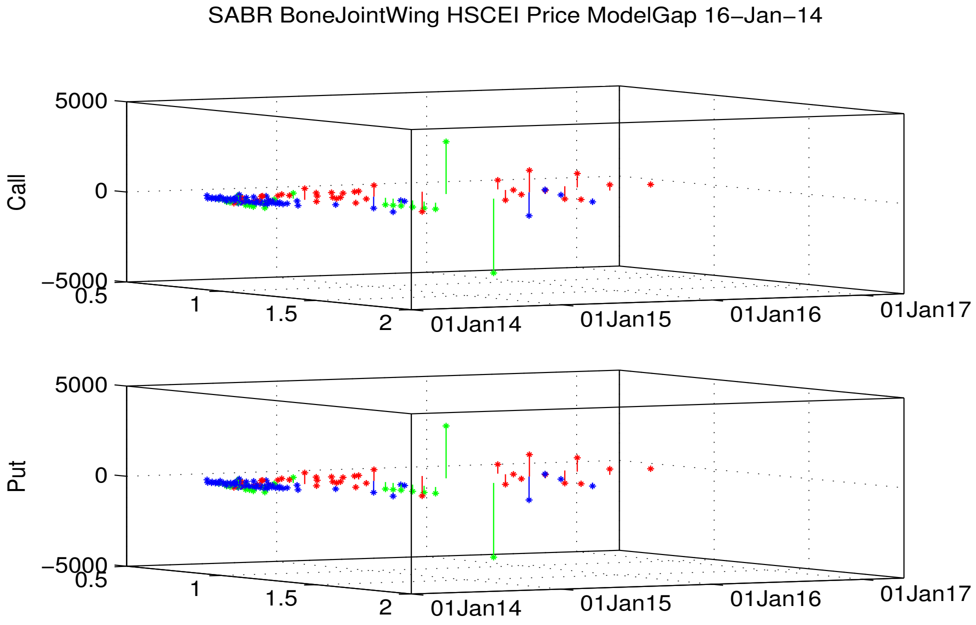


Figure B.43: SABR BoneWing HSCEI Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



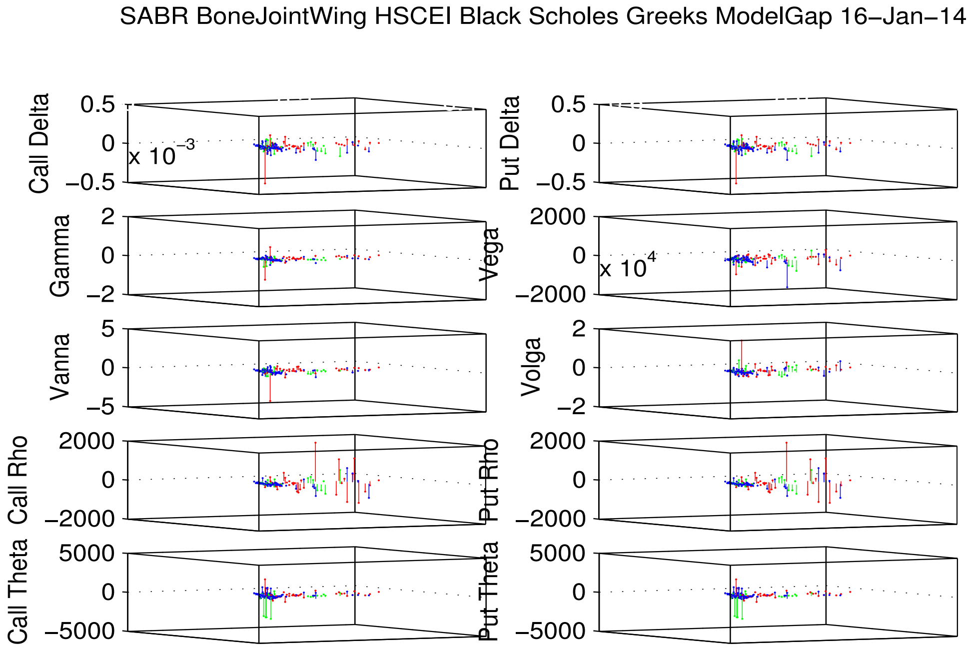
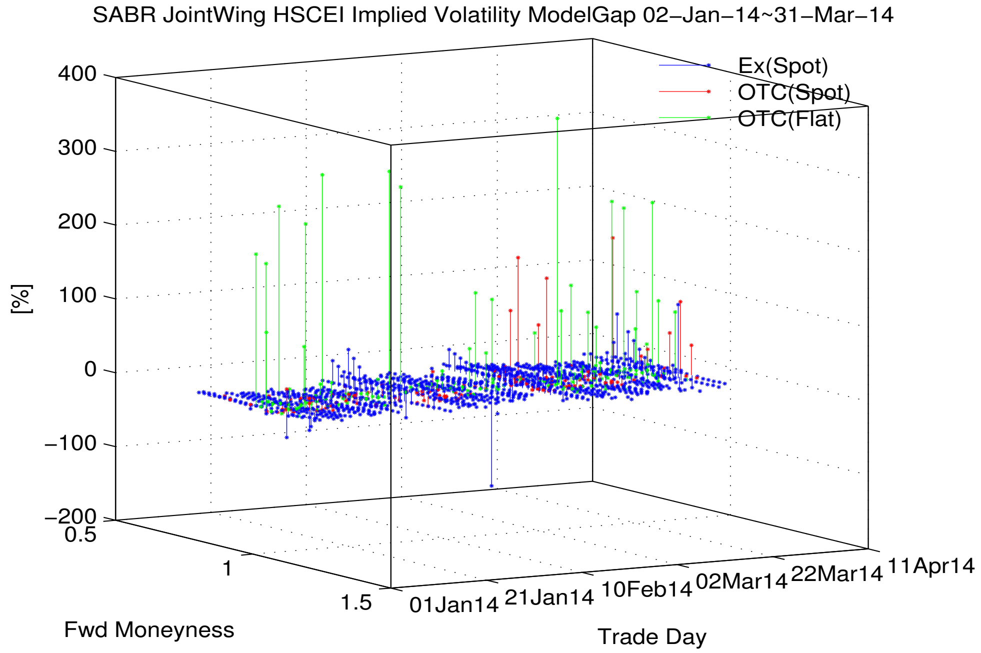


Figure B.44: SABR BoneWing HSCEI Price, Black Scholes Greeks ModelGap (16-Jan-14)



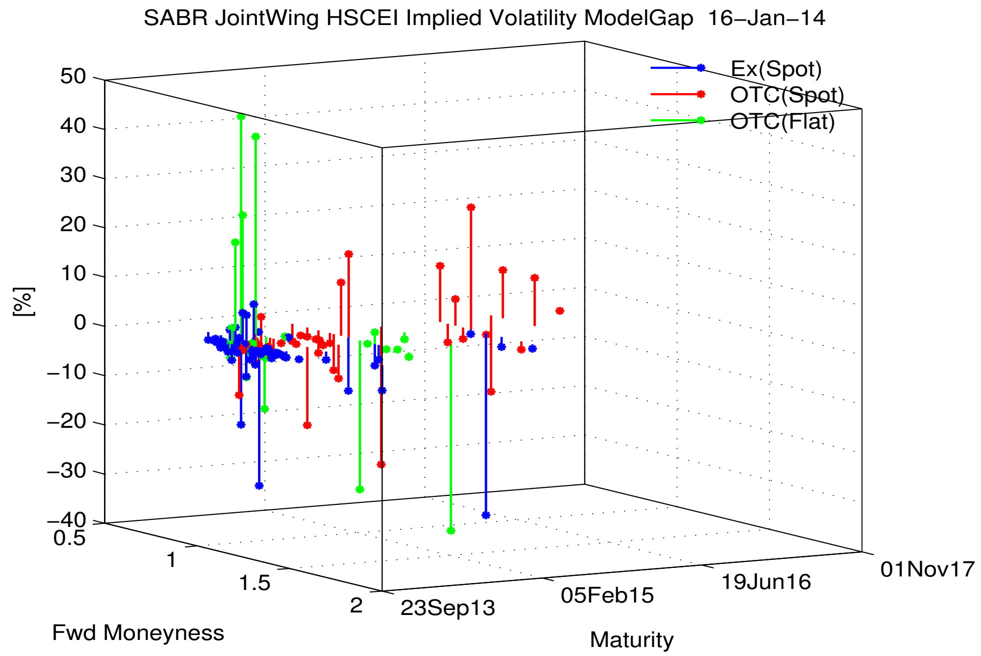
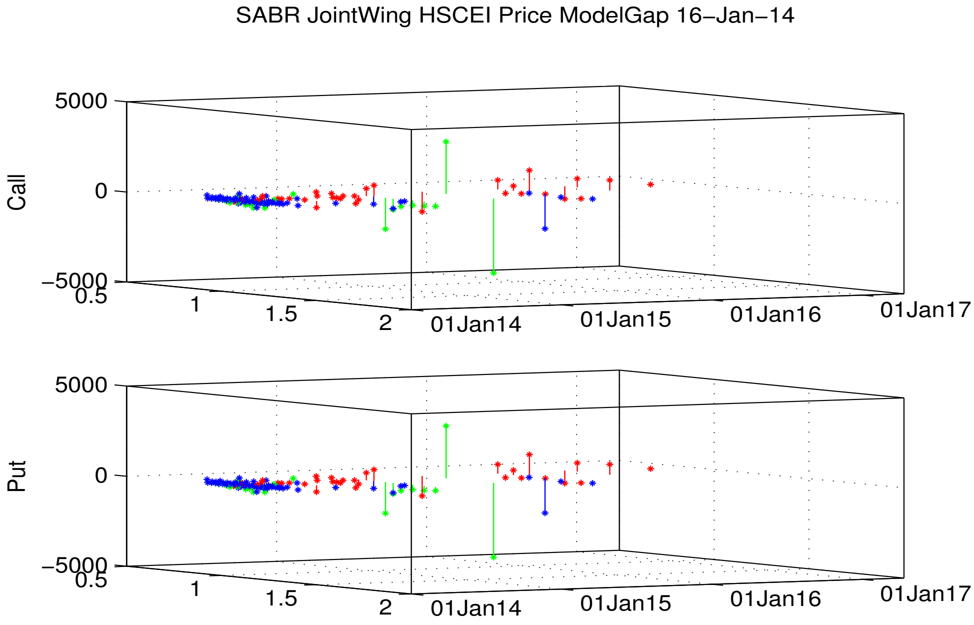


Figure B.45: SABR BoneWing HSCEI Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



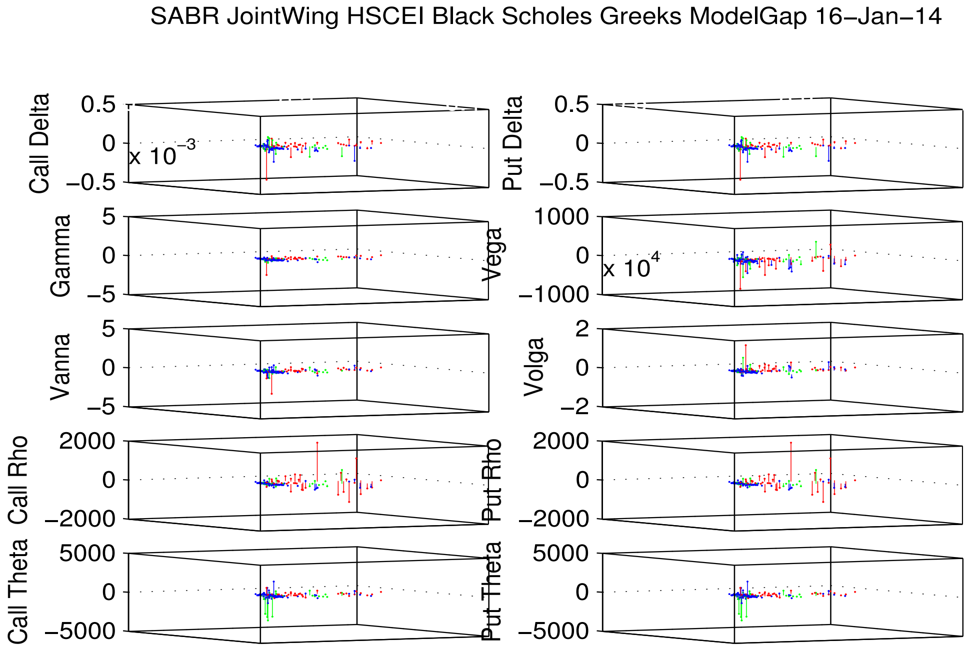
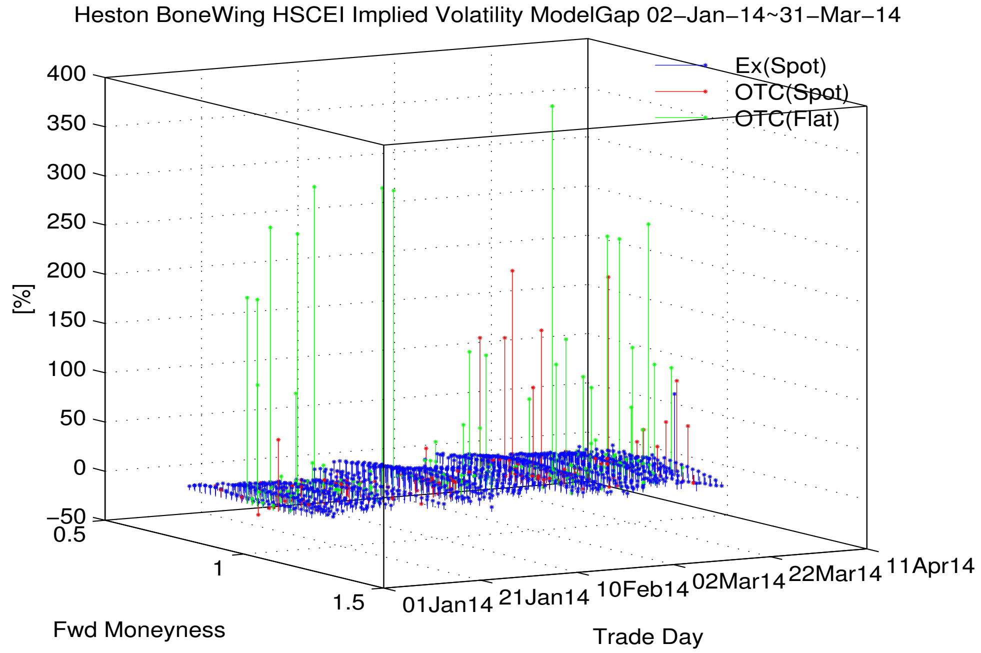


Figure B.46: SABR BoneWing HSCEI Price, Black Scholes Greeks ModelGap (16-Jan-14)



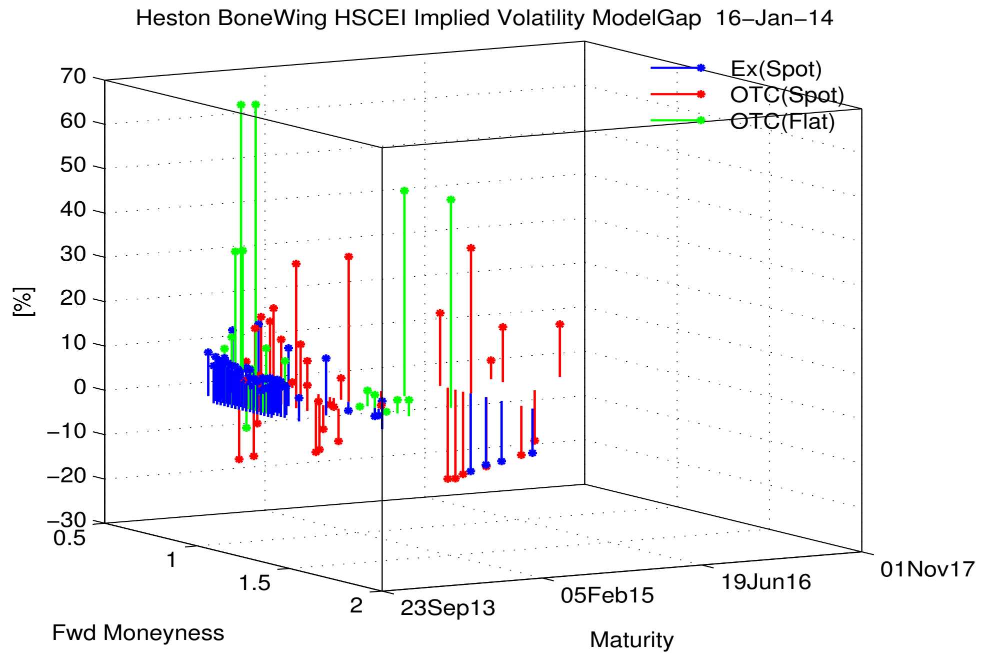
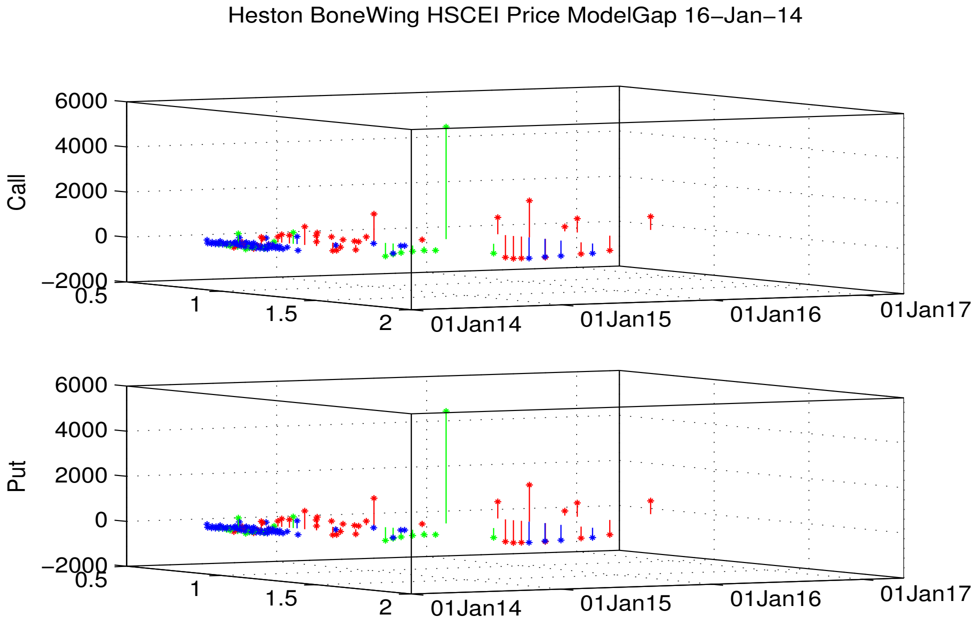


Figure B.47: Heston BoneWing HSCEI Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



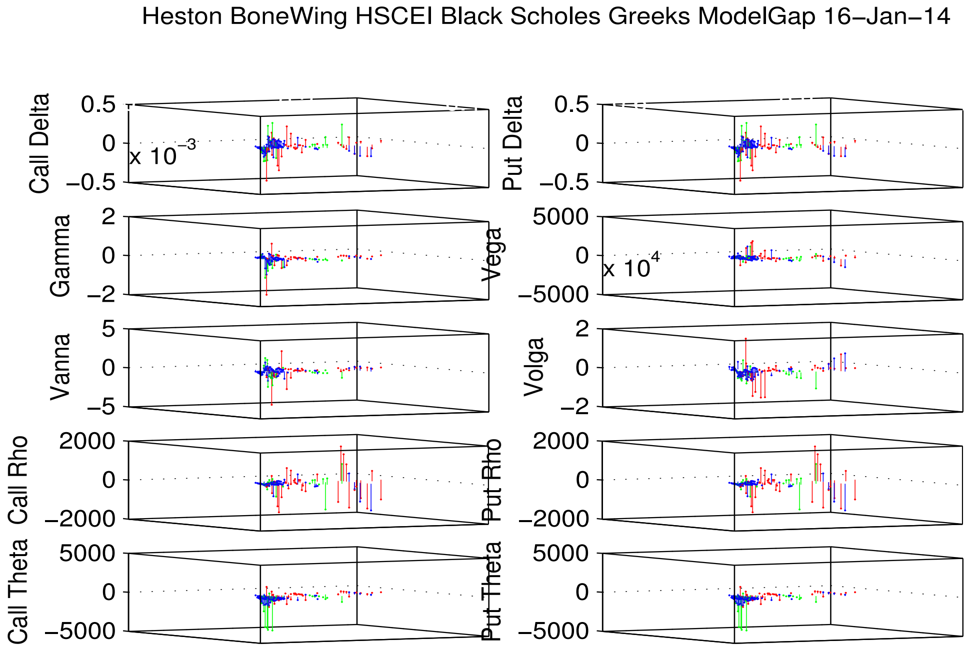
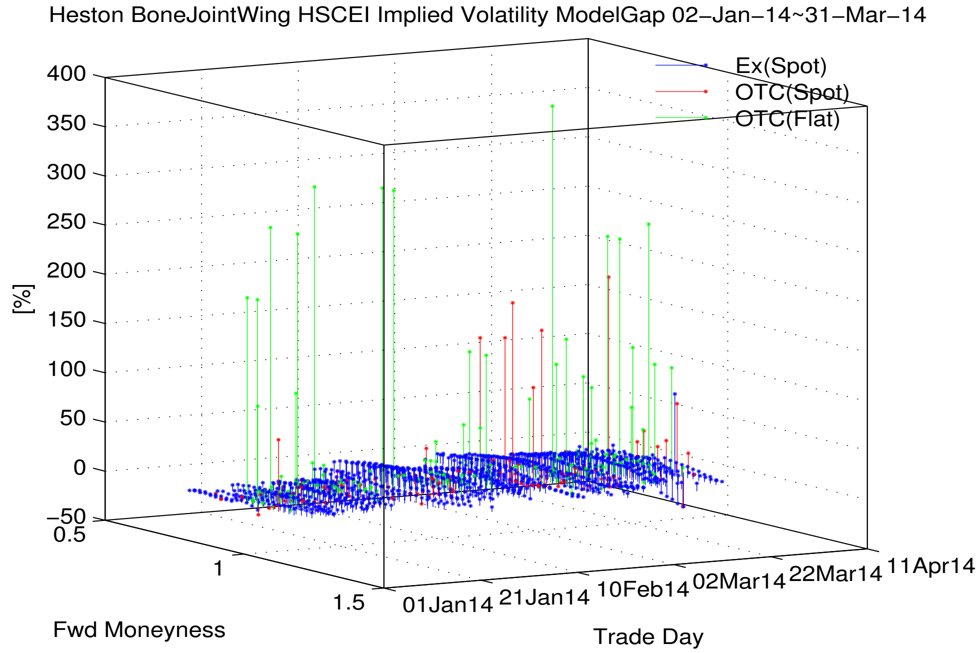


Figure B.48: Heston BoneWing HSCEI Price, Black Scholes Greeks ModelGap (16-Jan-14)



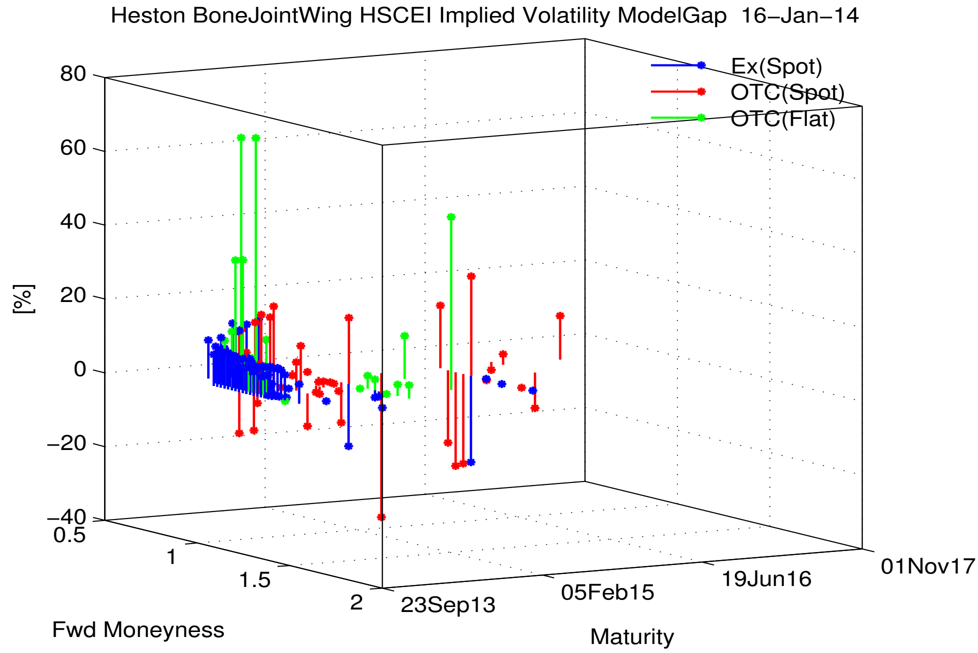
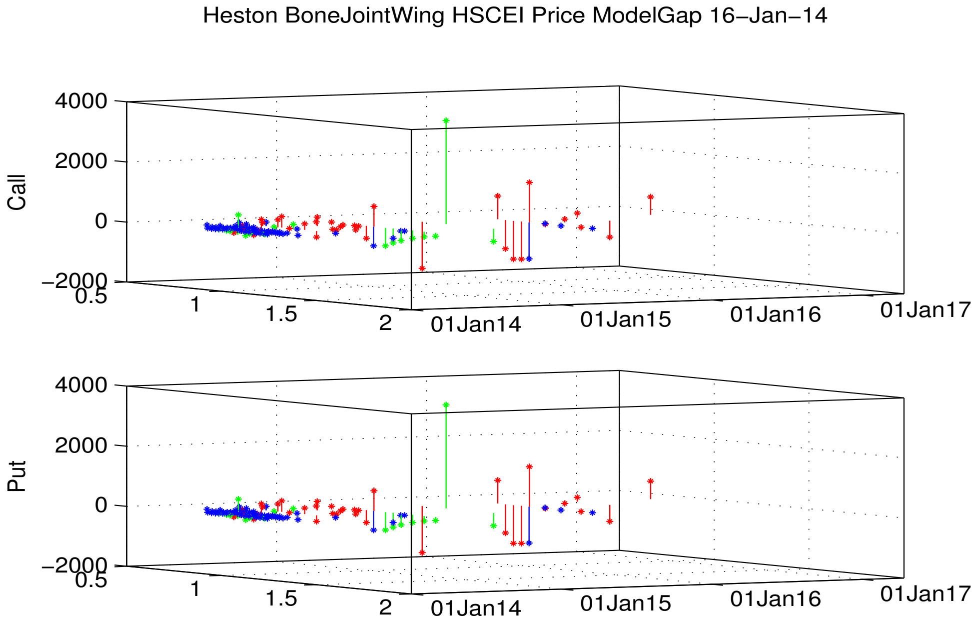


Figure B.49: Heston BoneWing HSCEI Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



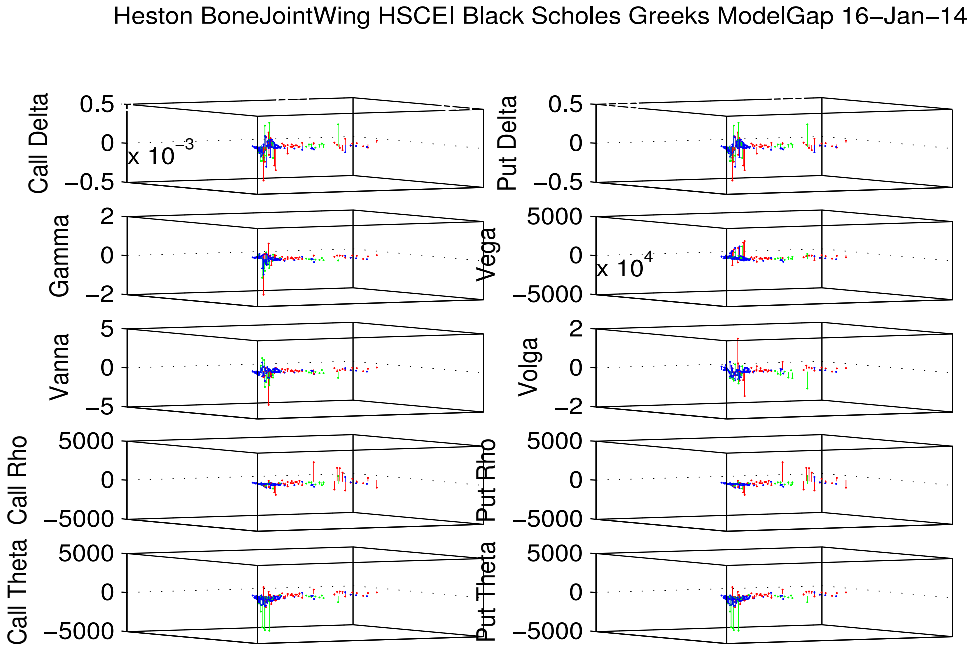
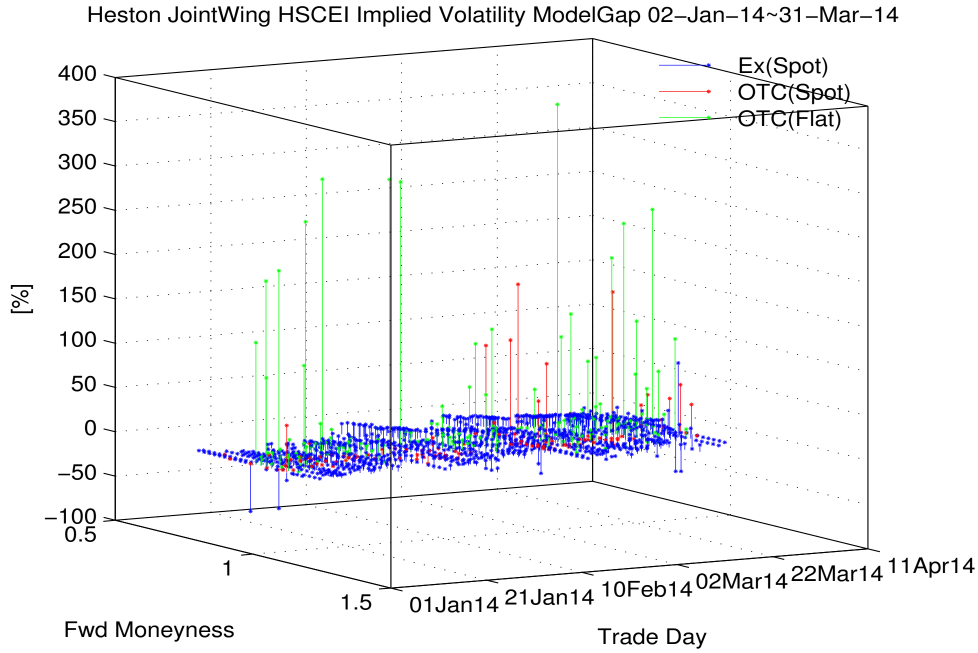


Figure B.50: Heston BoneWing HSCEI Price, Black Scholes Greeks ModelGap (16-Jan-14)



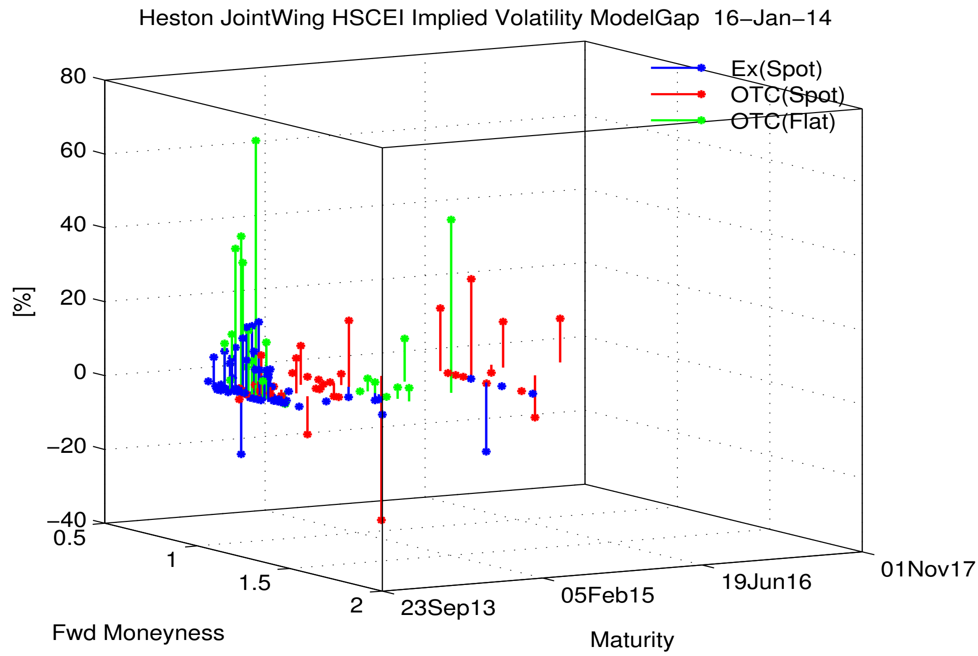
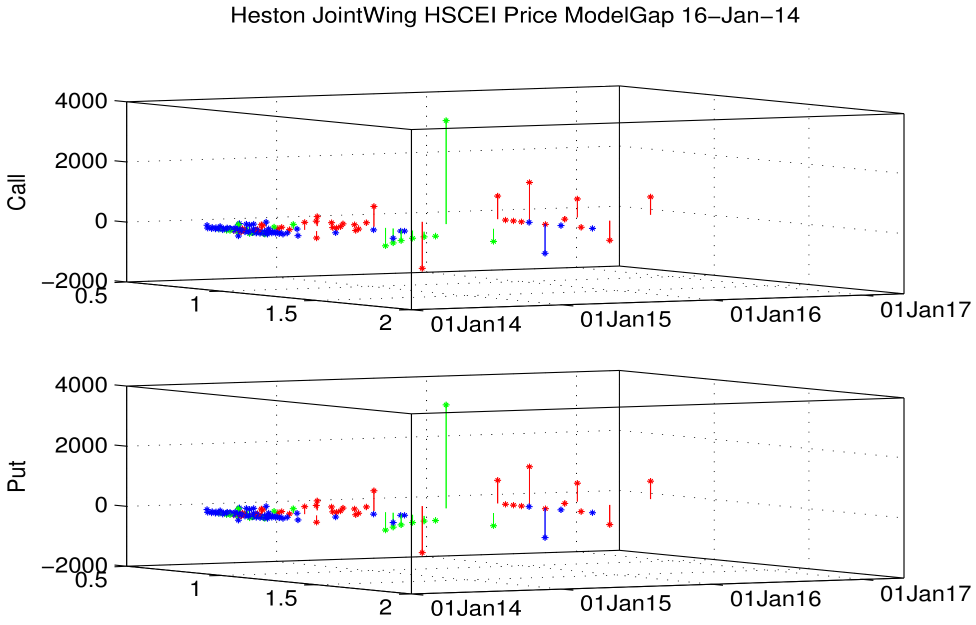


Figure B.51: Heston BoneWing HSCEI Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



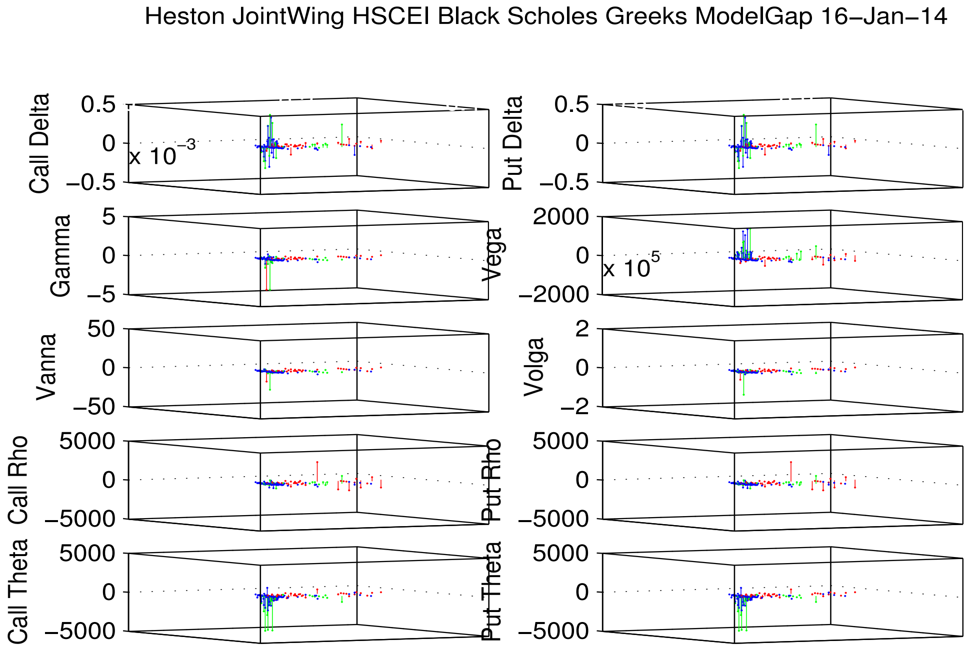


Figure B.52: Heston BoneWing HSCEI Price, Black Scholes Greeks ModelGap (16-Jan-14)

Table B.35: Root Mean Squared HSCEI Implied Volatility ModelGap (02-Jan-14∼31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
5.917E-01	4.132E-01	3.207E-01	4.164E-01	4.118E-01	3.946E-01

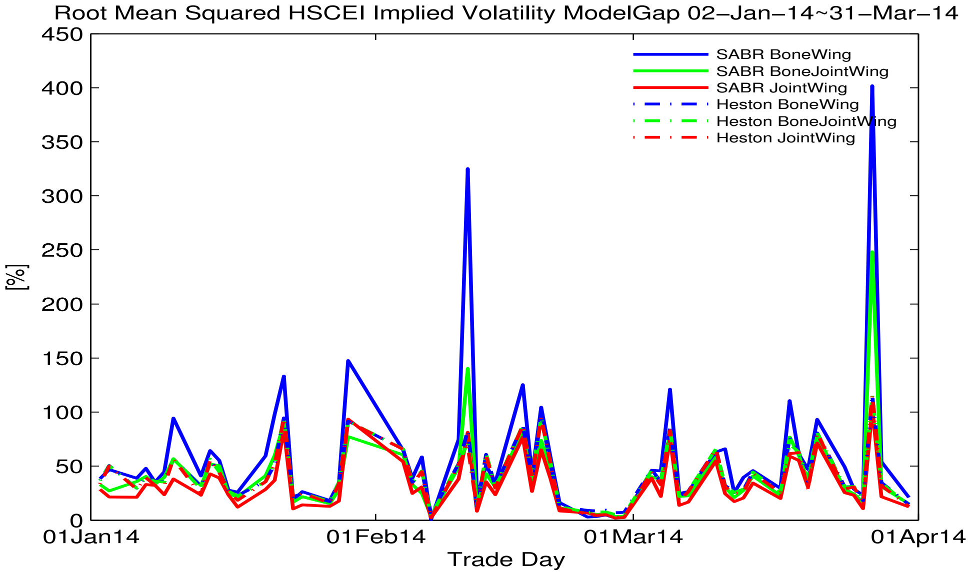


Figure B.53: Root Mean Squared HSCEI Implied Volatility ModelGap (02-Jan-14∼31-Mar-14)

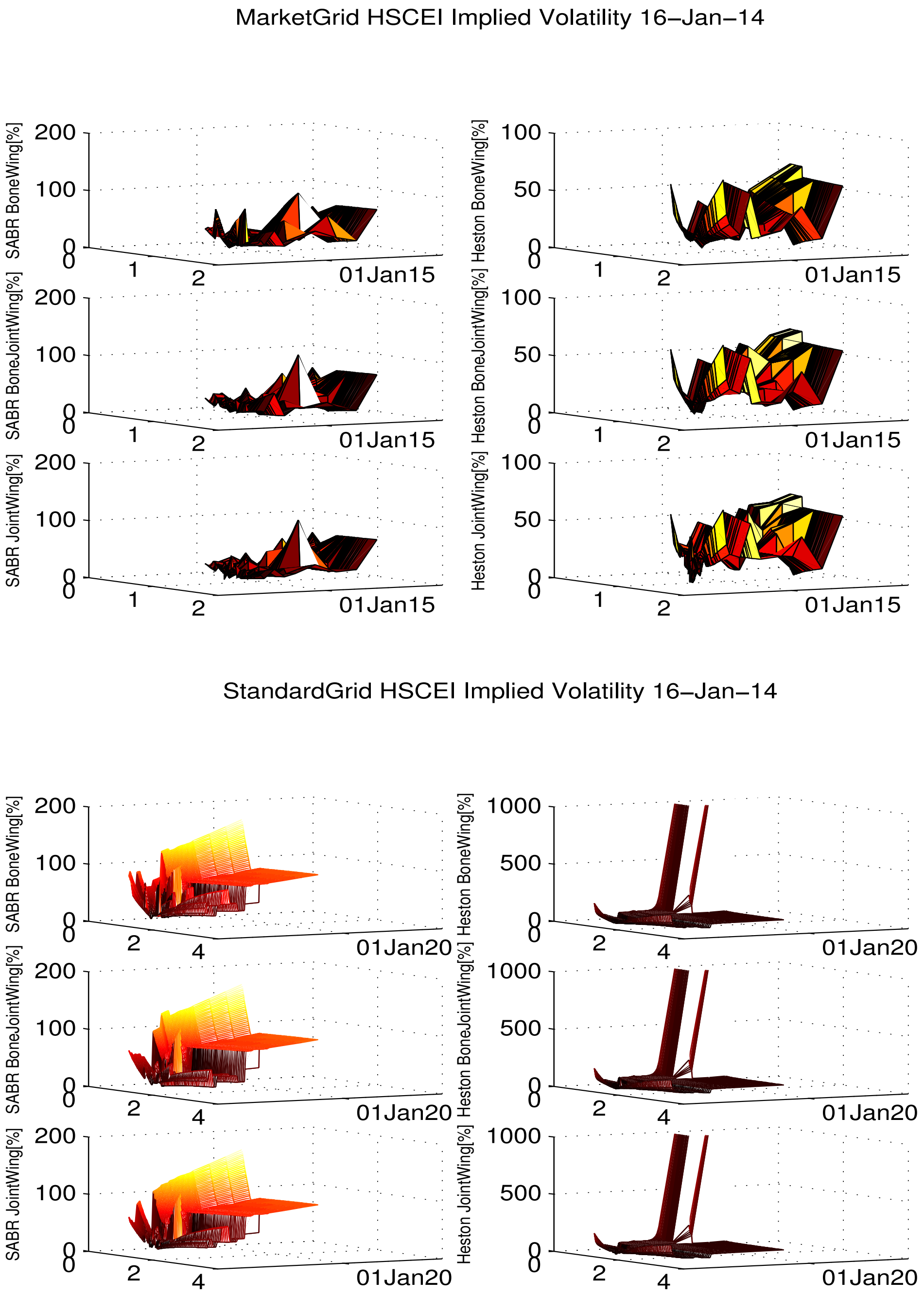
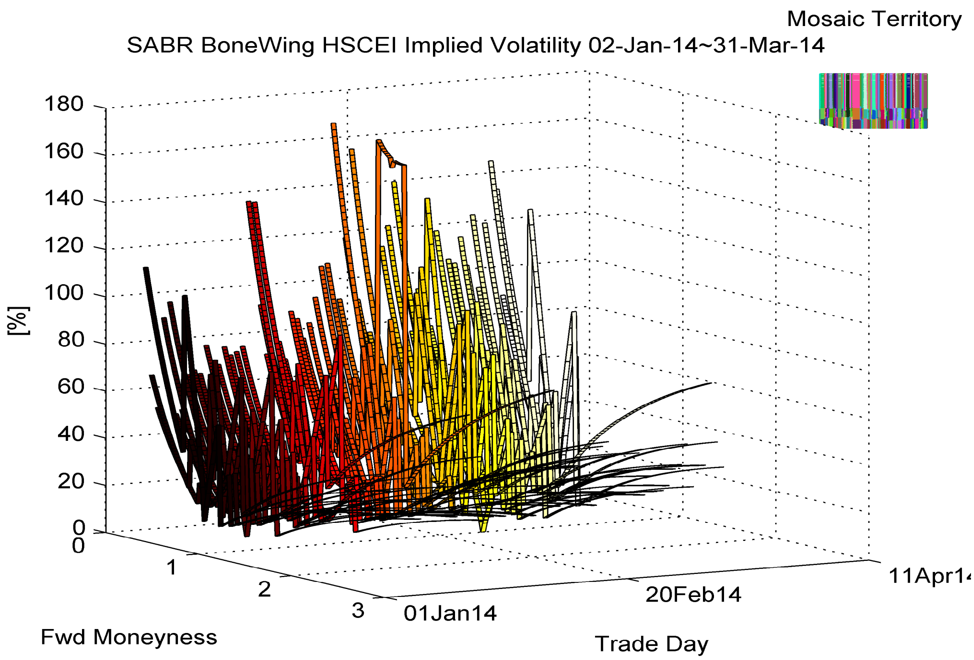


Figure B.54: HSCEI Market, Standard Grid Implied Volatility (16-Jan-14)



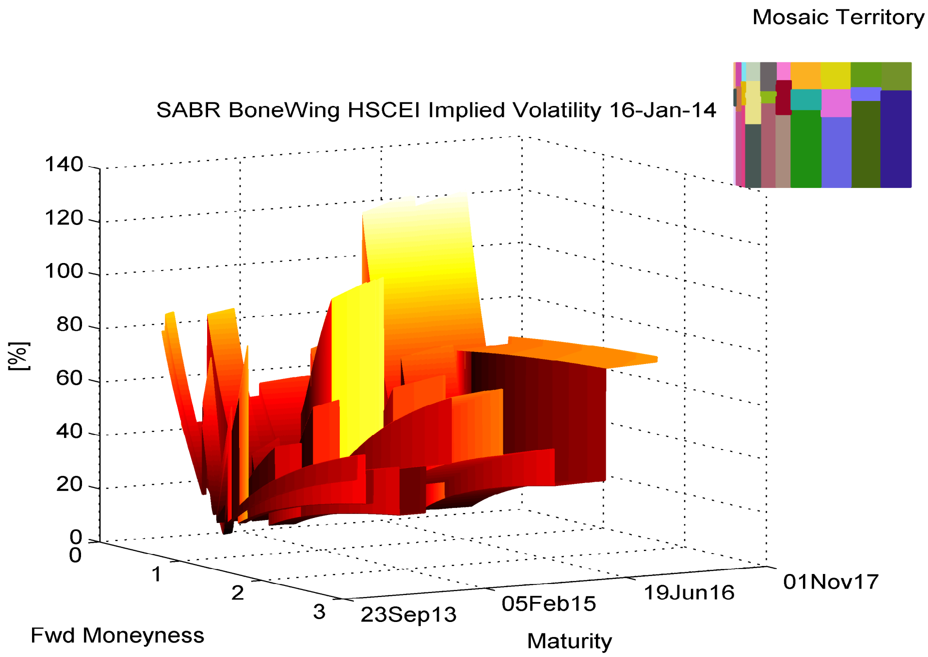
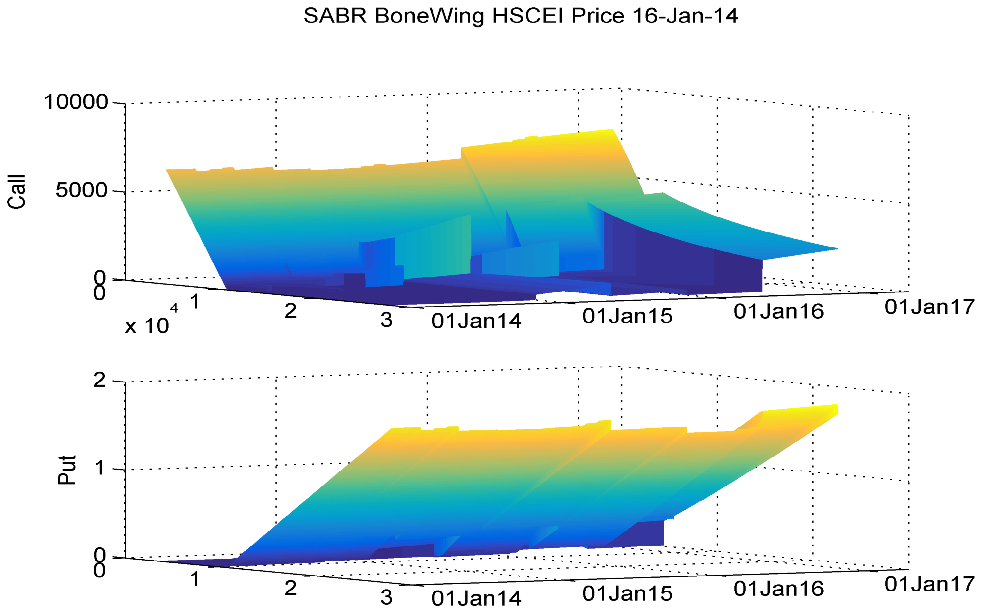


Figure B.55: SABR BoneWing HSCEI Implied Volatility (Nearest) (03-Jan-14∼31-Mar-14), (16-Jan-14)



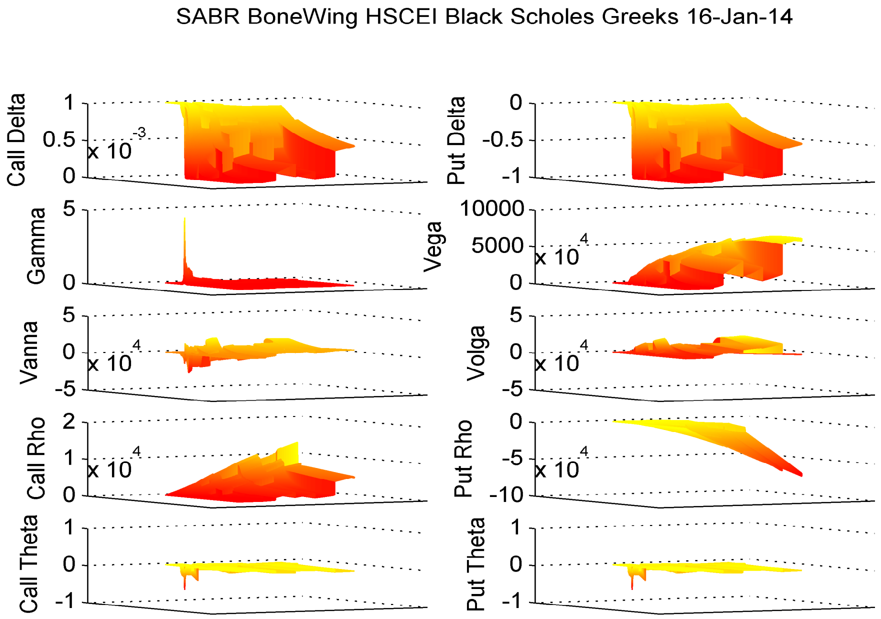
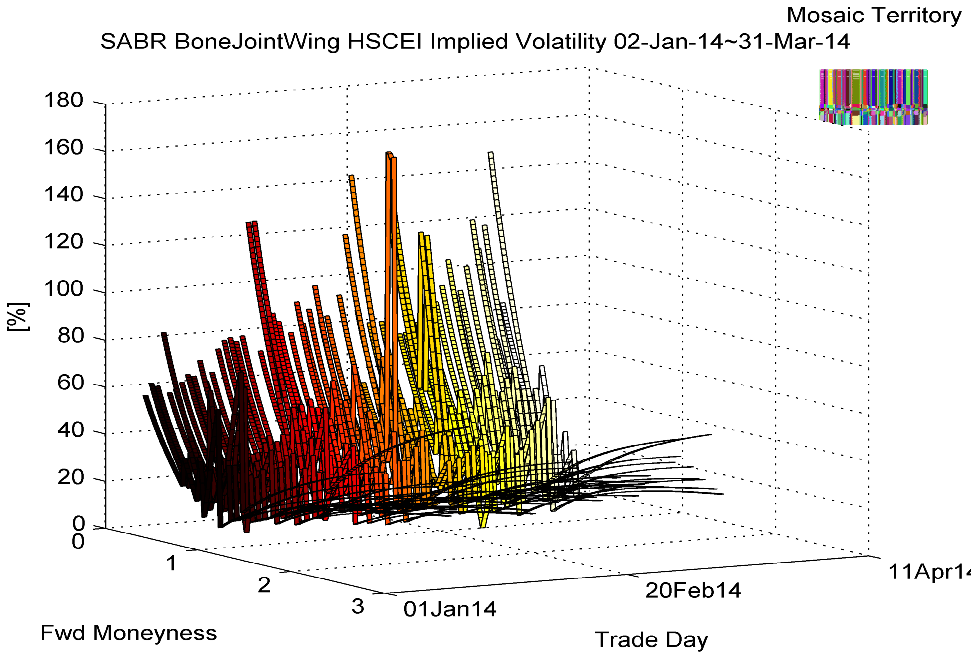


Figure B.56: SABR BoneWing HSCEI Price, Black Scholes Greeks (16-Jan-14)



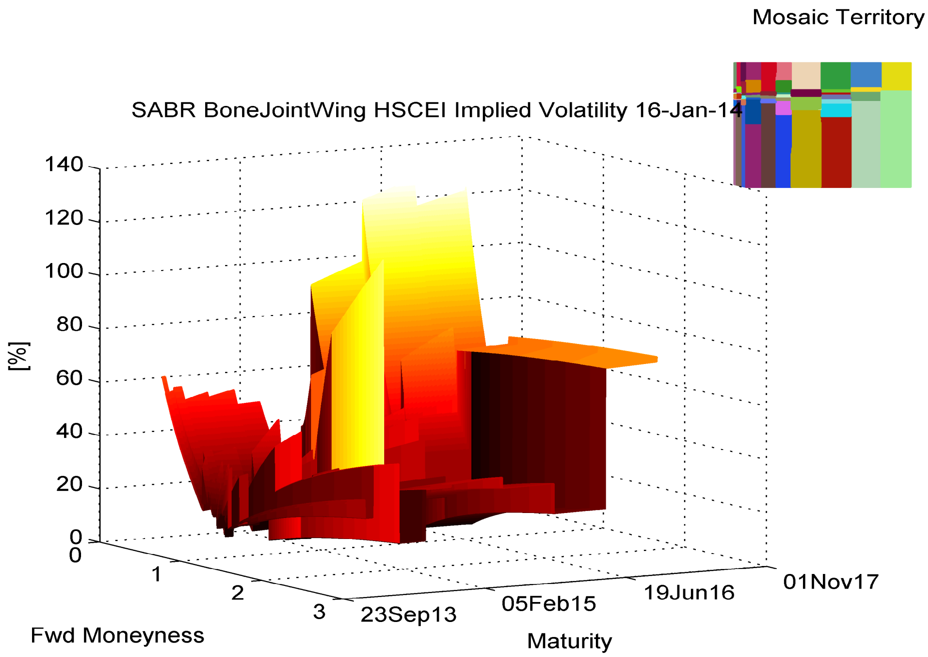
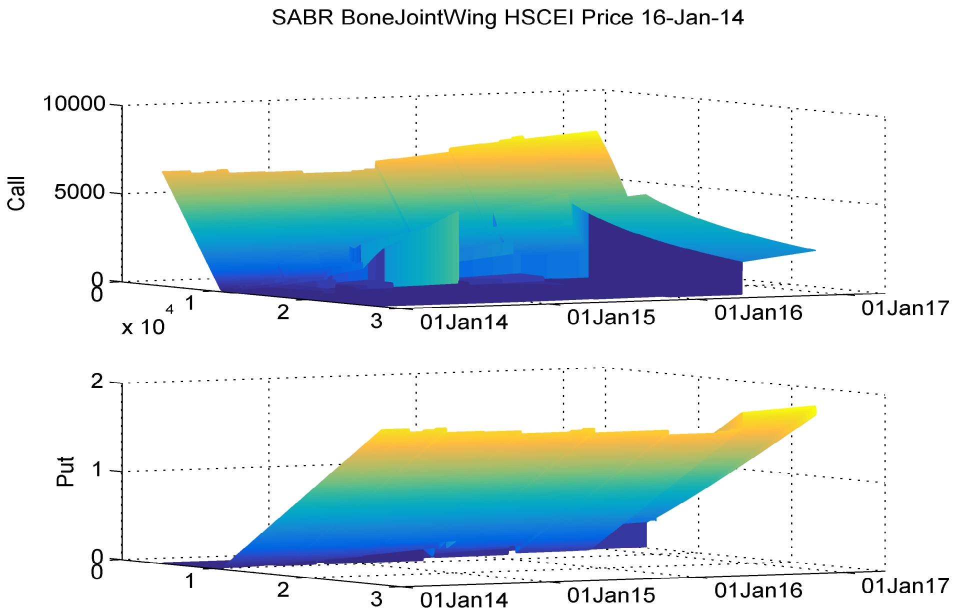


Figure B.57: SABR BoneWing HSCEI Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



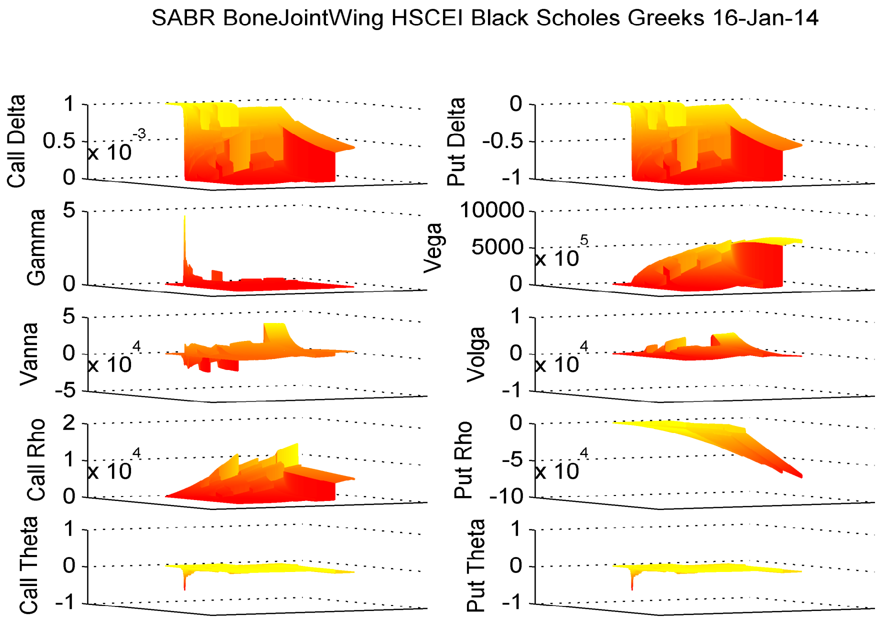
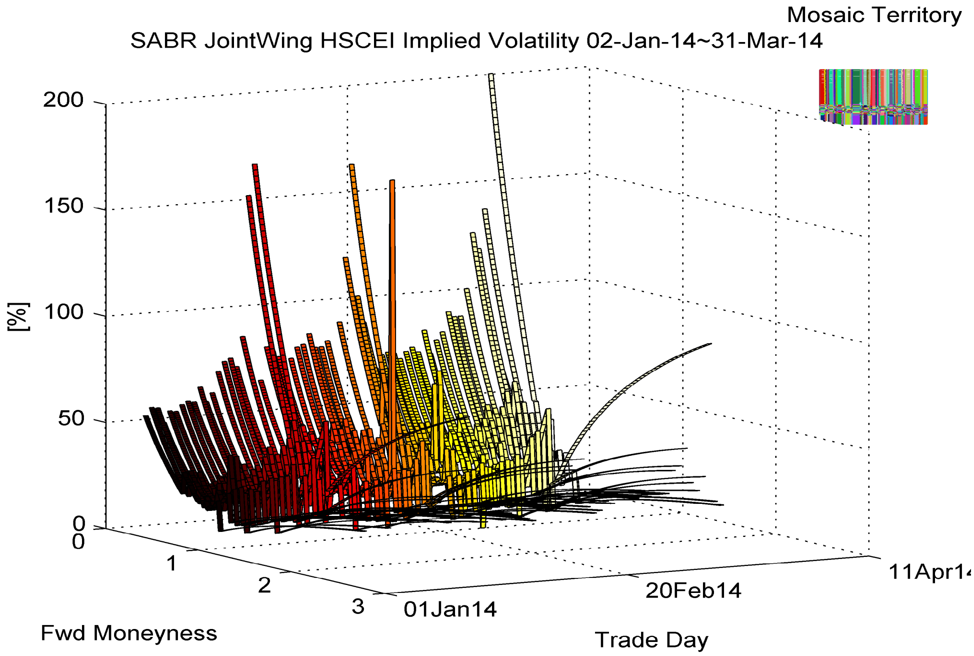


Figure B.58: SABR BoneWing HSCEI Price, Black Scholes Greeks (16-Jan-14)



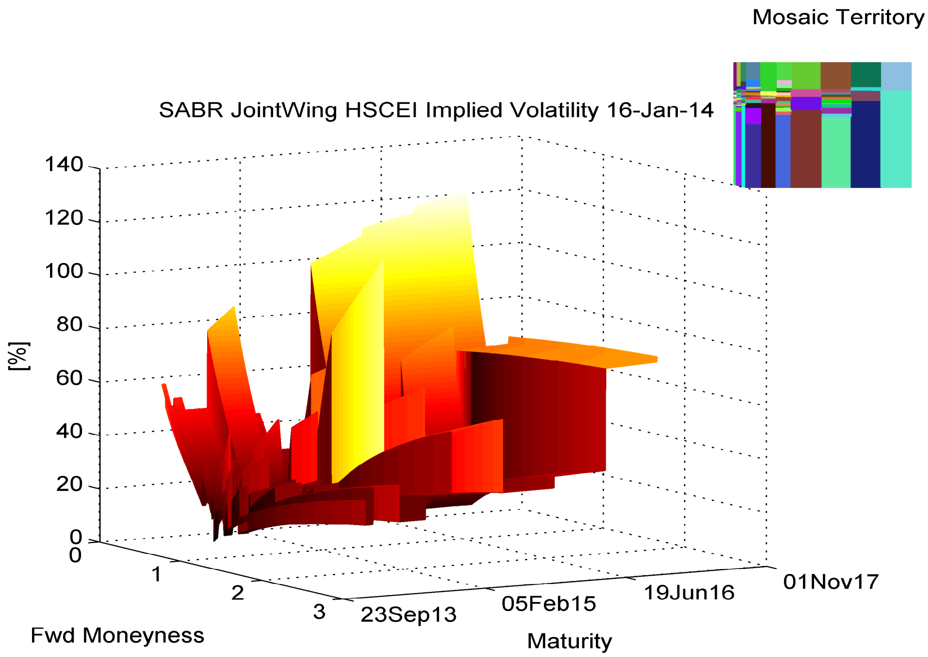


Figure B.59: SABR BoneWing HSCEI Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)

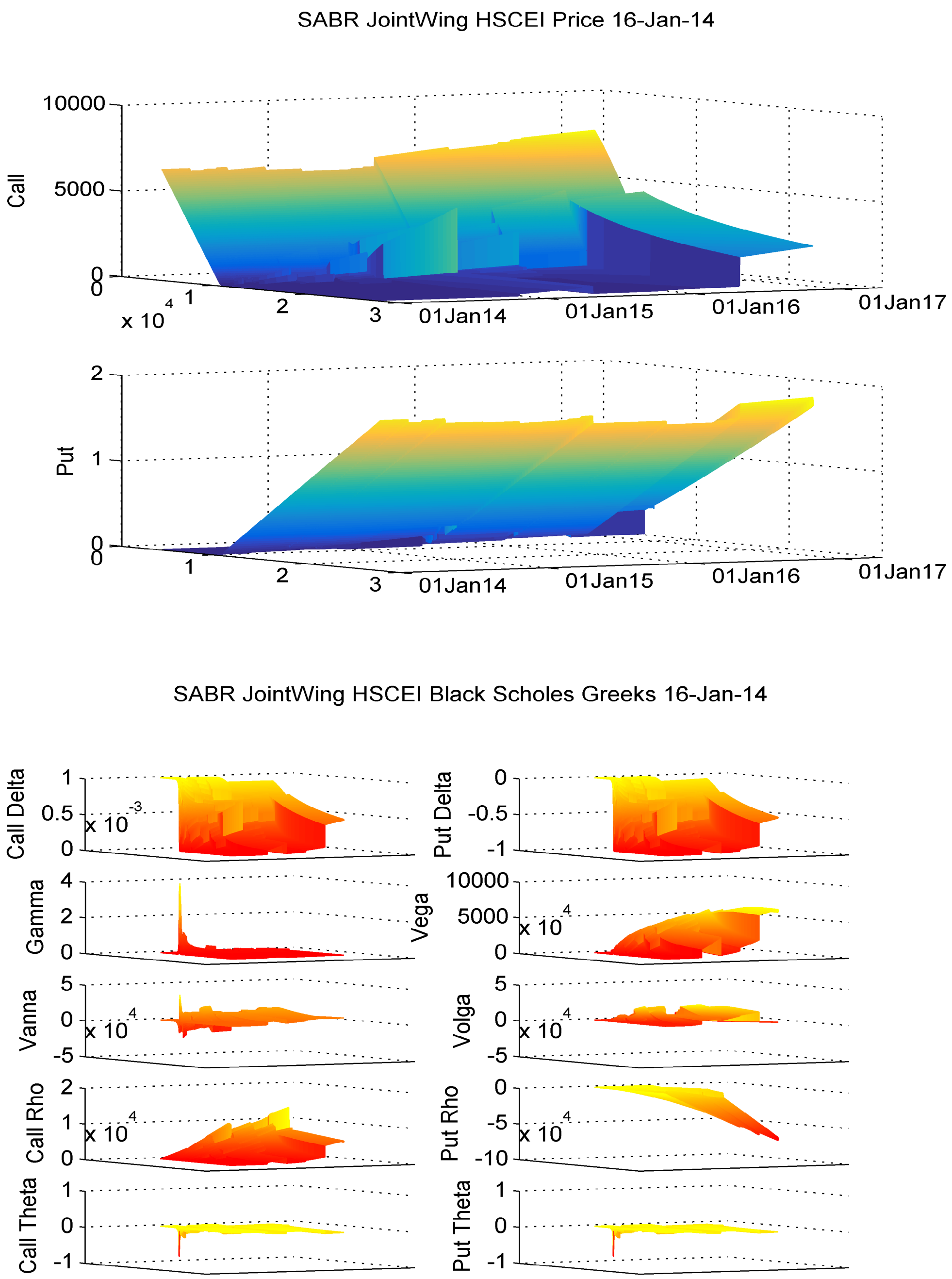
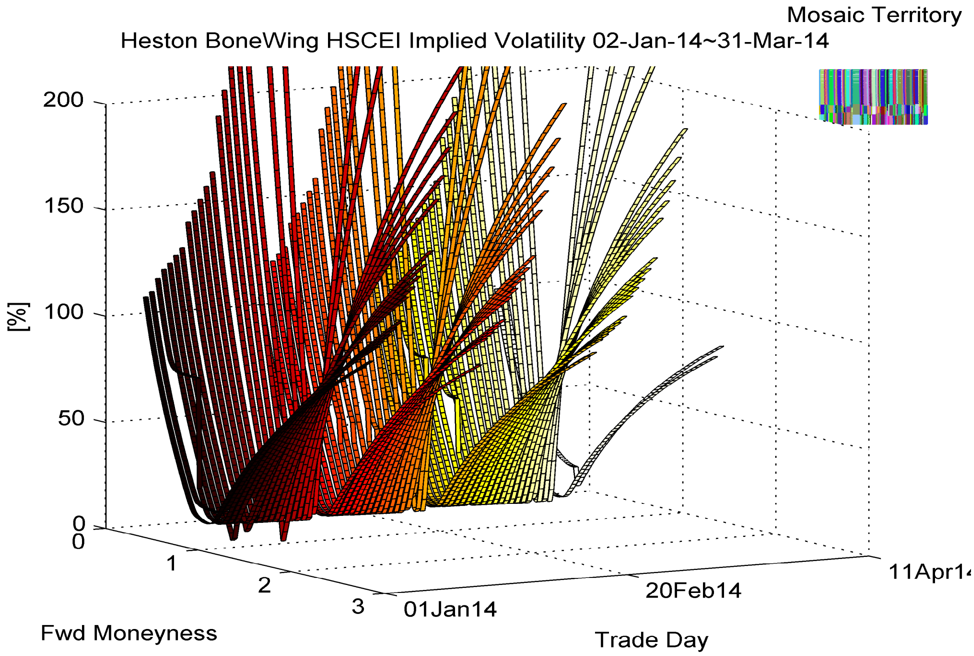


Figure B.60: SABR BoneWing HSCEI Price, Black Scholes Greeks (16-Jan-14)



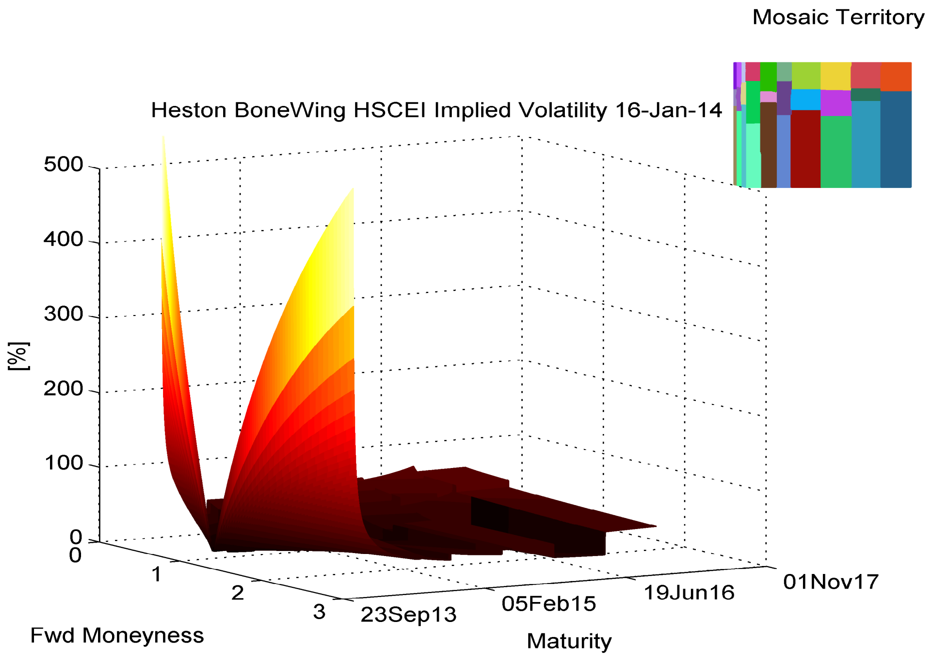
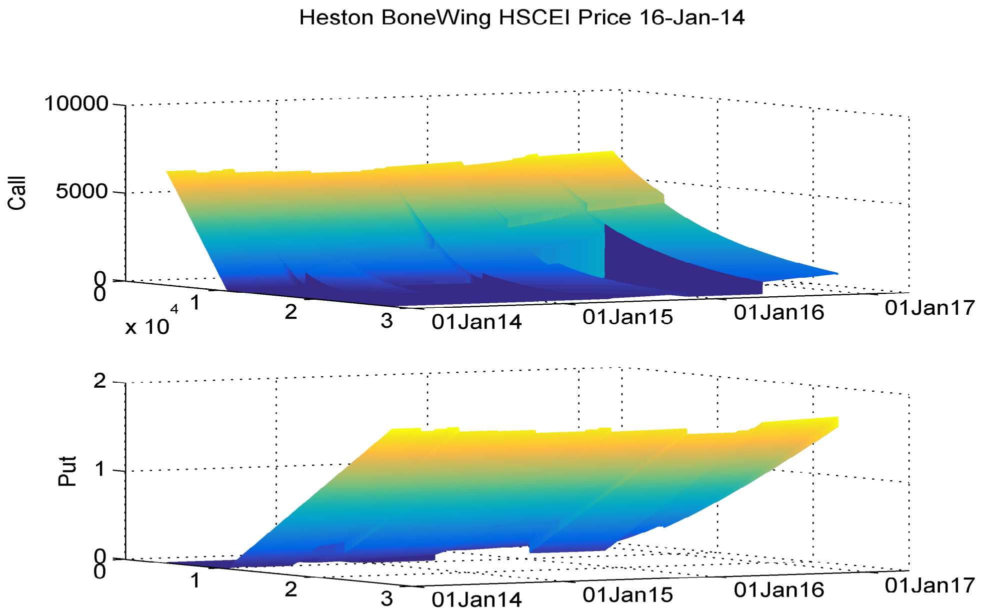


Figure B.61: Heston BoneWing HSCEI Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



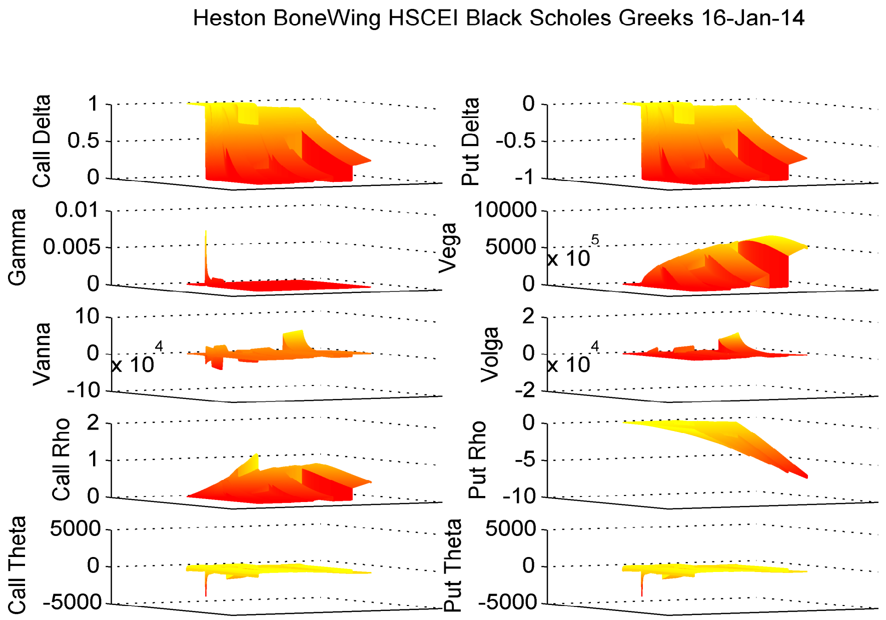


Figure B.62: Heston BoneWing HSCEI Price, Black Scholes Greeks (16-Jan-14)

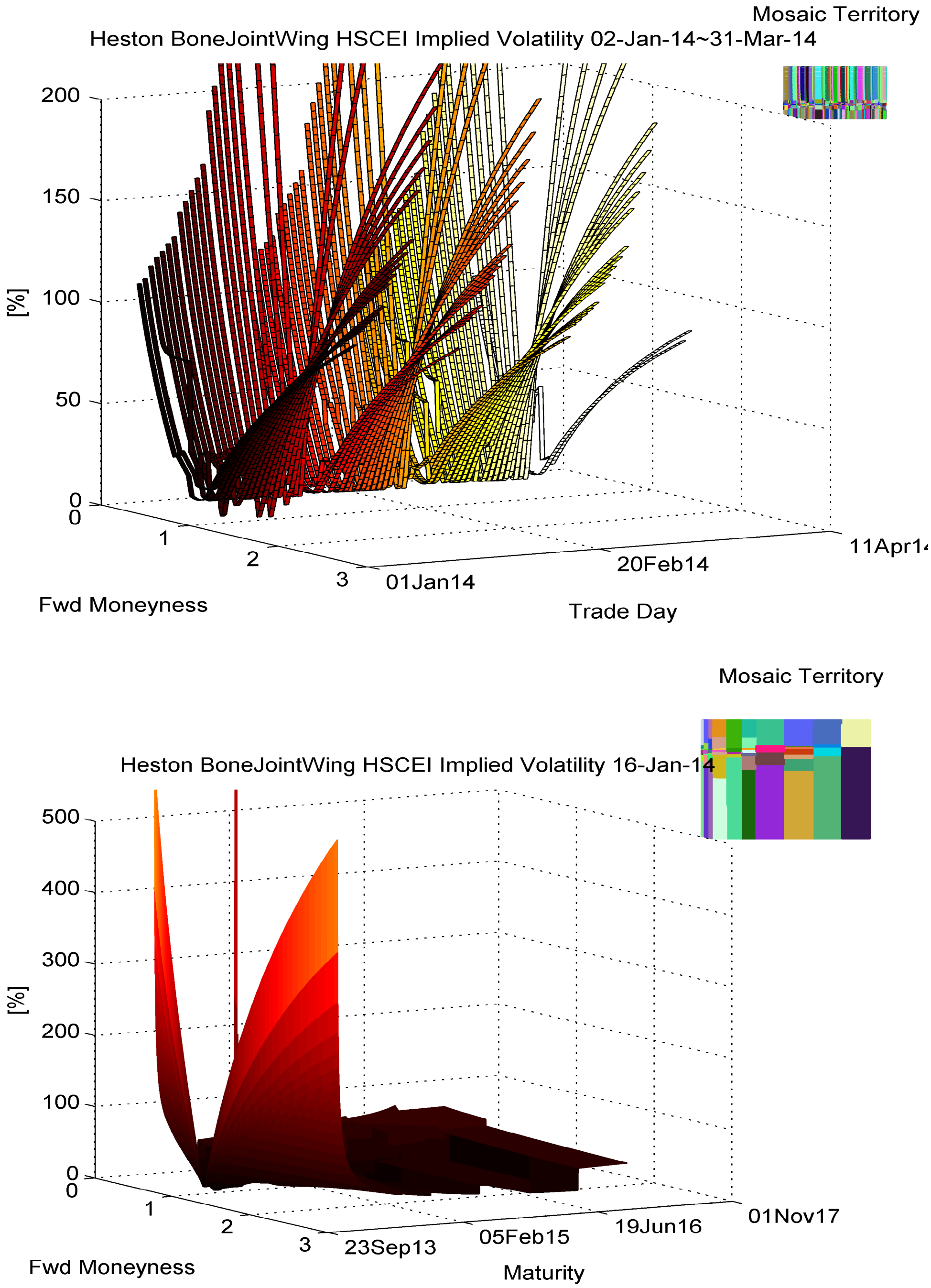
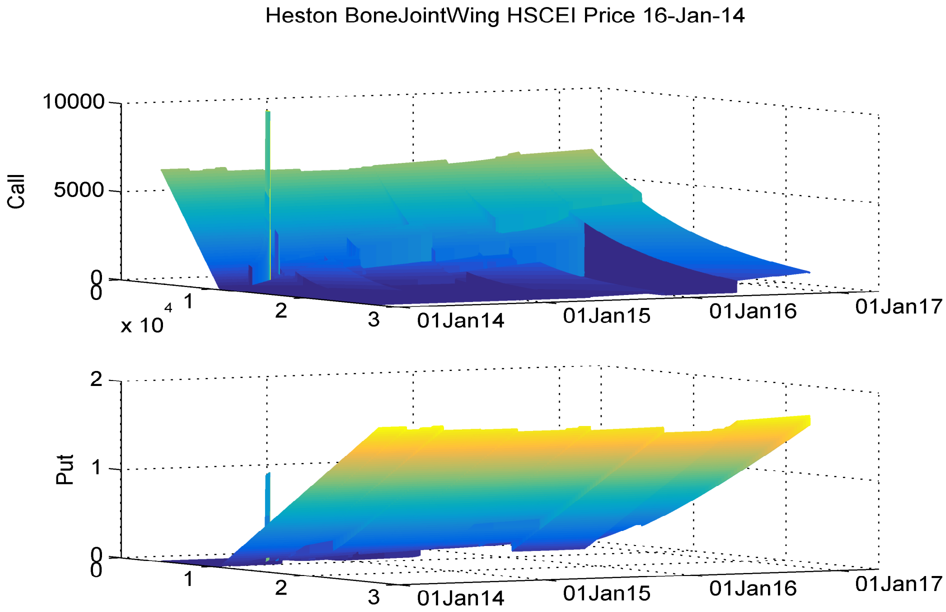


Figure B.63: Heston BoneWing HSCEI Implied Volatility (Nearest) (03-Jan-14∼31-Mar-14), (16-Jan-14)



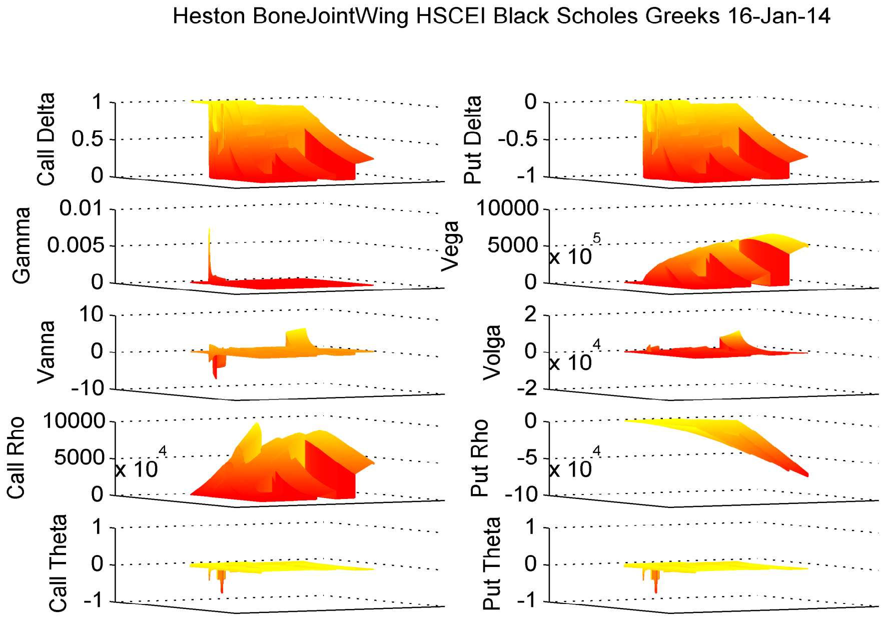
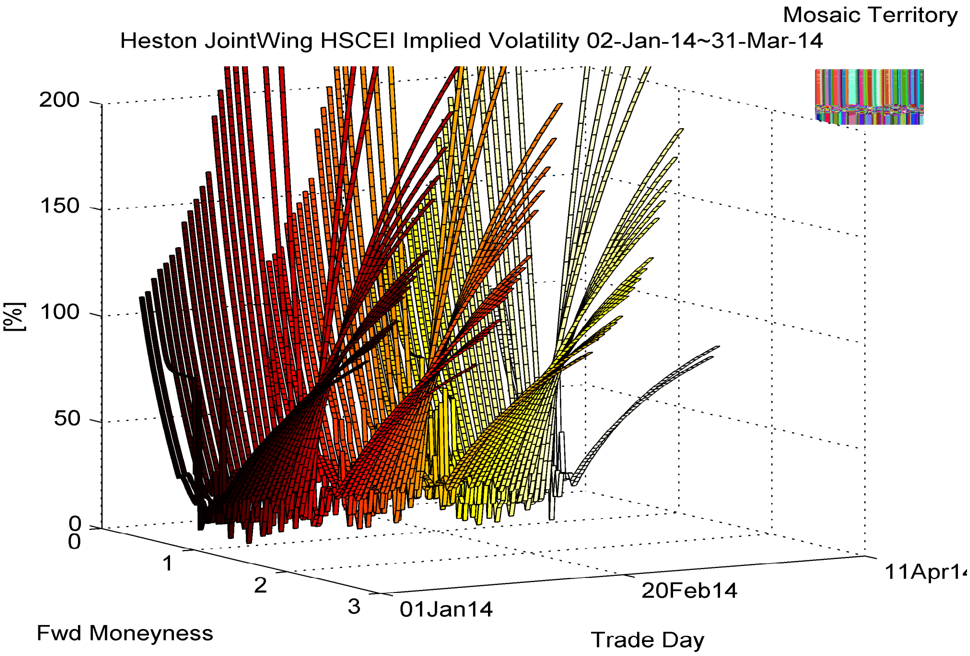


Figure B.64: Heston BoneWing HSCEI Price, Black Scholes Greeks (16-Jan-14)



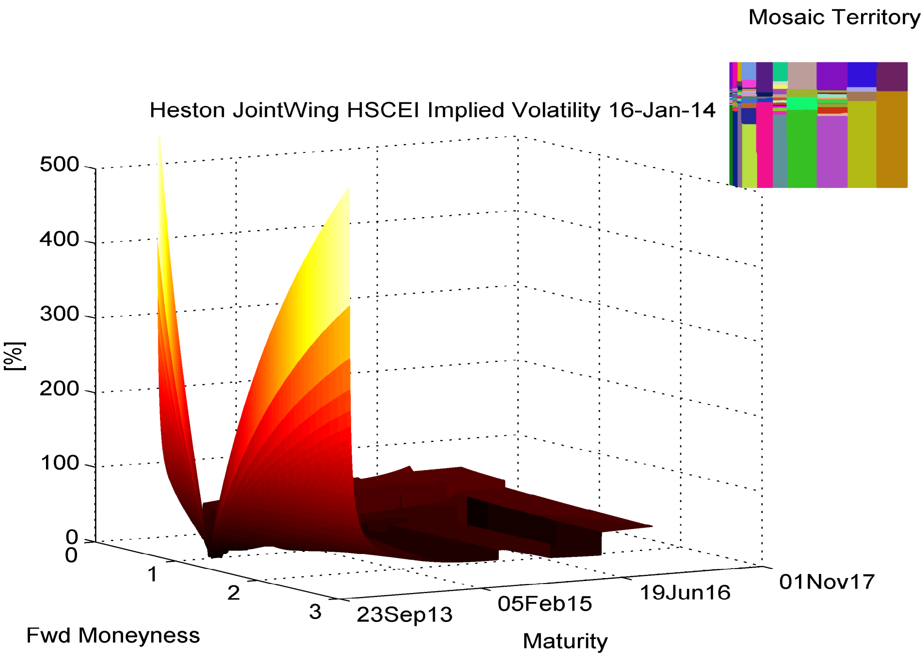
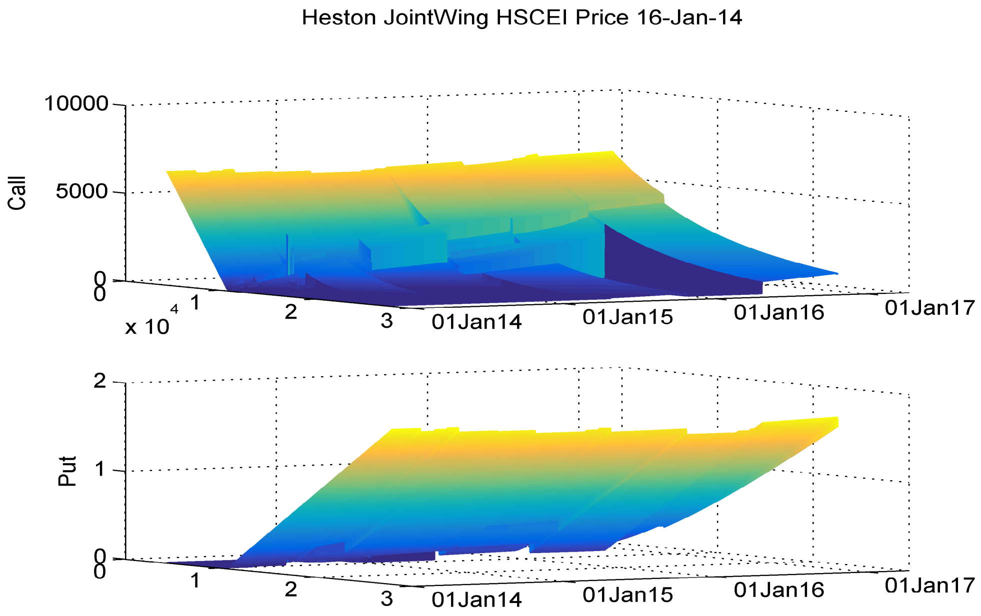


Figure B.65: Heston BoneWing HSCEI Implied Volatility (Nearest) (03-Jan-14∼31-Mar-14), (16-Jan-14)



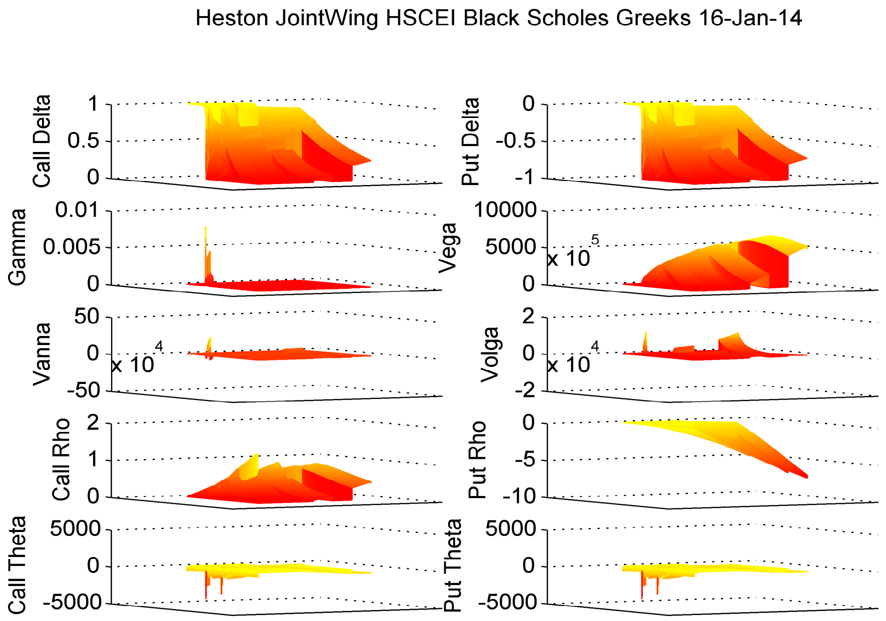
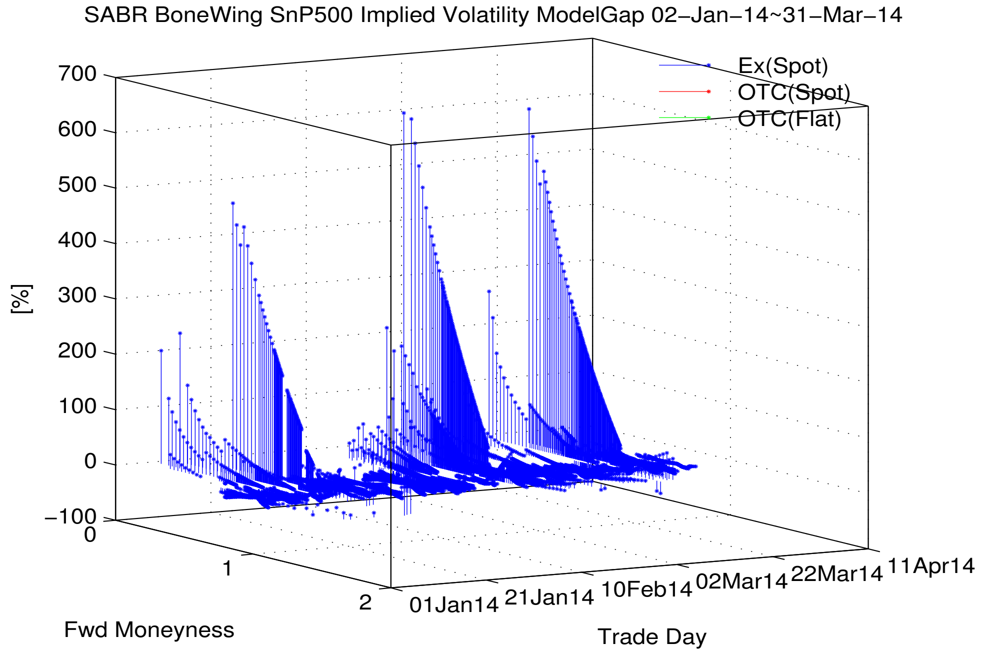


Figure B.66: Heston BoneWing HSCEI Price, Black Scholes Greeks (16-Jan-14)



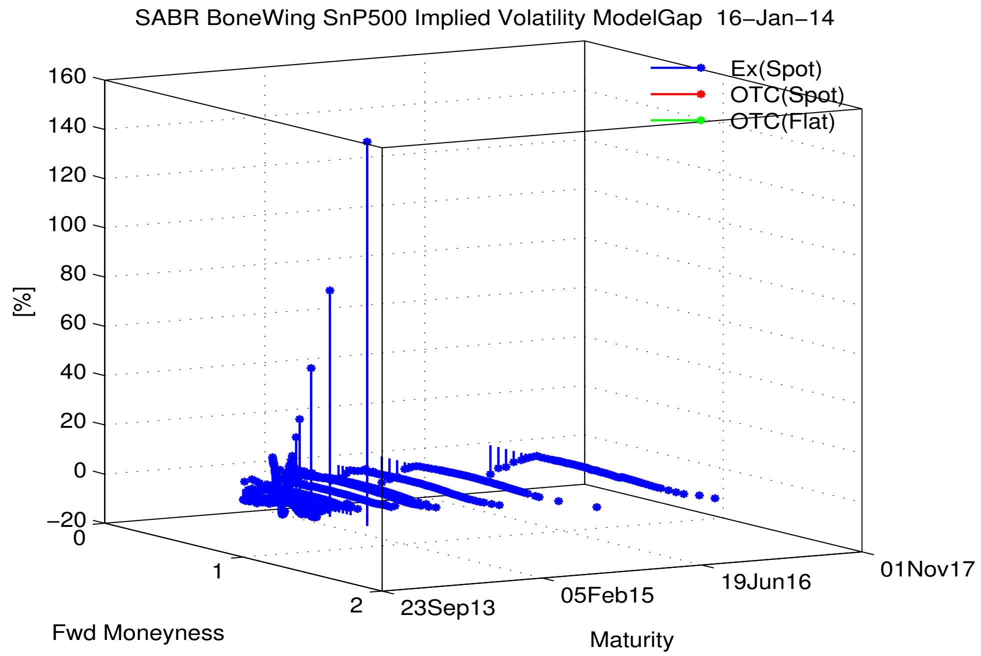
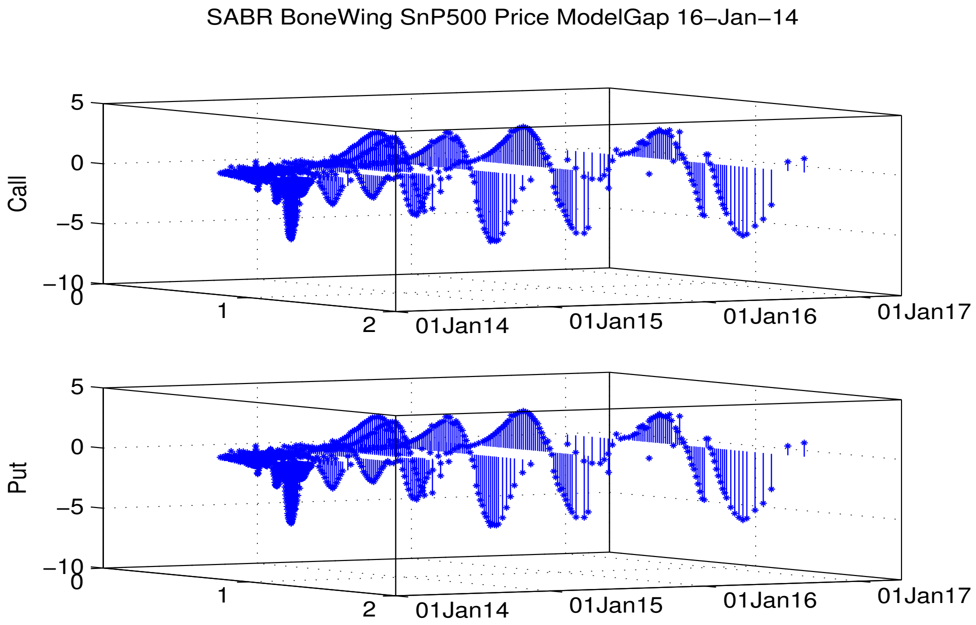


Figure B.67: SABR BoneWing SnP500 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



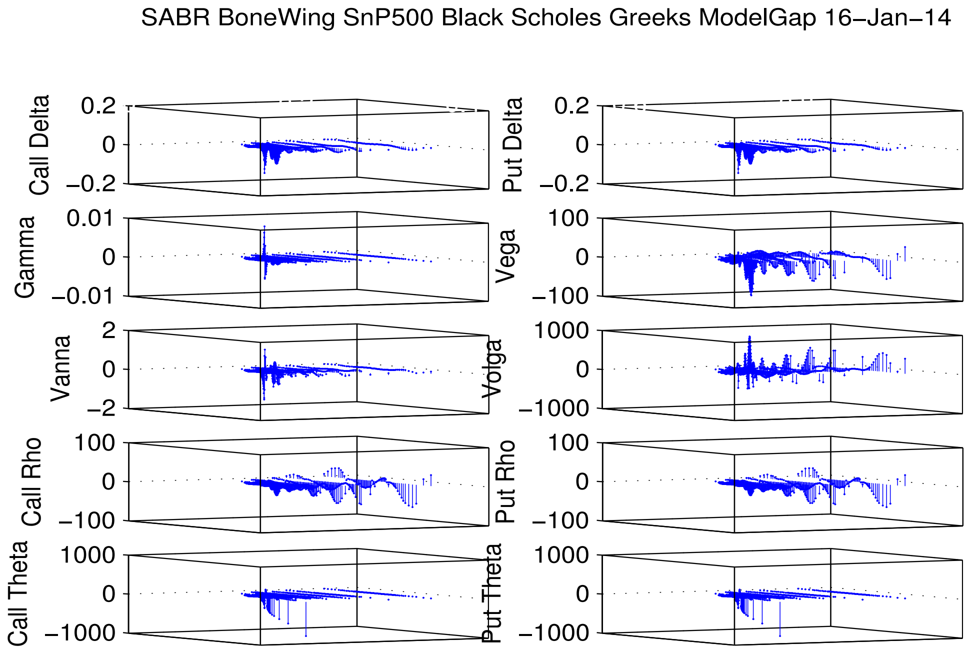
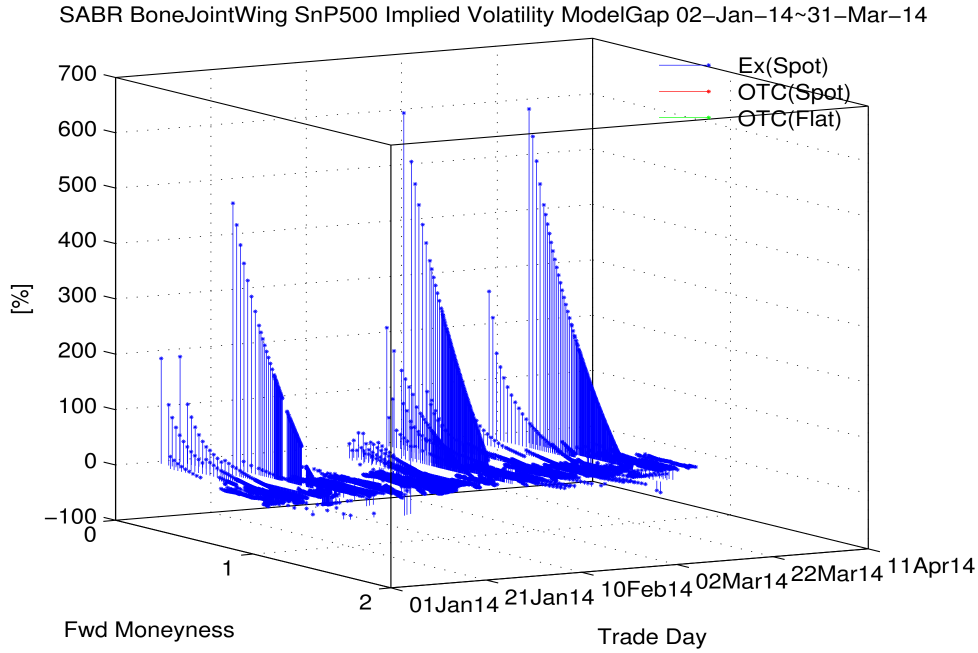


Figure B.68: SABR BoneWing SnP500 Price, Black Scholes Greeks ModelGap (16-Jan-14)



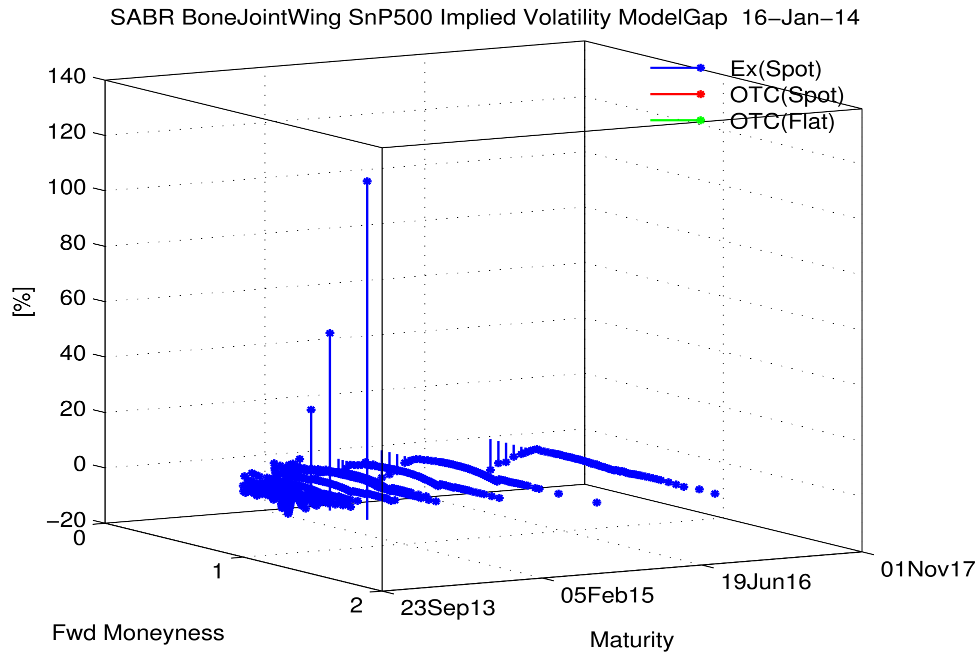


Figure B.69: SABR BoneWing SnP500 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)

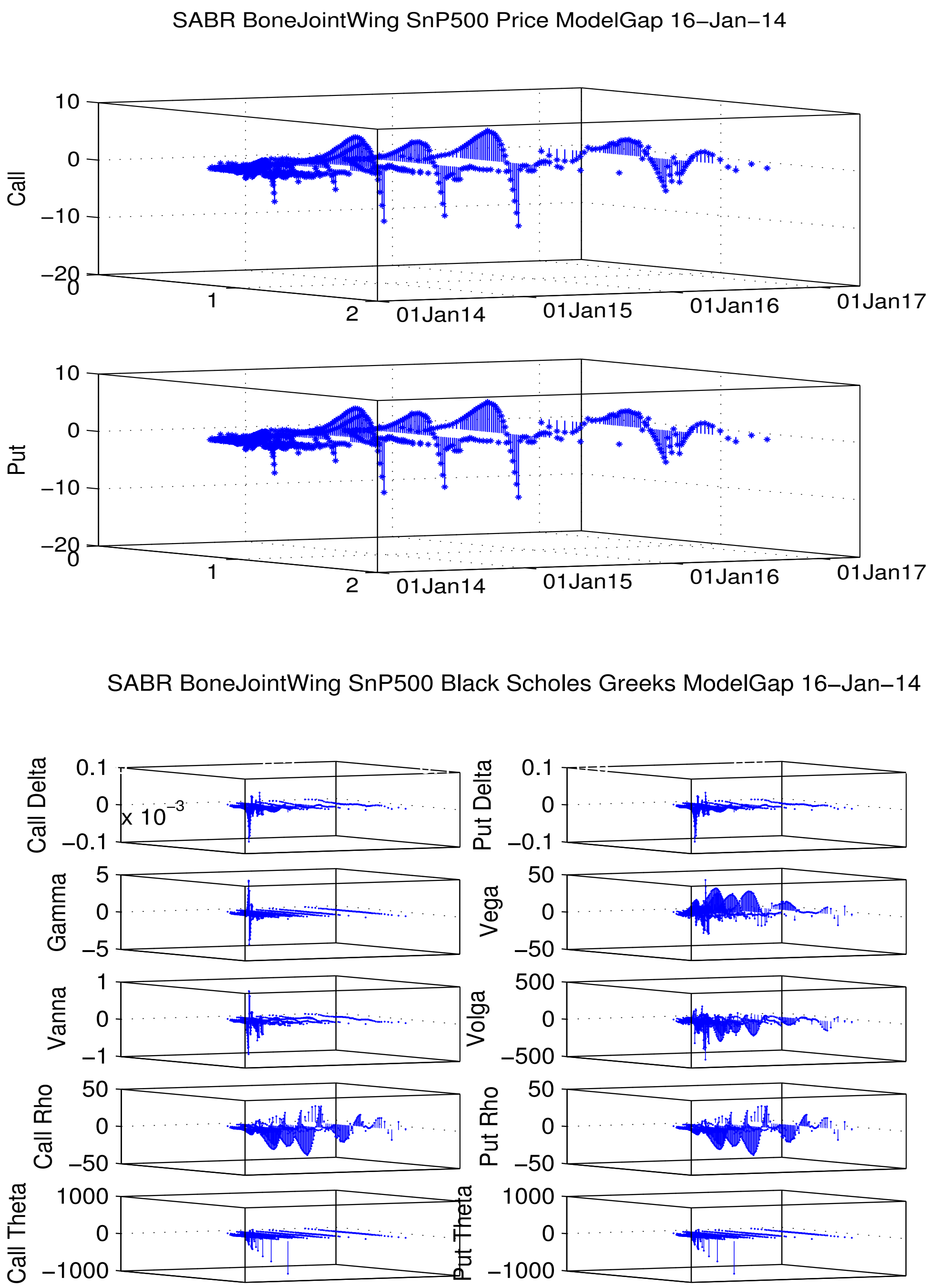


Figure B.70: SABR BoneWing SnP500 Price, Black Scholes Greeks ModelGap (16-Jan-14)

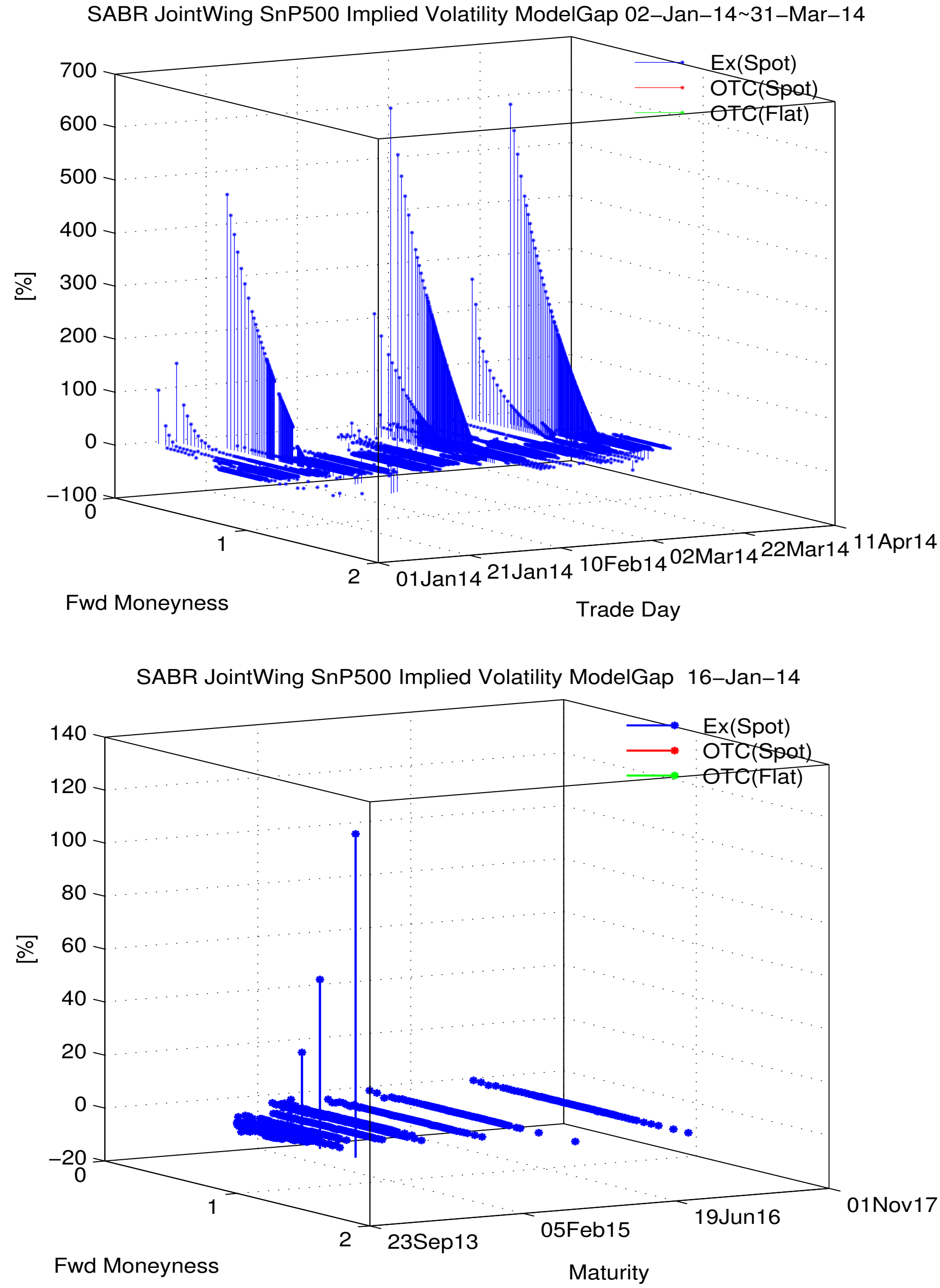
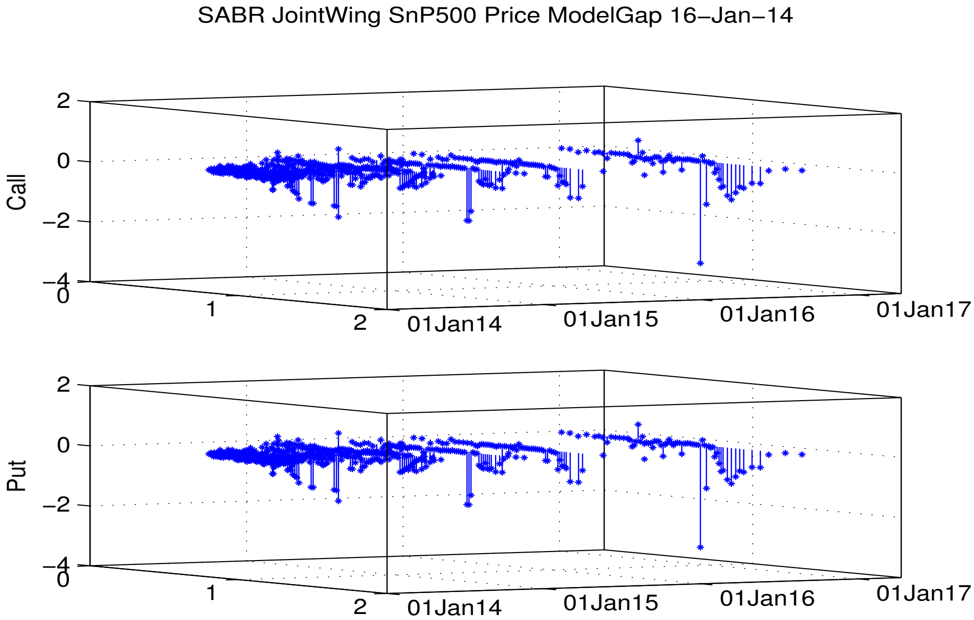


Figure B.71: SABR BoneWing SnP500 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



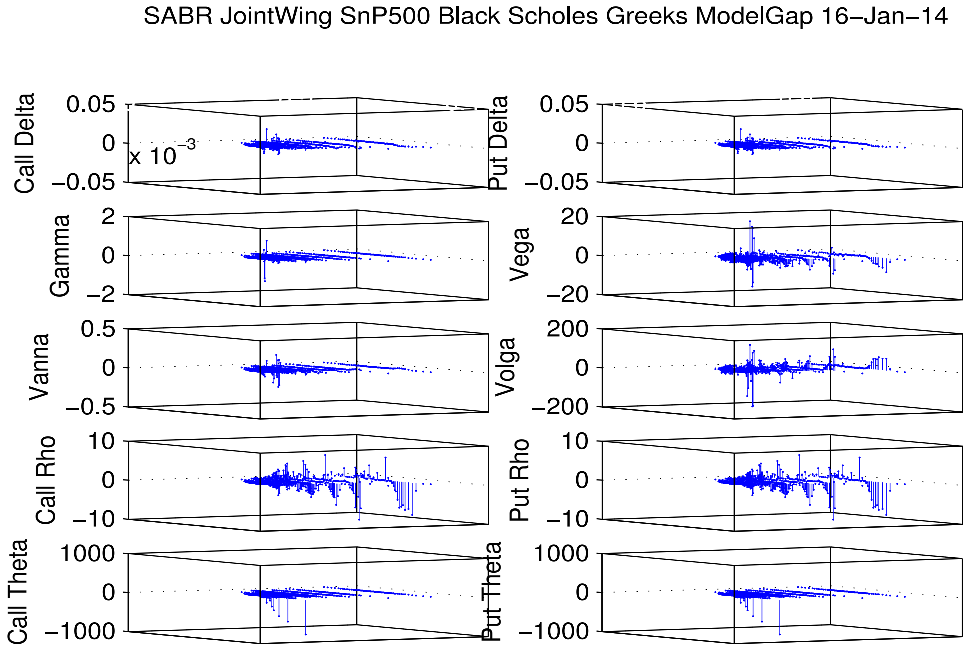
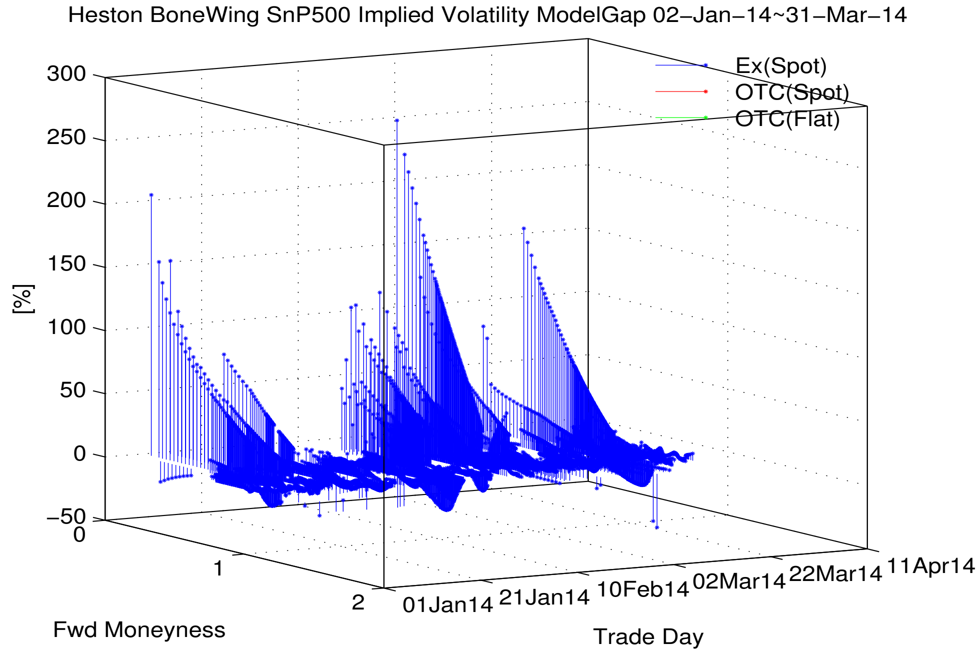


Figure B.72: SABR BoneWing SnP500 Price, Black Scholes Greeks ModelGap (16-Jan-14)



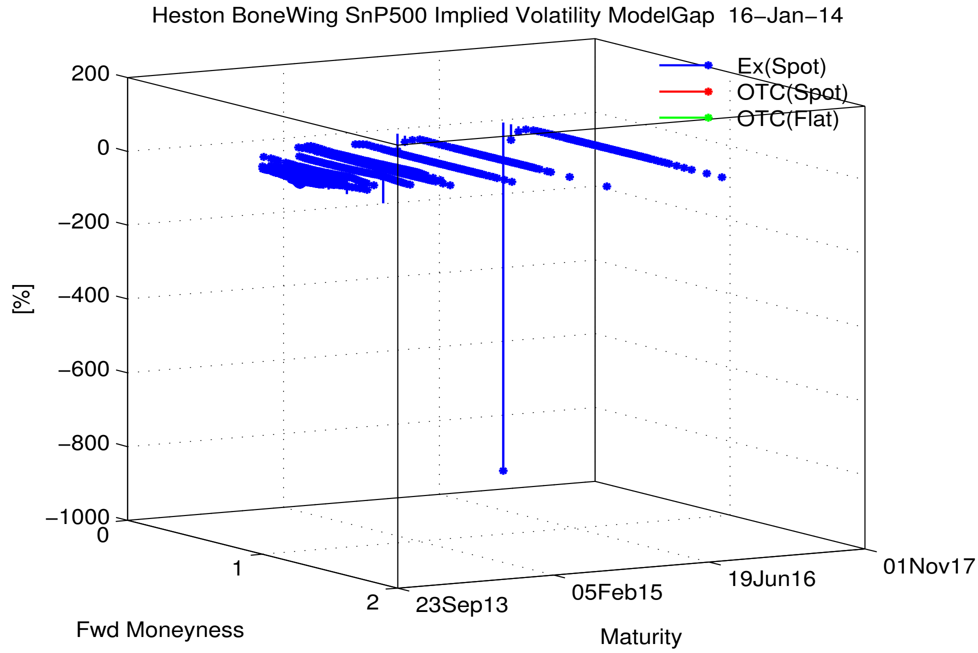
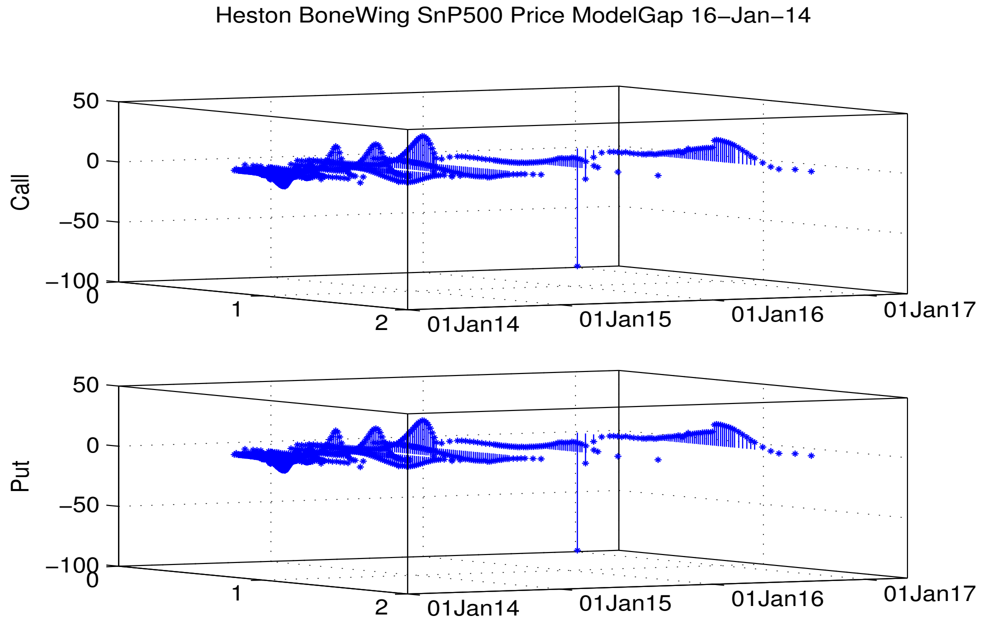


Figure B.73: Heston BoneWing SnP500 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



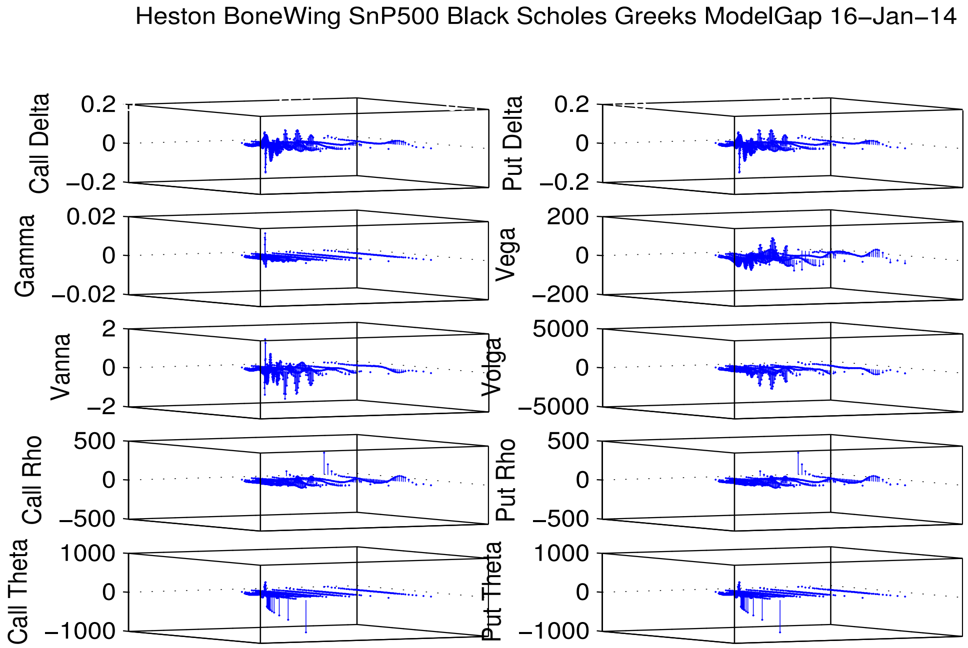
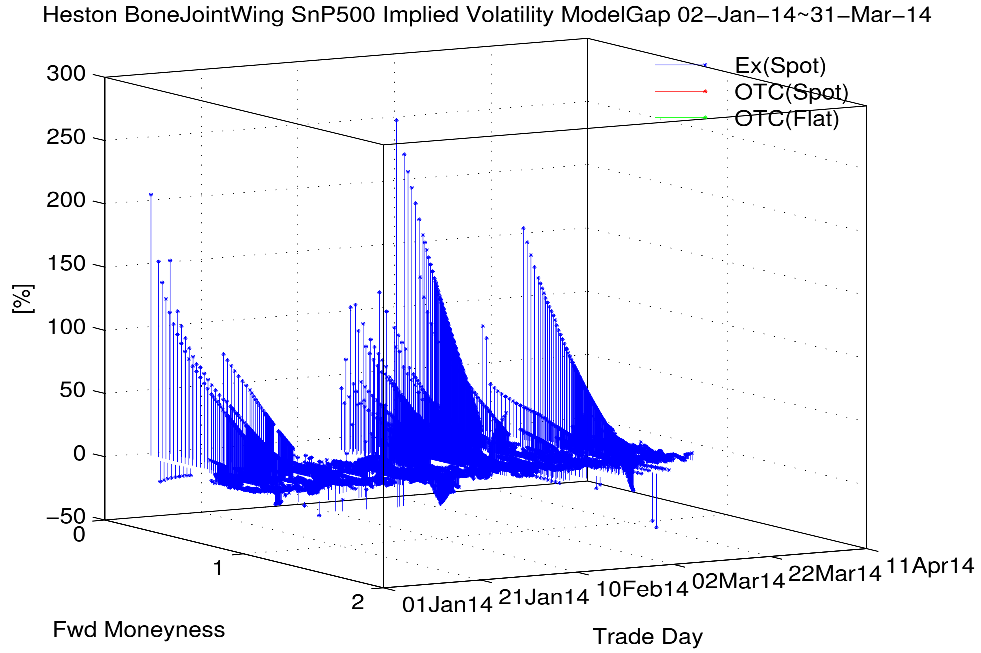


Figure B.74: Heston BoneWing SnP500 Price, Black Scholes Greeks ModelGap (16-Jan-14)



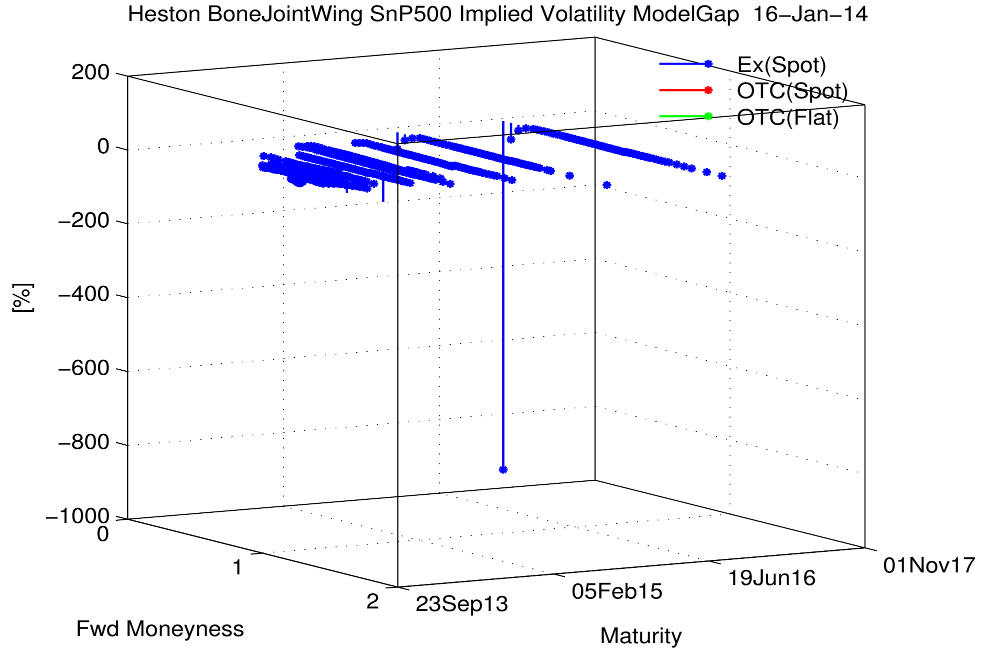


Figure B.75: Heston BoneWing SnP500 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)

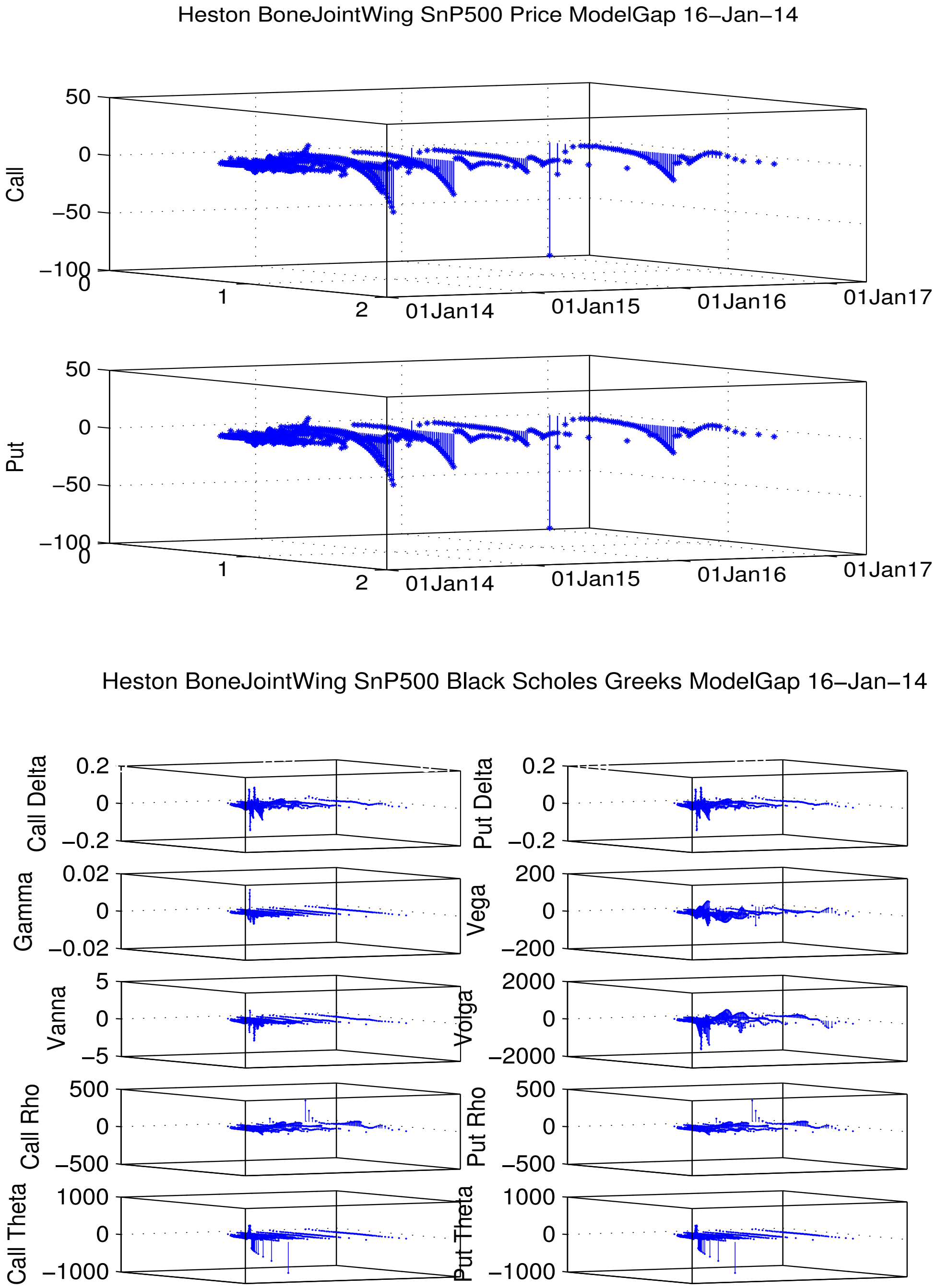
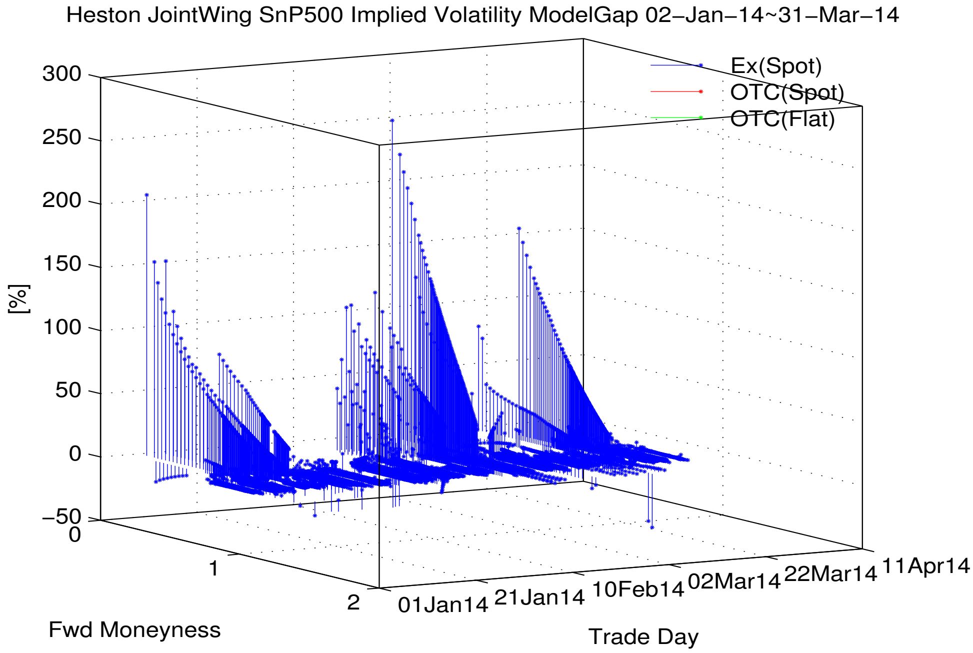


Figure B.76: Heston BoneWing SnP500 Price, Black Scholes Greeks ModelGap (16-Jan-14)



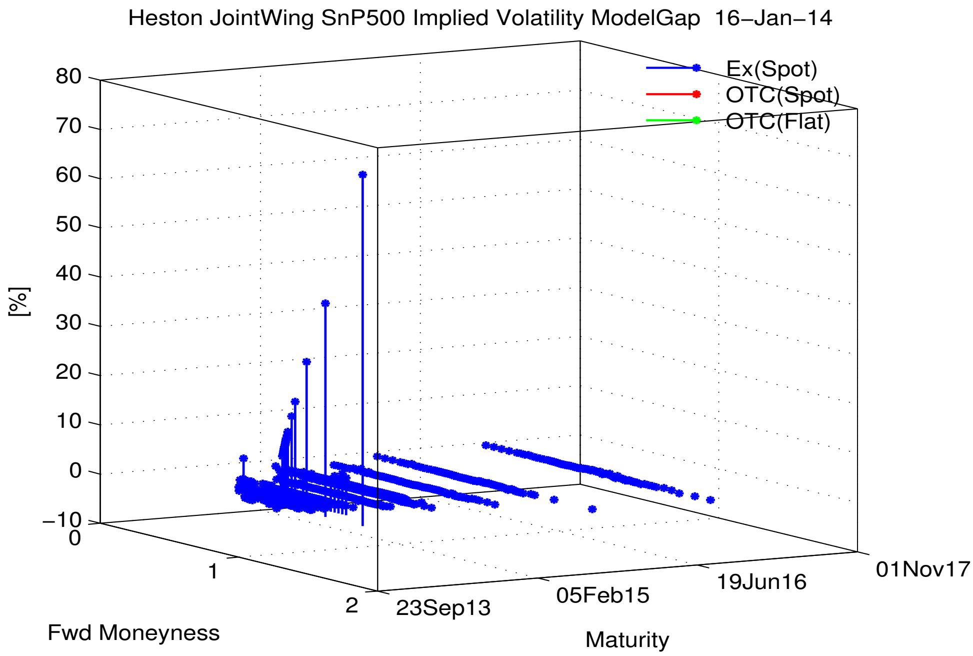
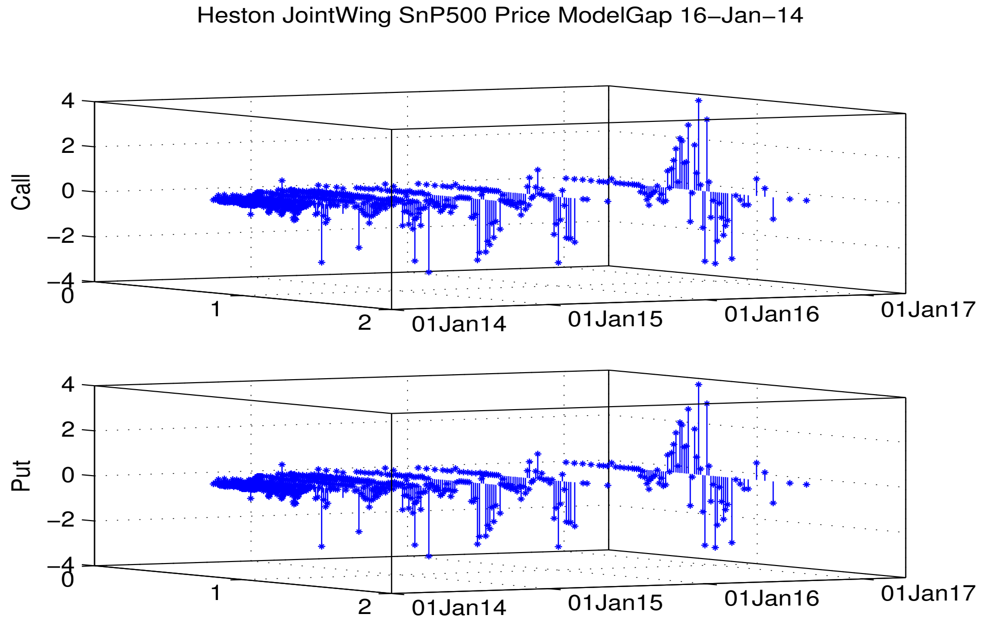


Figure B.77: Heston BoneWing SnP500 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



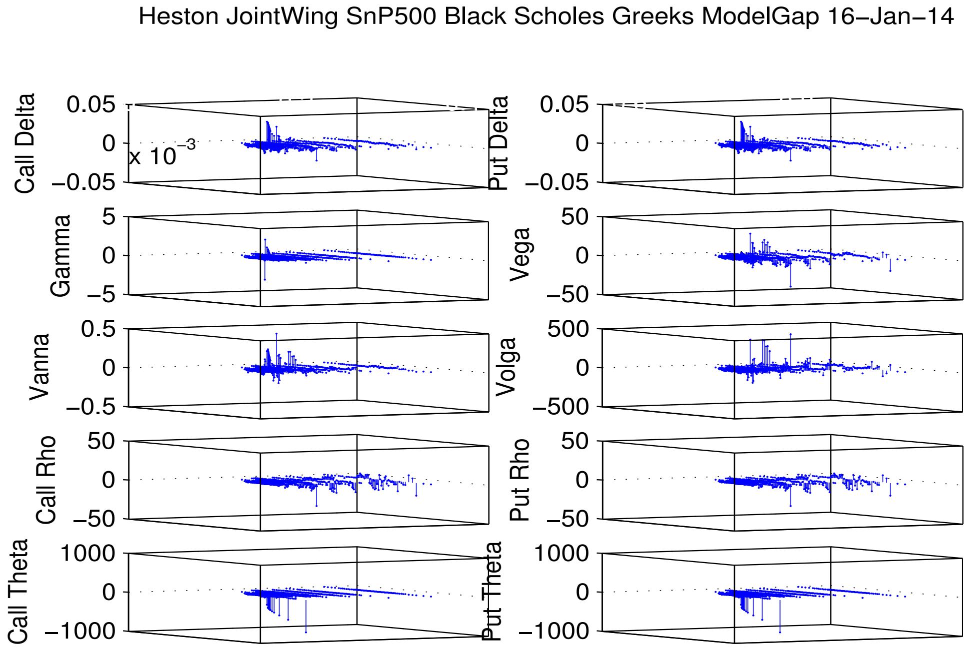


Figure B.78: Heston BoneWing SnP500 Price, Black Scholes Greeks ModelGap (16-Jan-14)

Table B.36: Root Mean Squared SnP500 Implied Volatility ModelGap (02-Jan-14~31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
8.670E-02	7.432E-02	5.187E-02	1.724E-01	1.711E-01	3.791E-02

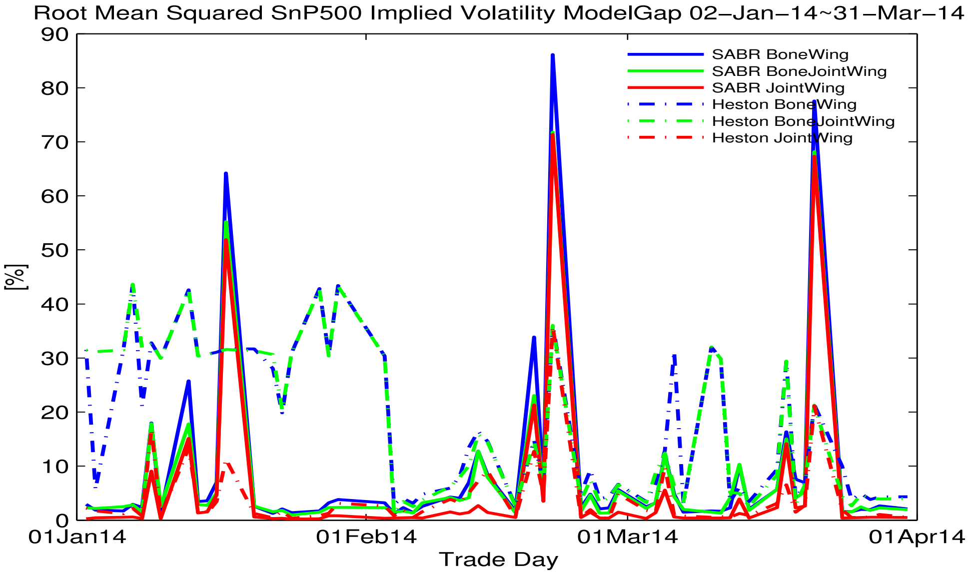


Figure B.79: Root Mean Squared SnP500 Implied Volatility ModelGap (02-Jan-14~31-Mar-14)

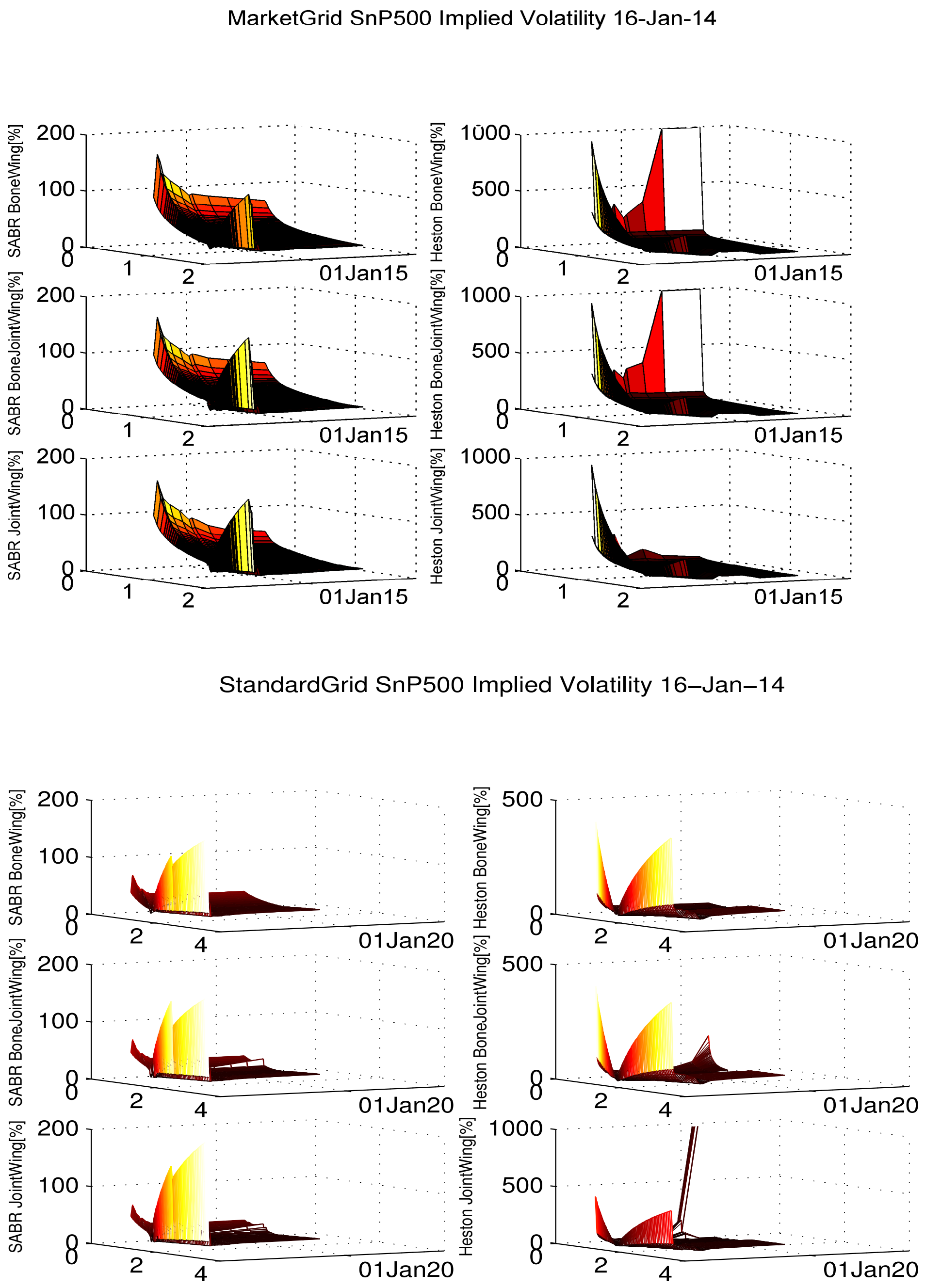
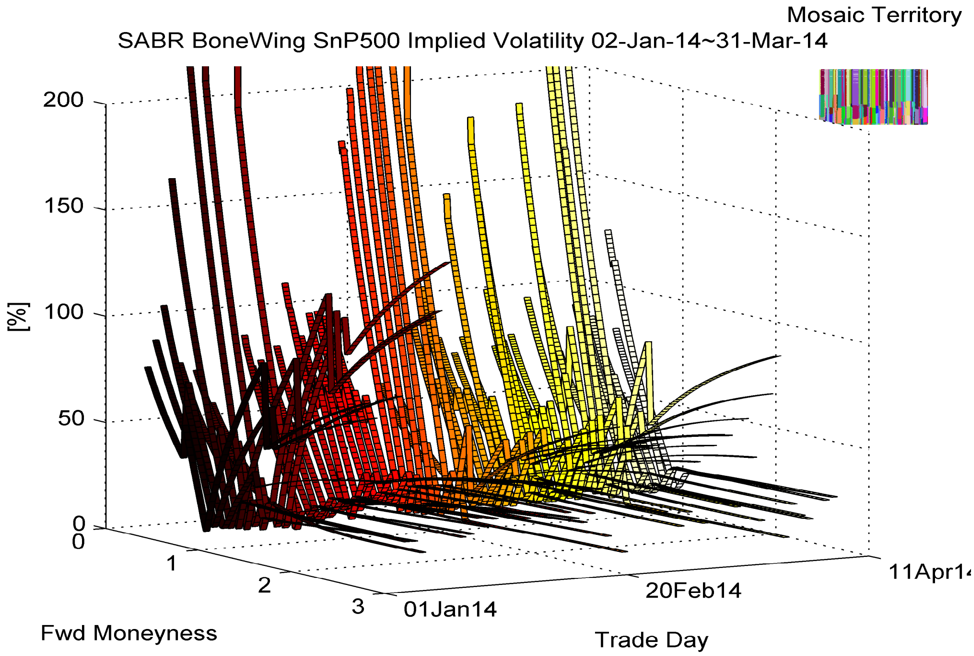


Figure B.80: SnP500 Market, Standard Grid Implied Volatility (16-Jan-14)



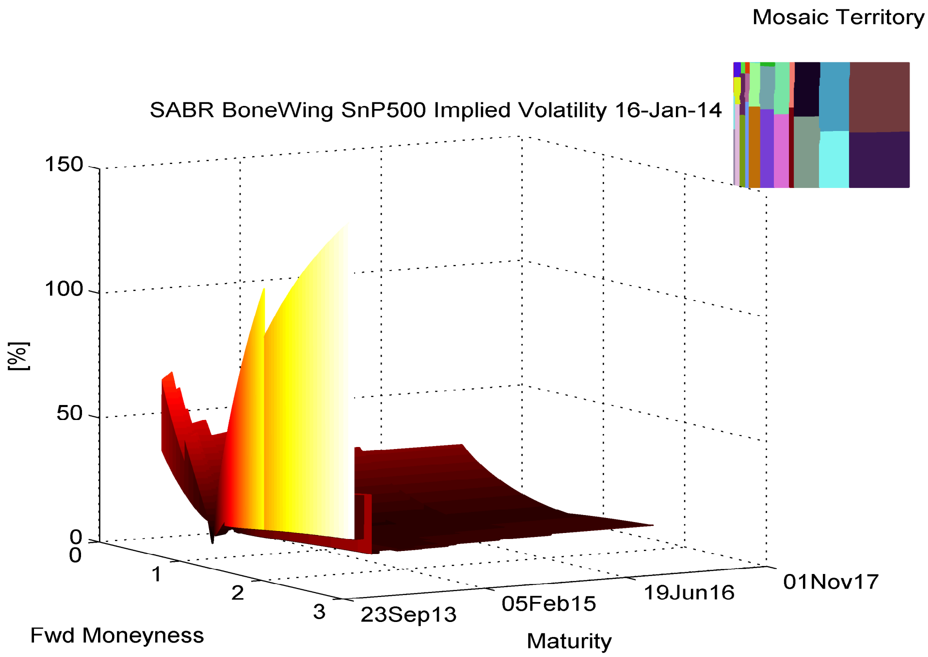


Figure B.81: SABR BoneWing SnP500 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)

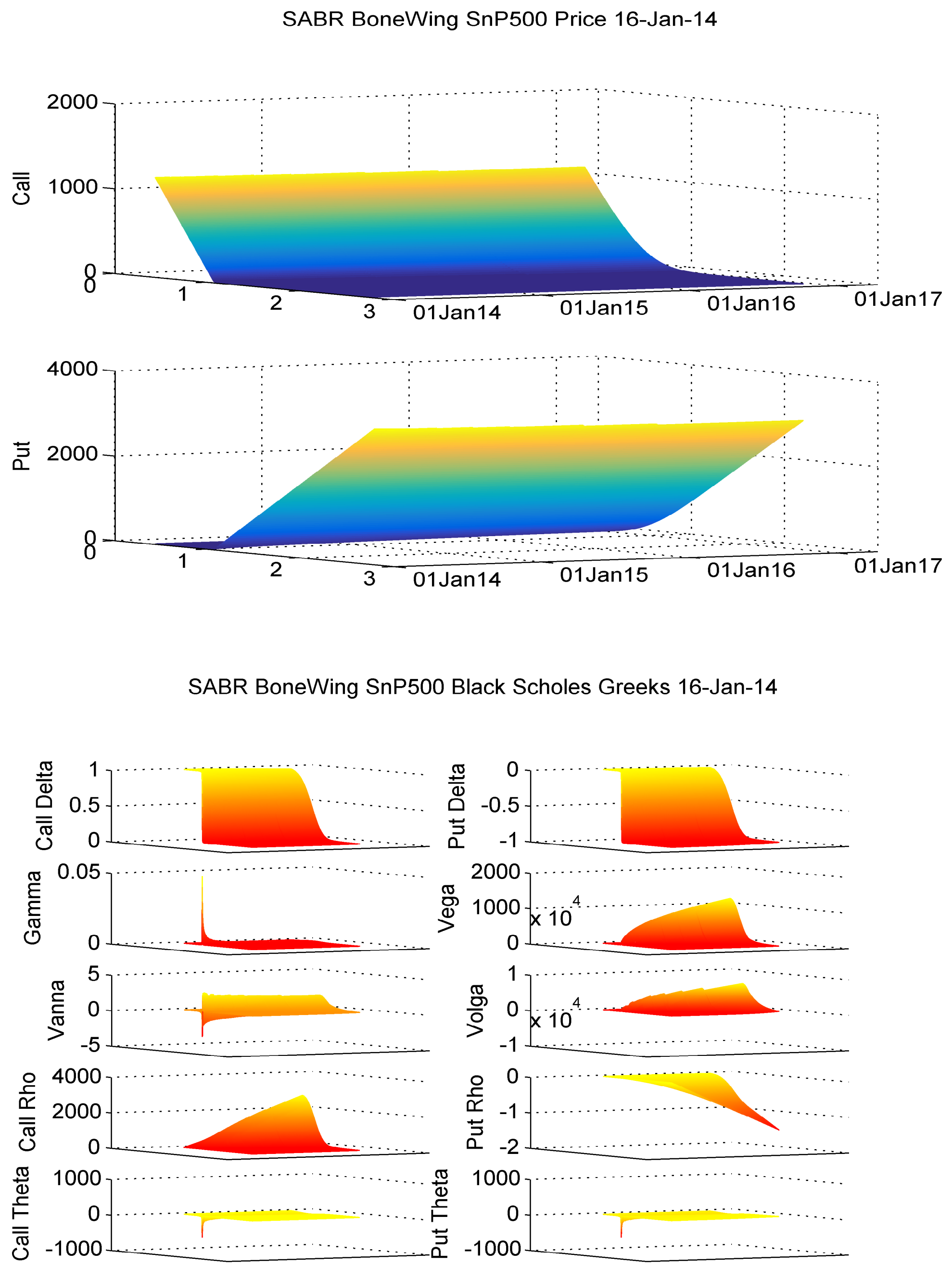
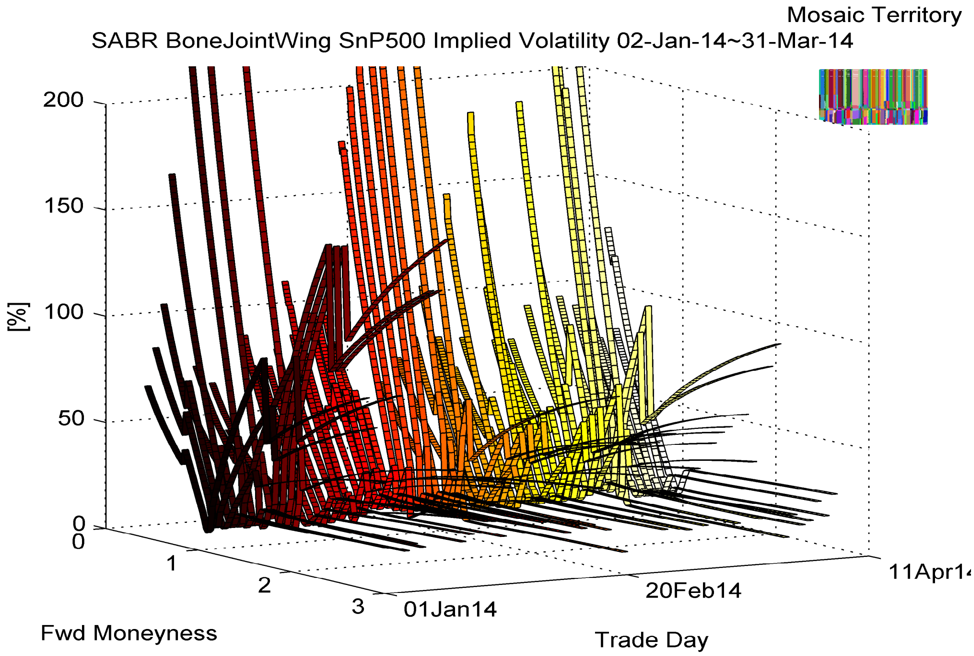


Figure B.82: SABR BoneWing SnP500 Price, Black Scholes Greeks (16-Jan-14)



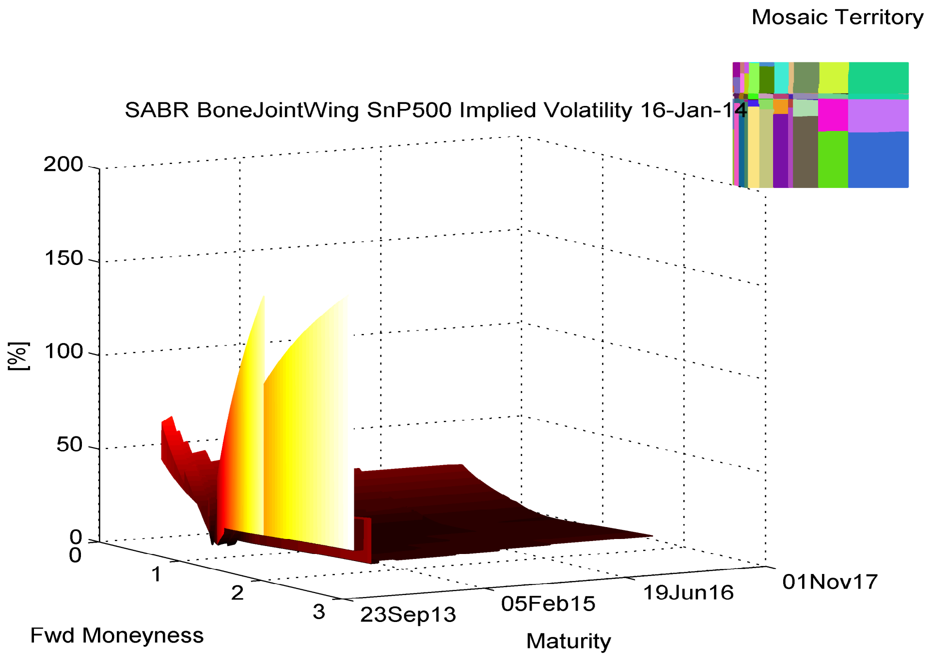
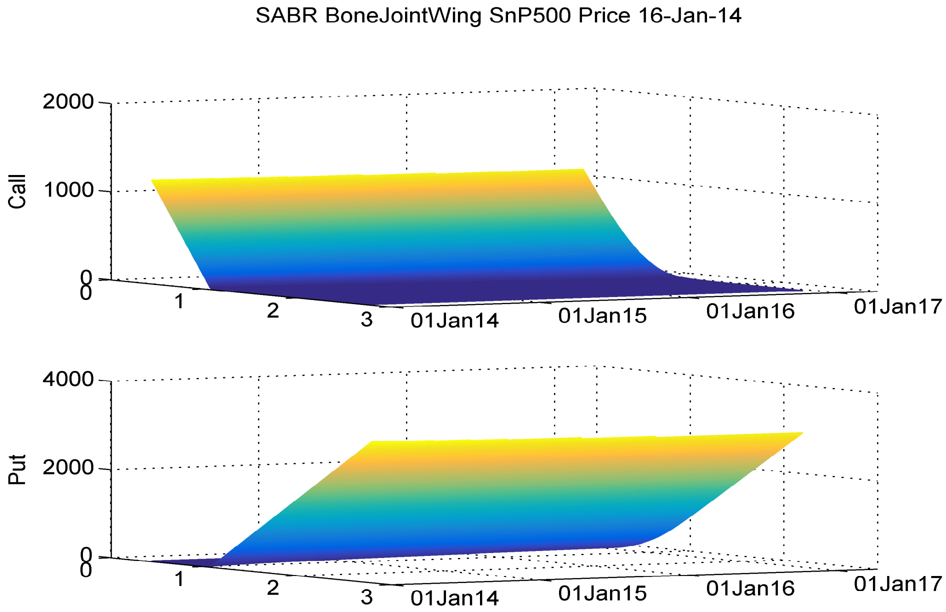


Figure B.83: SABR BoneWing SnP500 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



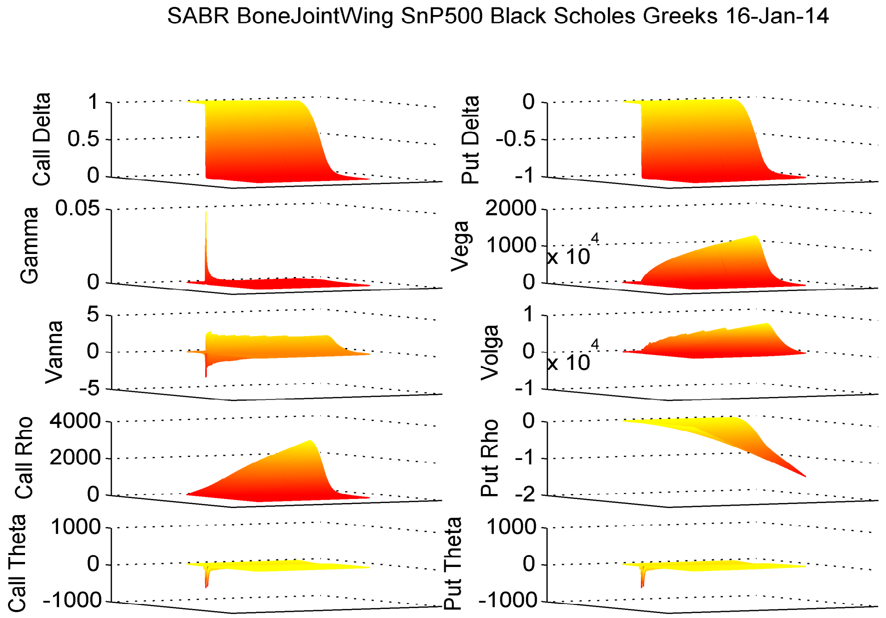
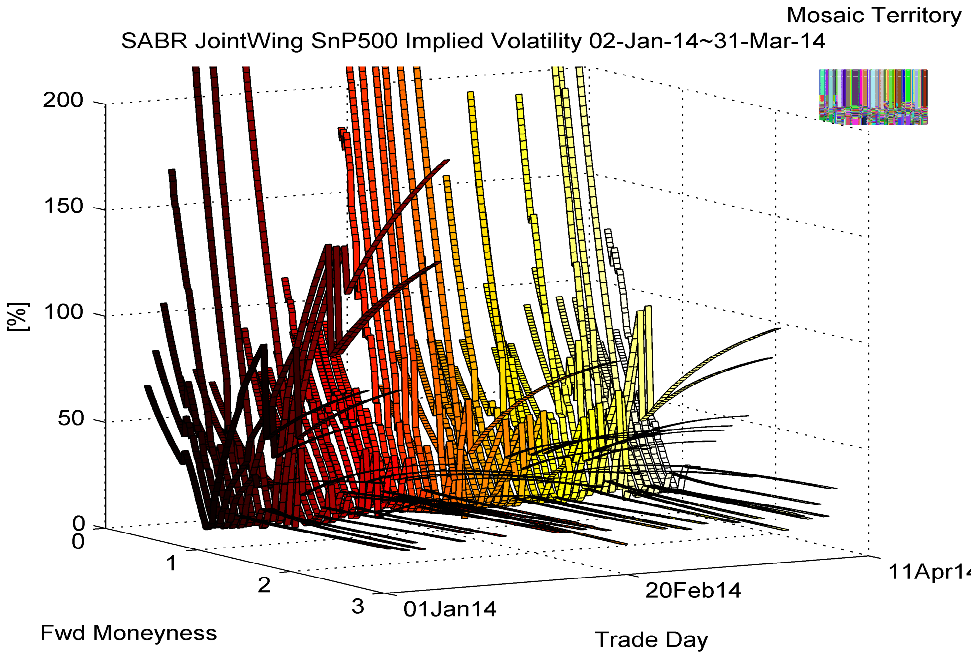


Figure B.84: SABR BoneWing SnP500 Price, Black Scholes Greeks (16-Jan-14)



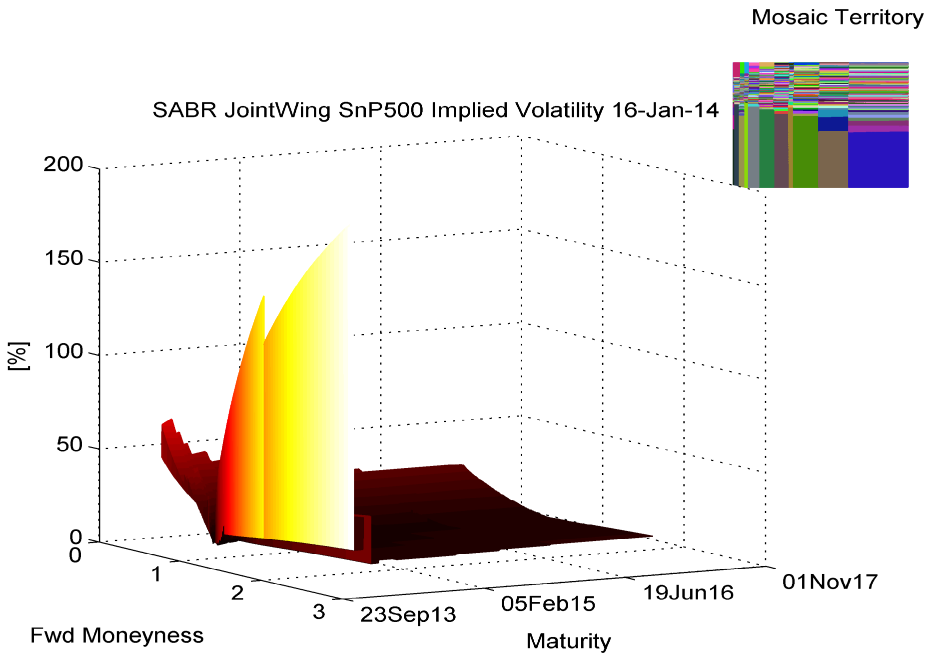


Figure B.85: SABR BoneWing SnP500 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)

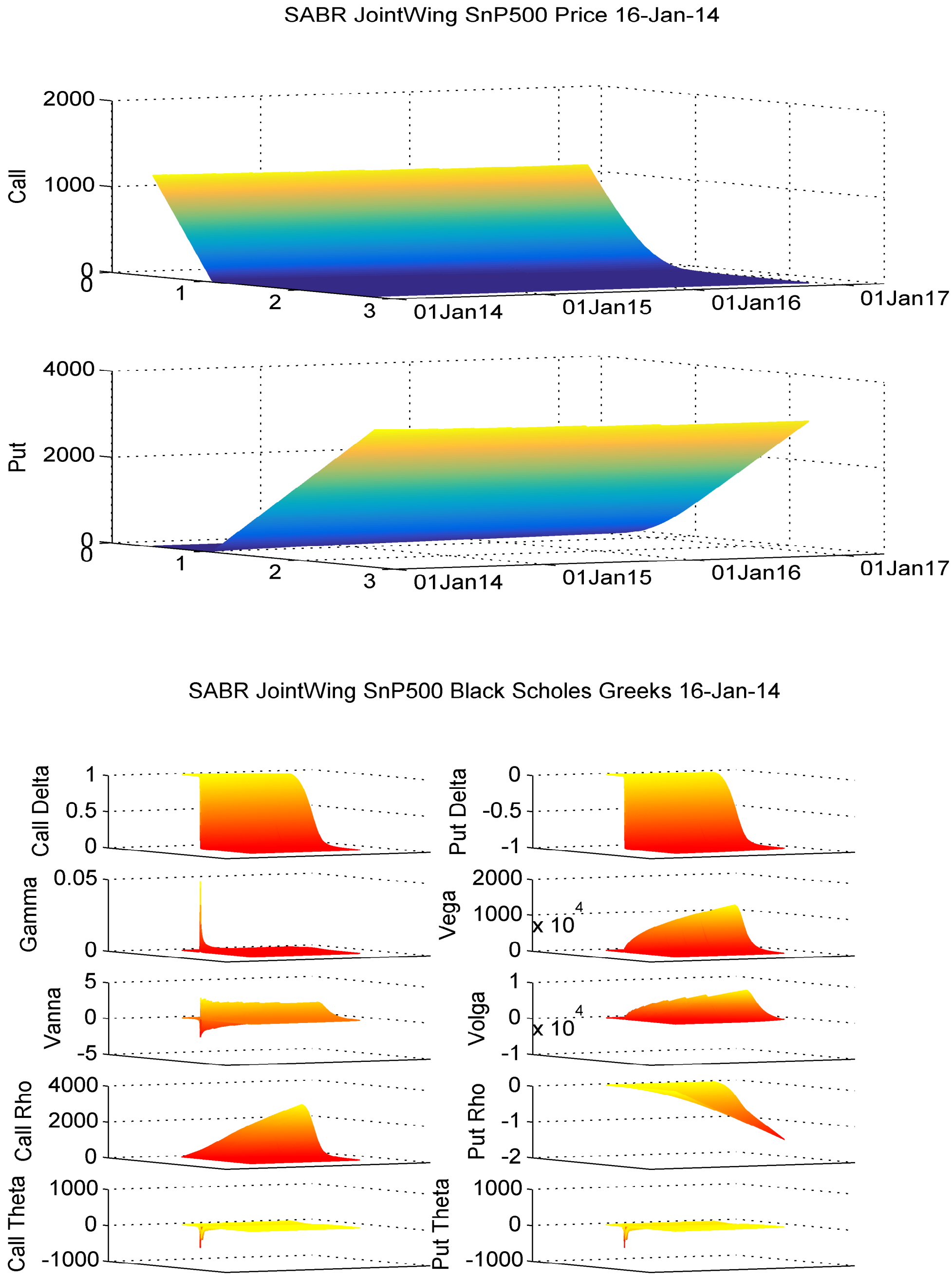
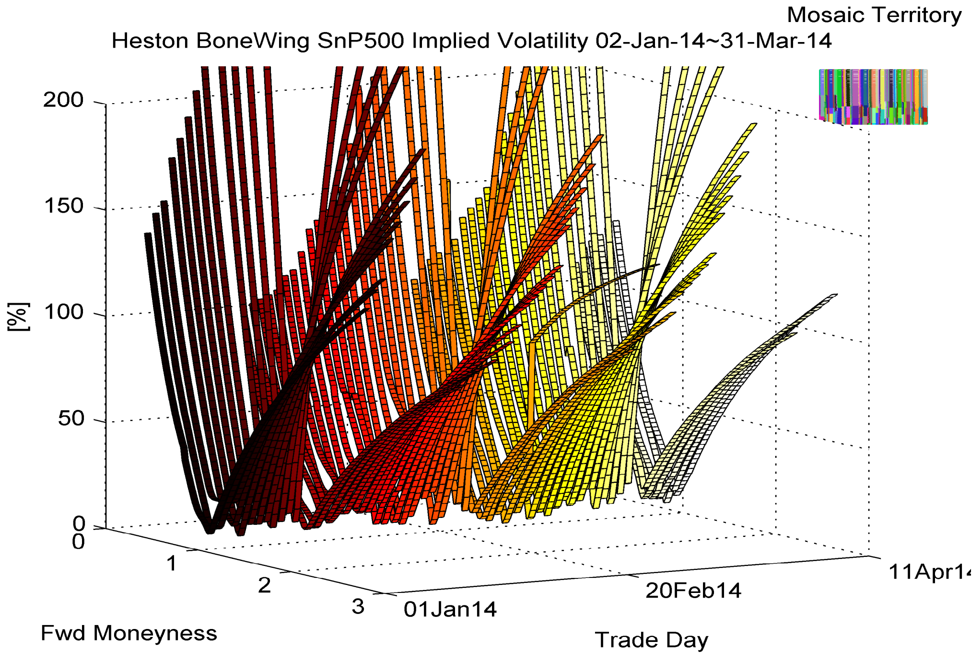


Figure B.86: SABR BoneWing SnP500 Price, Black Scholes Greeks (16-Jan-14)



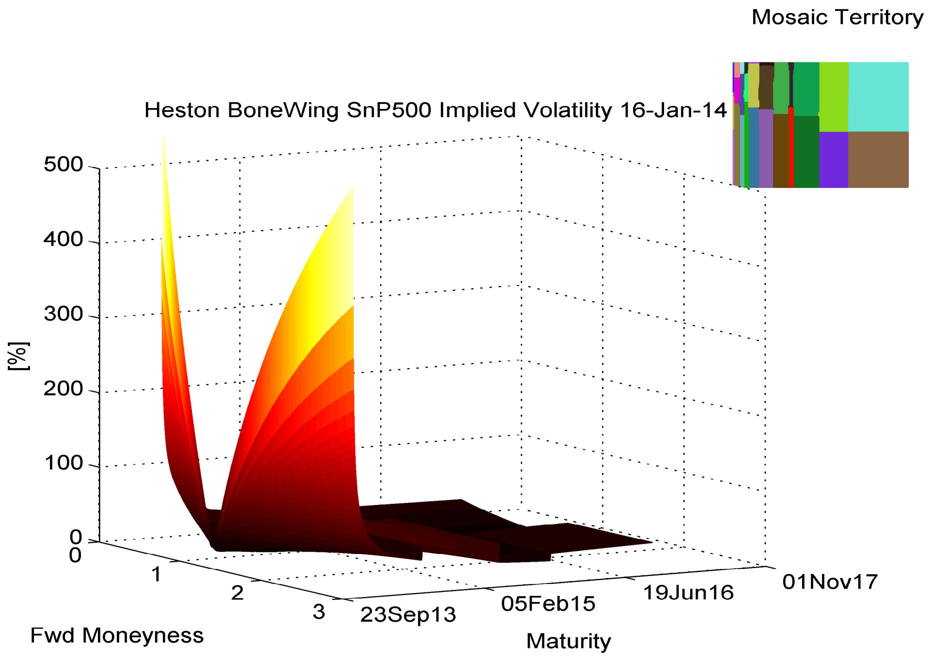
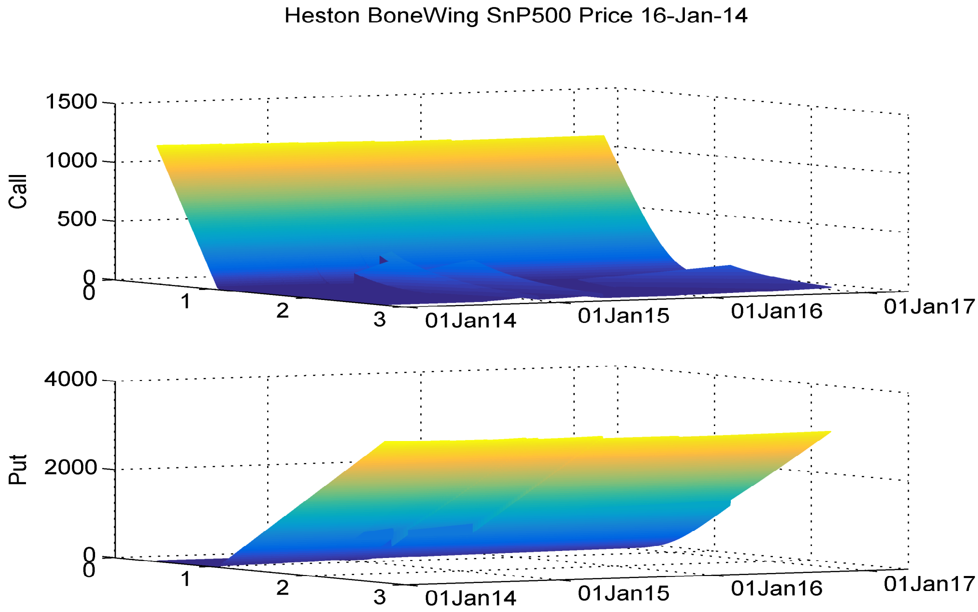


Figure B.87: Heston BoneWing SnP500 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



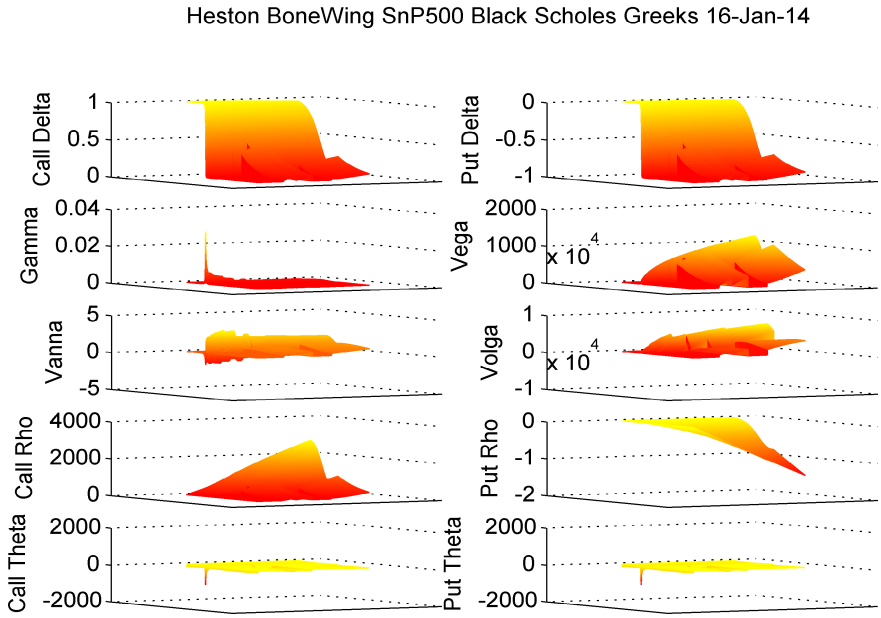
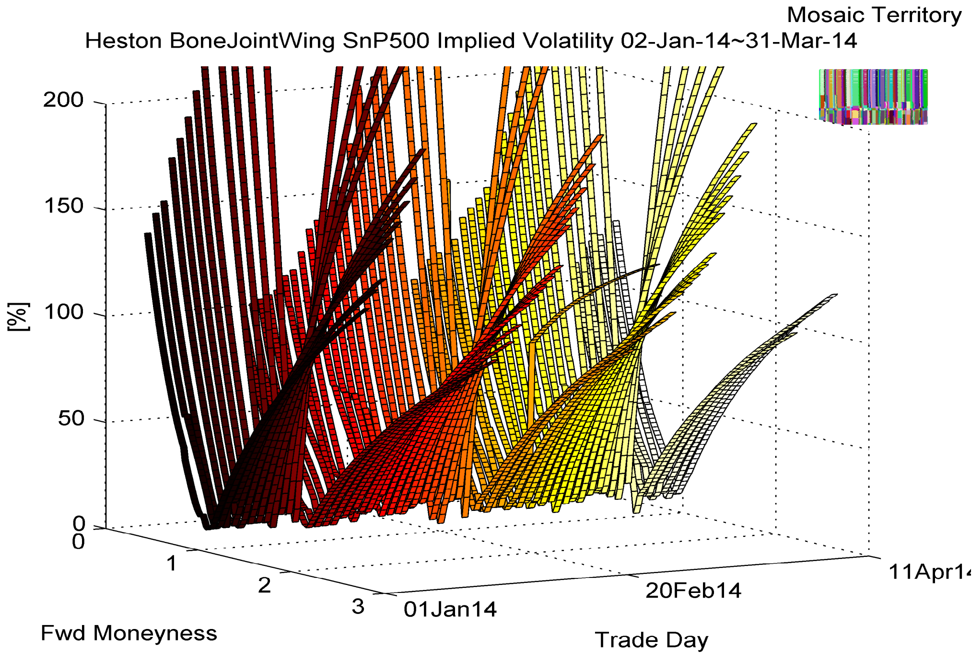


Figure B.88: Heston BoneWing SnP500 Price, Black Scholes Greeks (16-Jan-14)



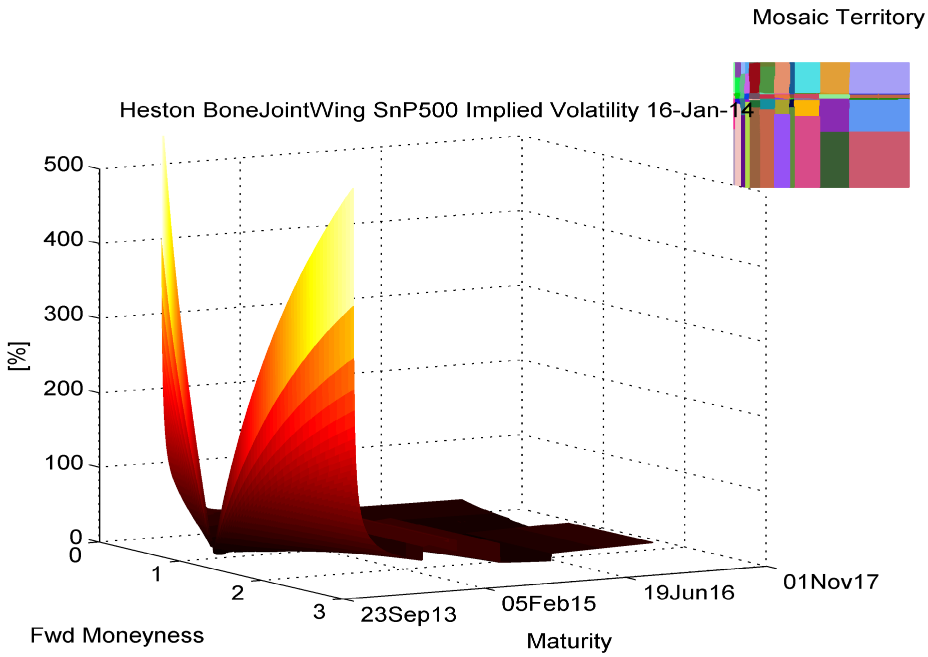
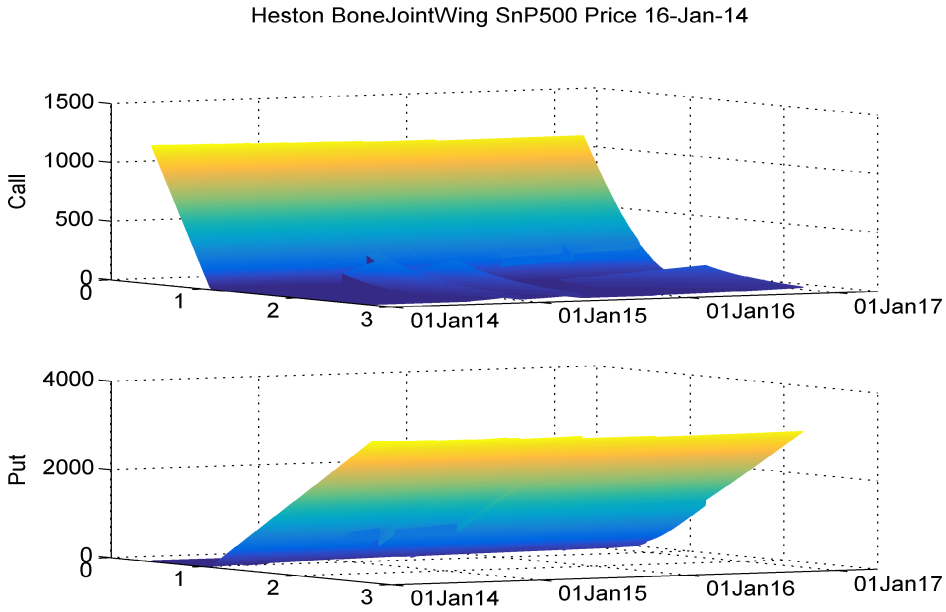


Figure B.89: Heston BoneWing SnP500 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



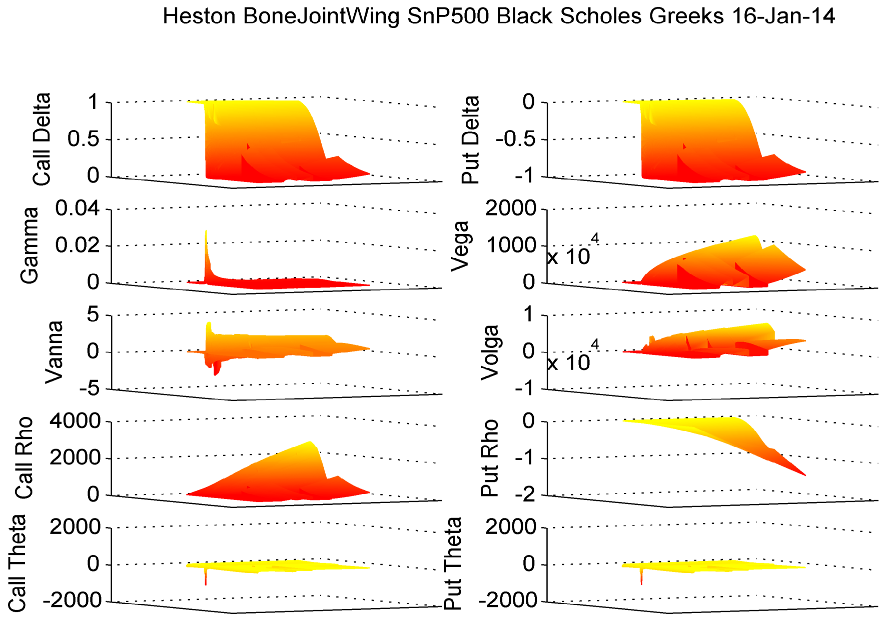
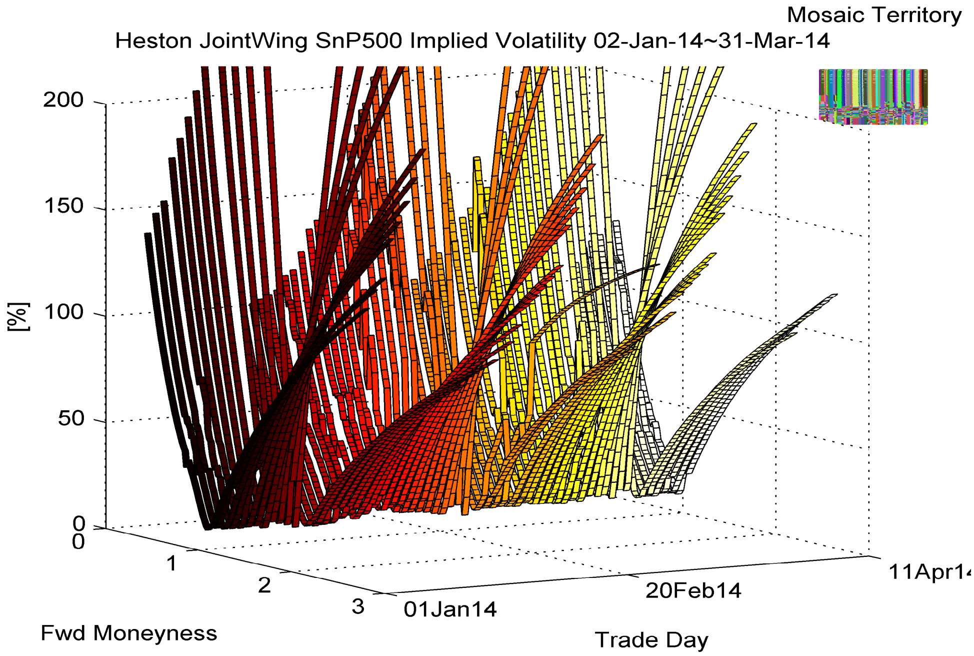


Figure B.90: Heston BoneWing SnP500 Price, Black Scholes Greeks (16-Jan-14)



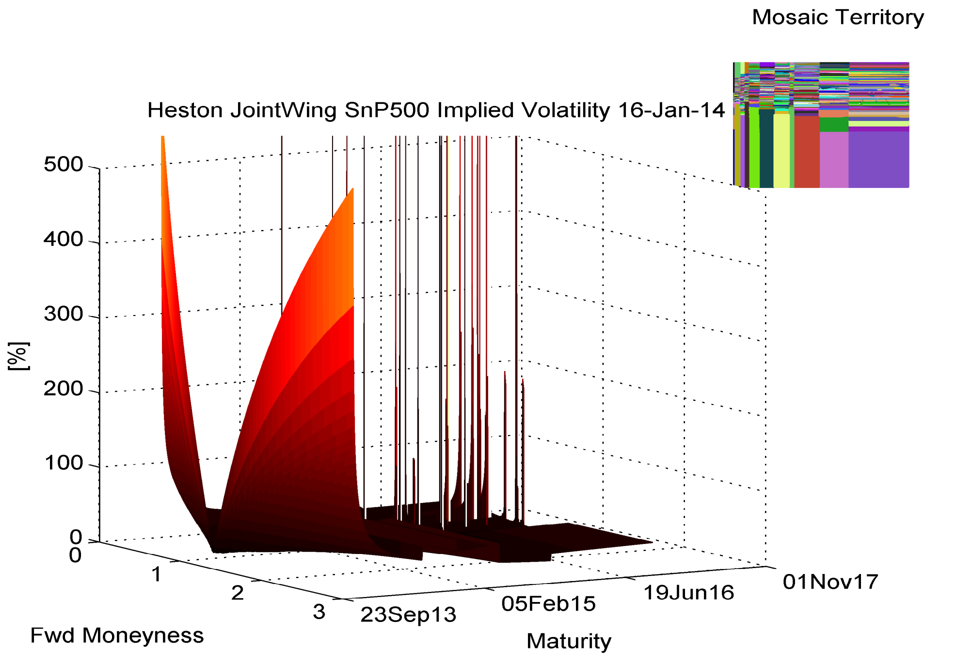


Figure B.91: Heston BoneWing SnP500 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)

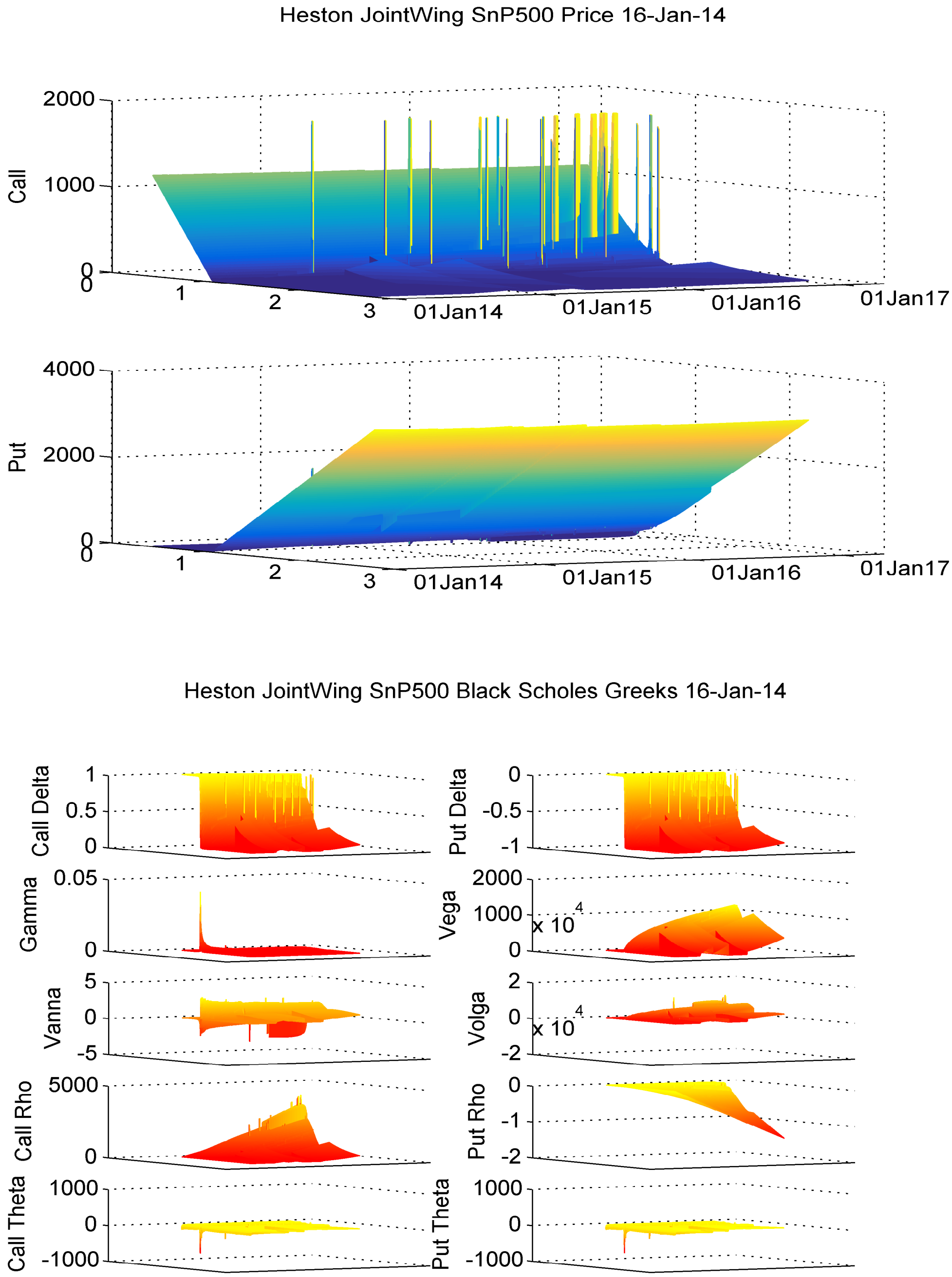
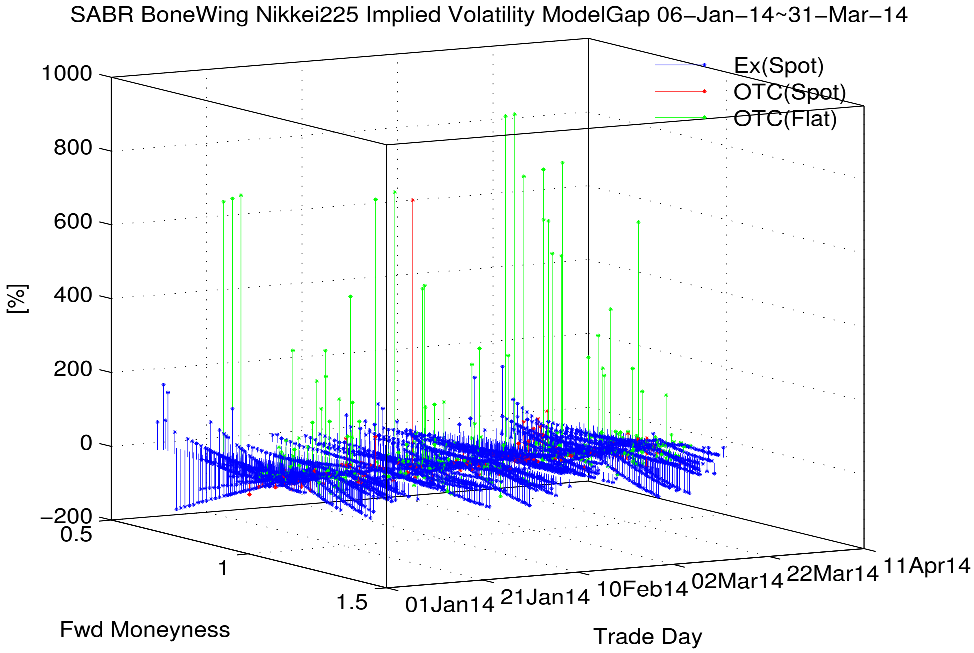


Figure B.92: Heston BoneWing SnP500 Price, Black Scholes Greeks (16-Jan-14)



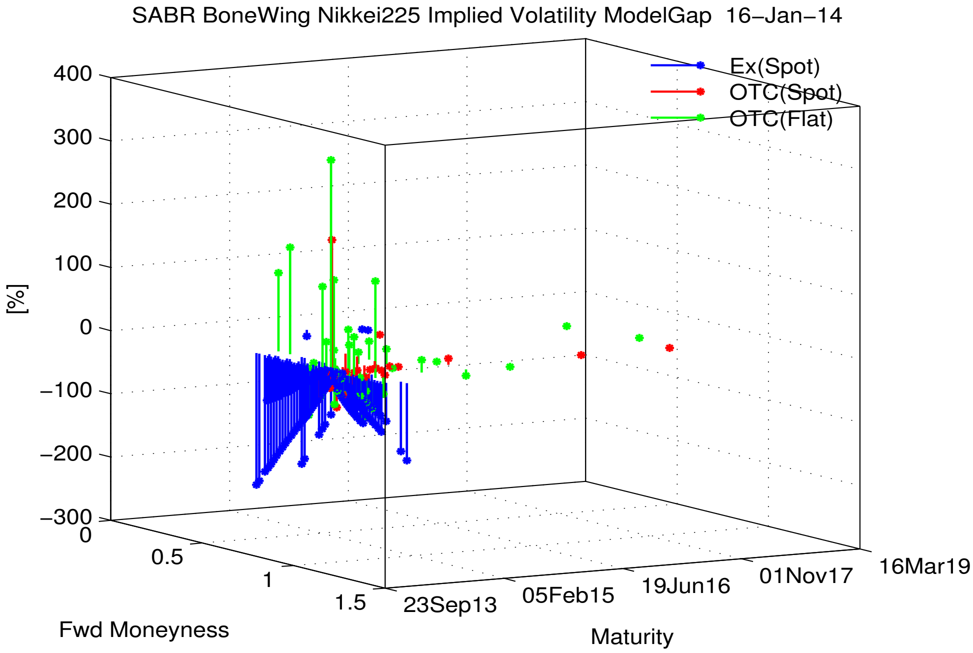


Figure B.93: SABR BoneWing Nikkei225 Implied Volatility (Nearest) Model-Gap (03-Jan-14∼31-Mar-14), (16-Jan-14)

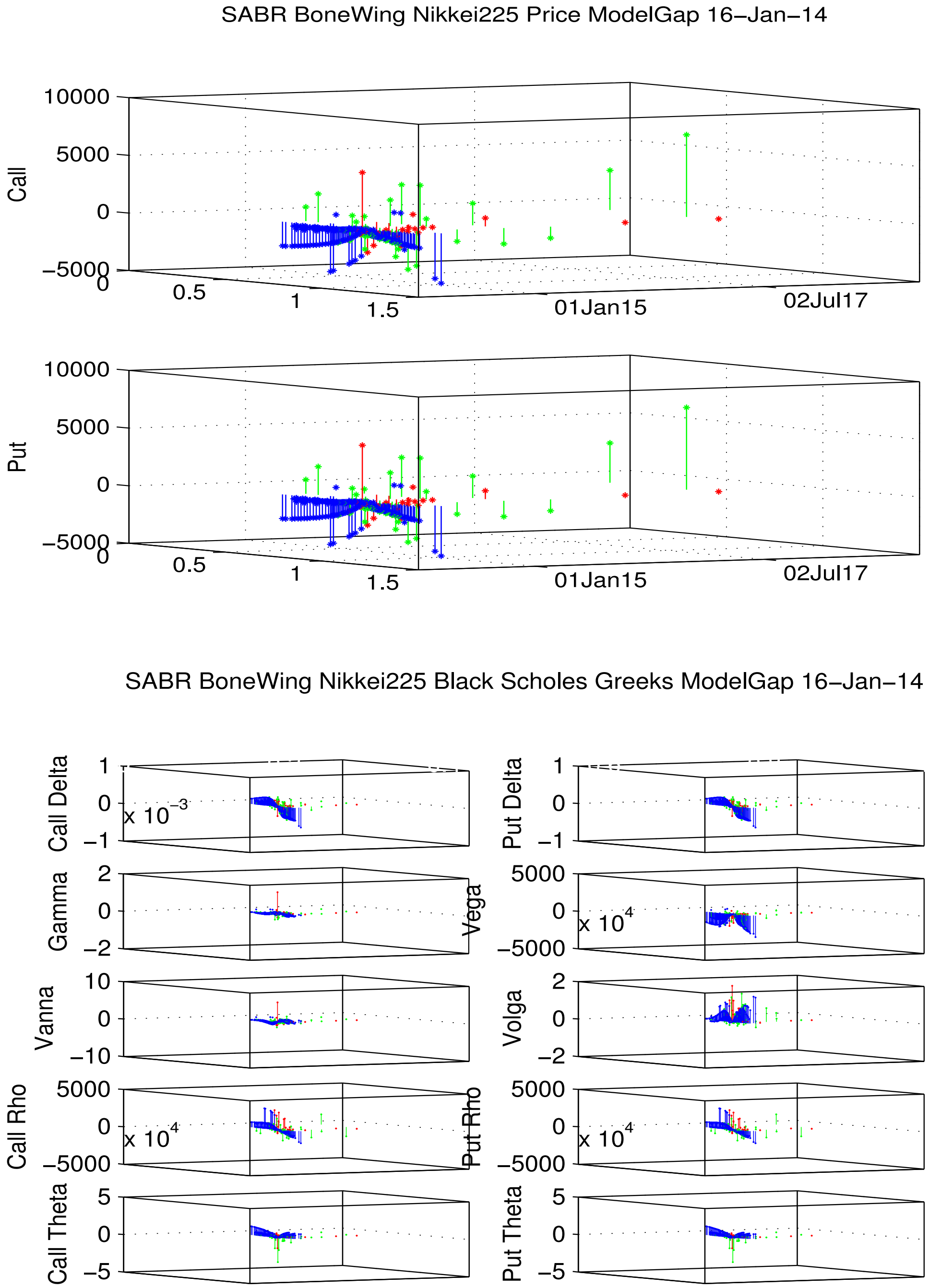
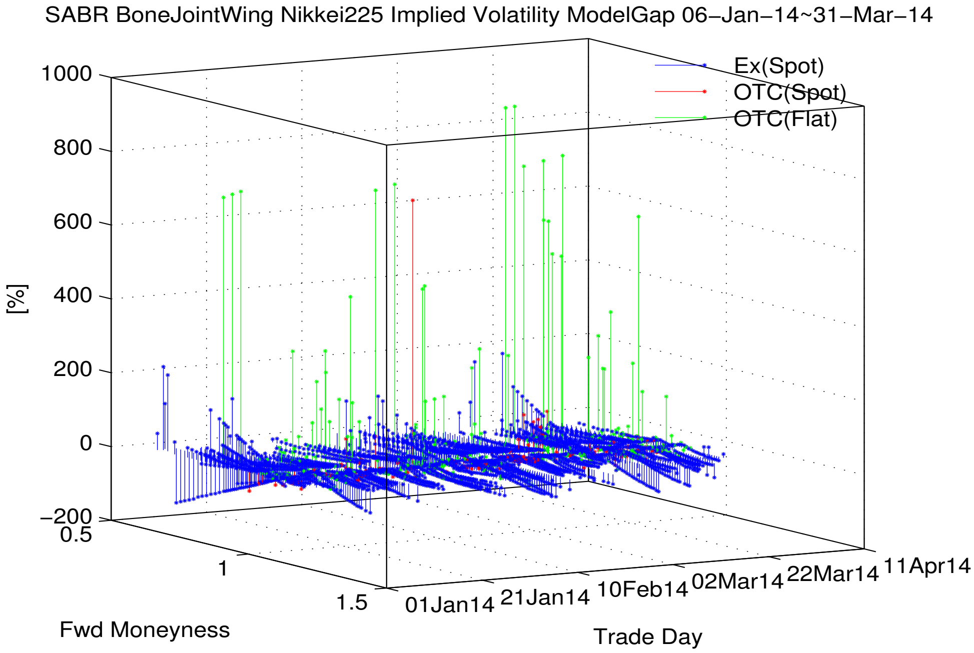


Figure B.94: SABR BoneWing Nikkei225 Price, Black Scholes Greeks ModelGap (16-Jan-14)



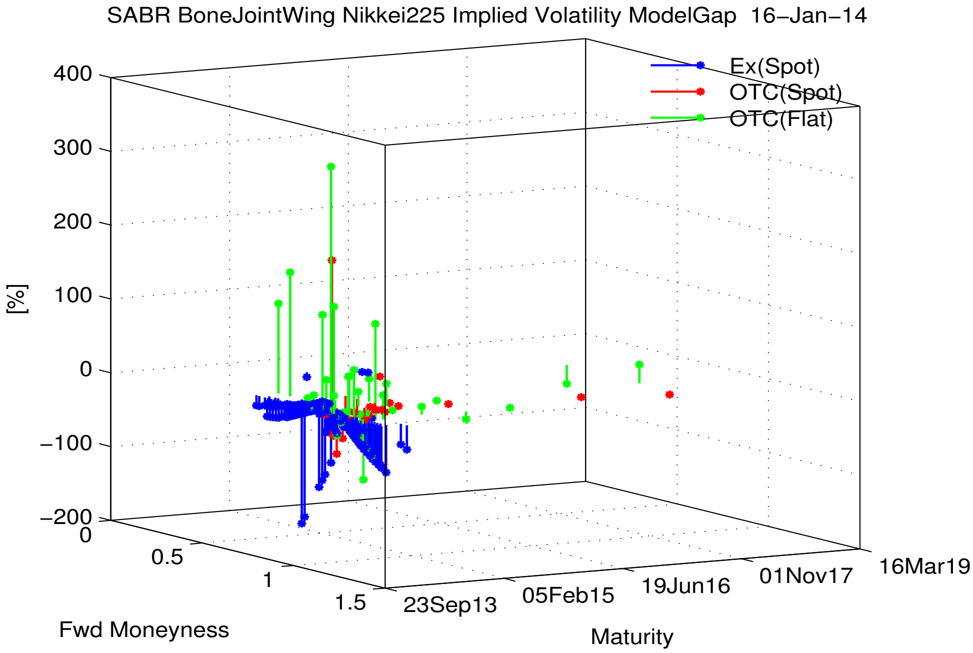


Figure B.95: SABR BoneWing Nikkei225 Implied Volatility (Nearest) Model-Gap (03-Jan-14∼31-Mar-14), (16-Jan-14)

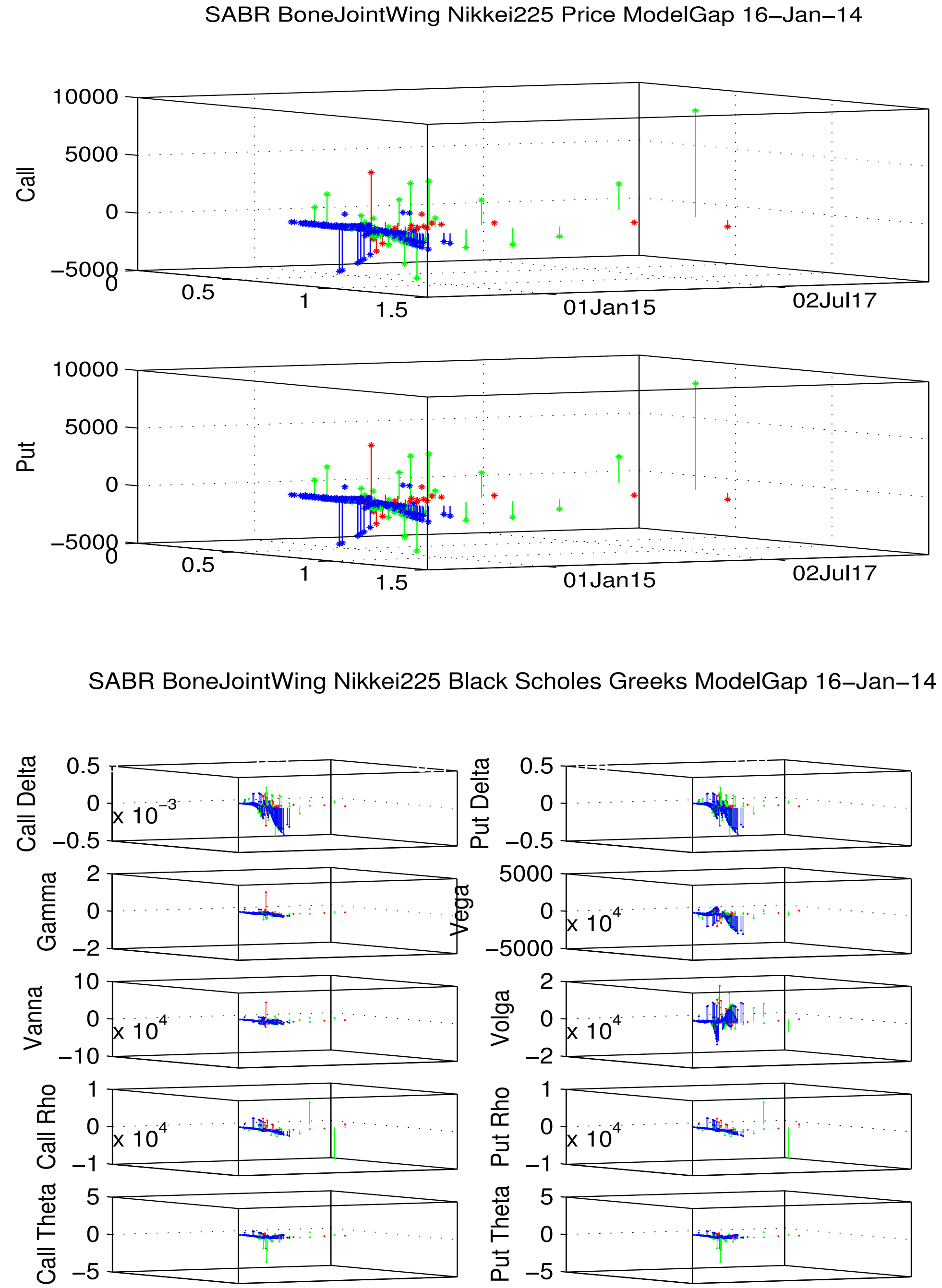


Figure B.96: SABR BoneWing Nikkei225 Price, Black Scholes Greeks ModelGap (16-Jan-14)

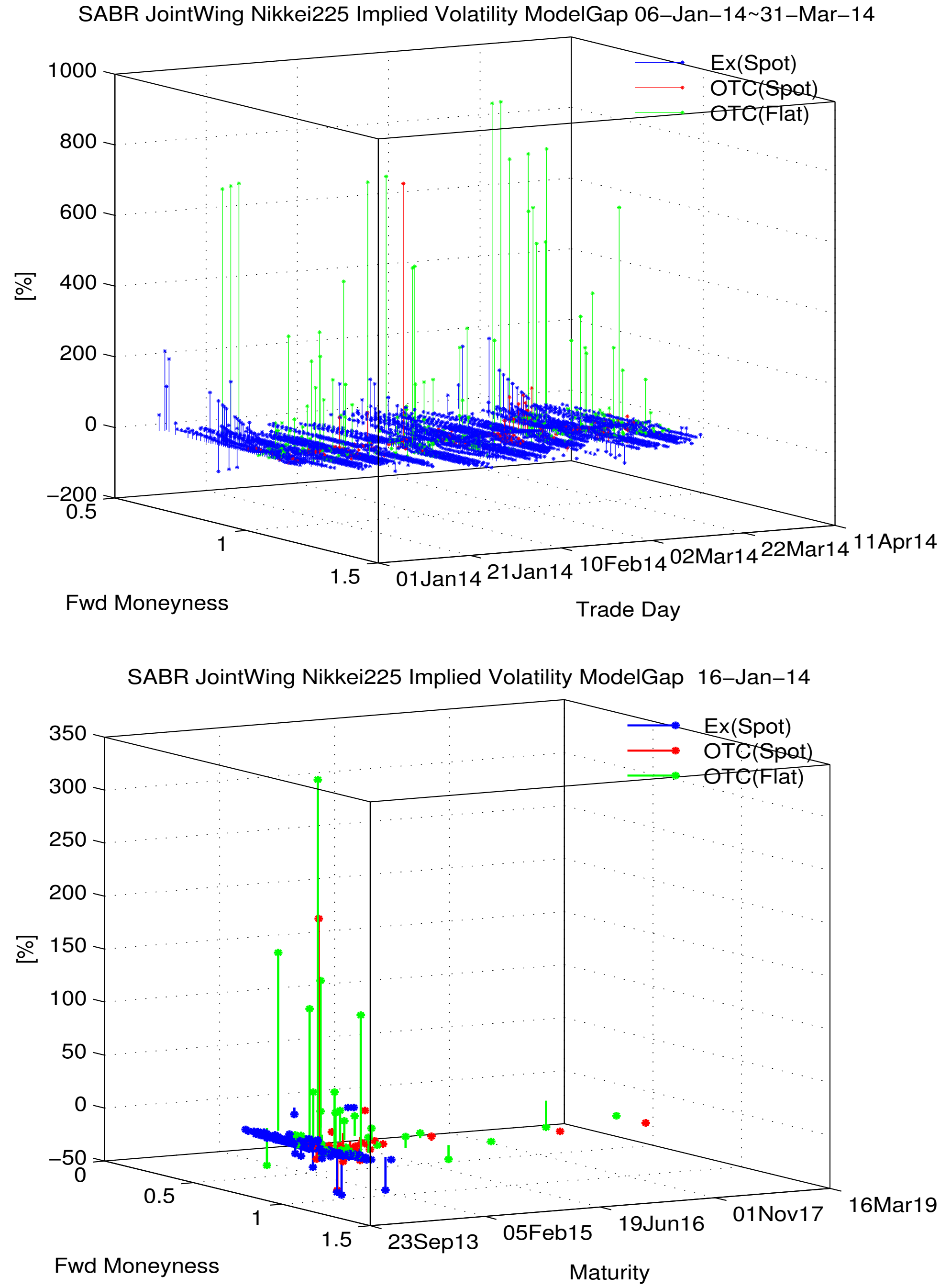
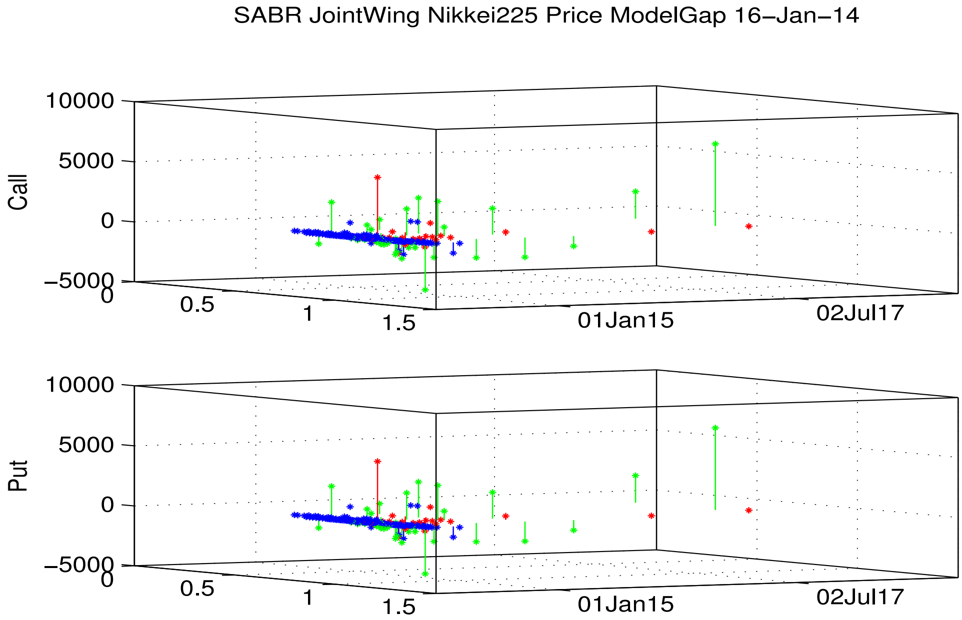


Figure B.97: SABR BoneWing Nikkei225 Implied Volatility (Nearest) Model-Gap (03-Jan-14~31-Mar-14), (16-Jan-14)



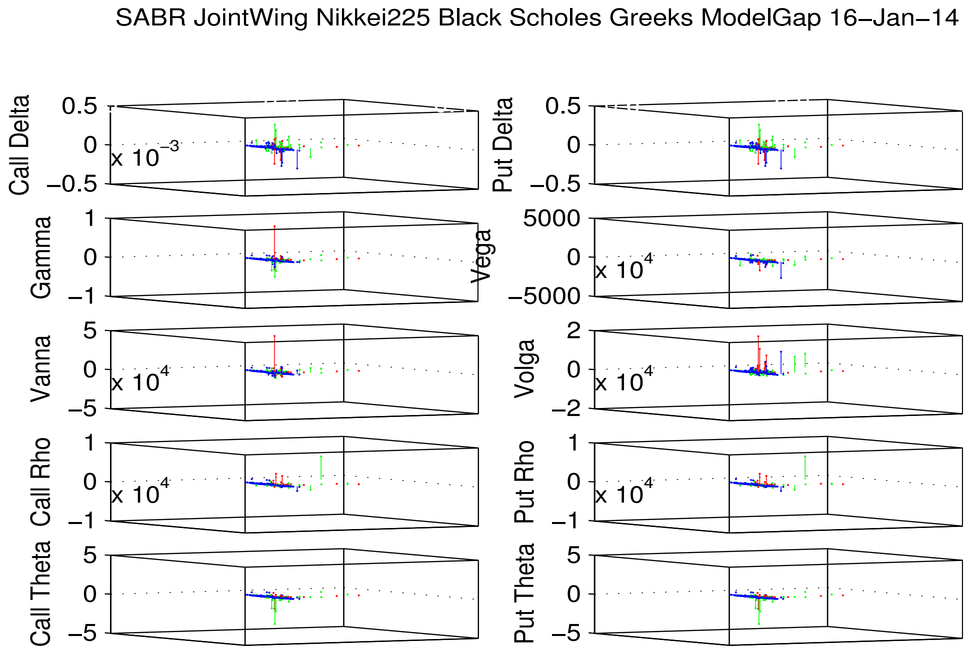
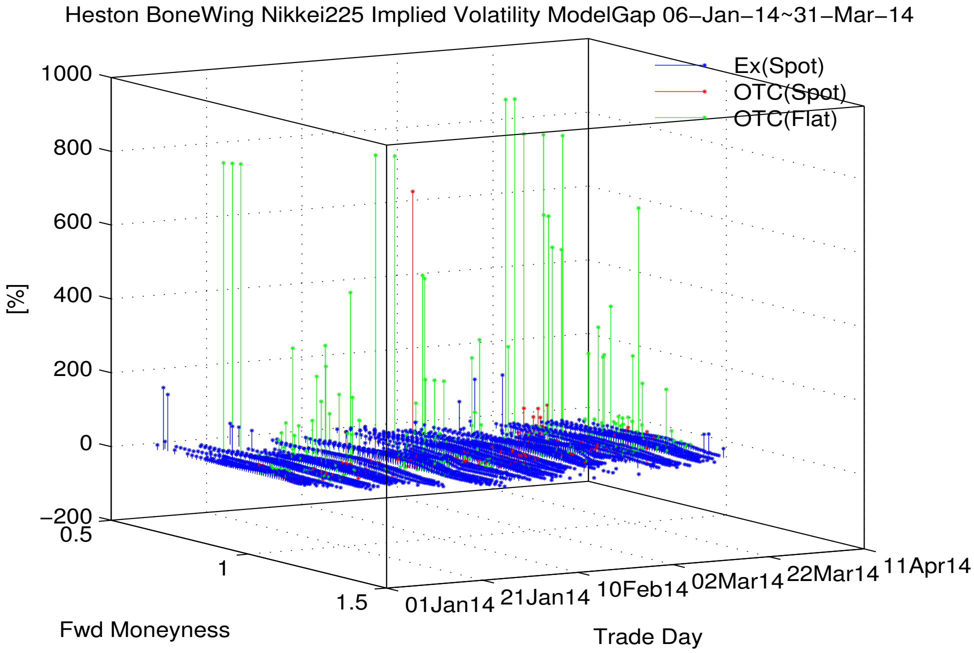


Figure B.98: SABR BoneWing Nikkei225 Price, Black Scholes Greeks Model-Gap (16-Jan-14)



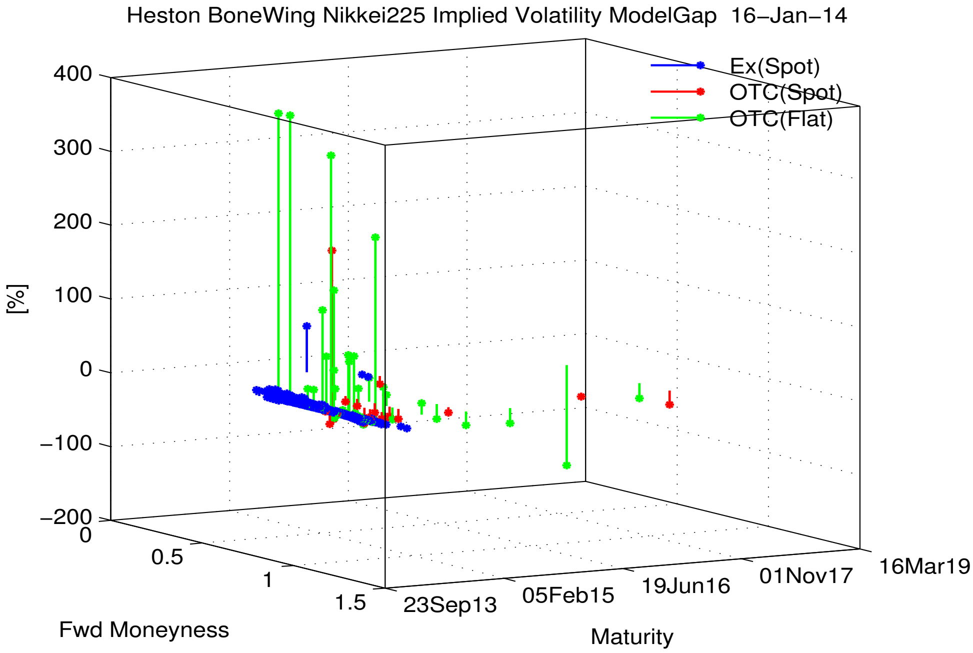
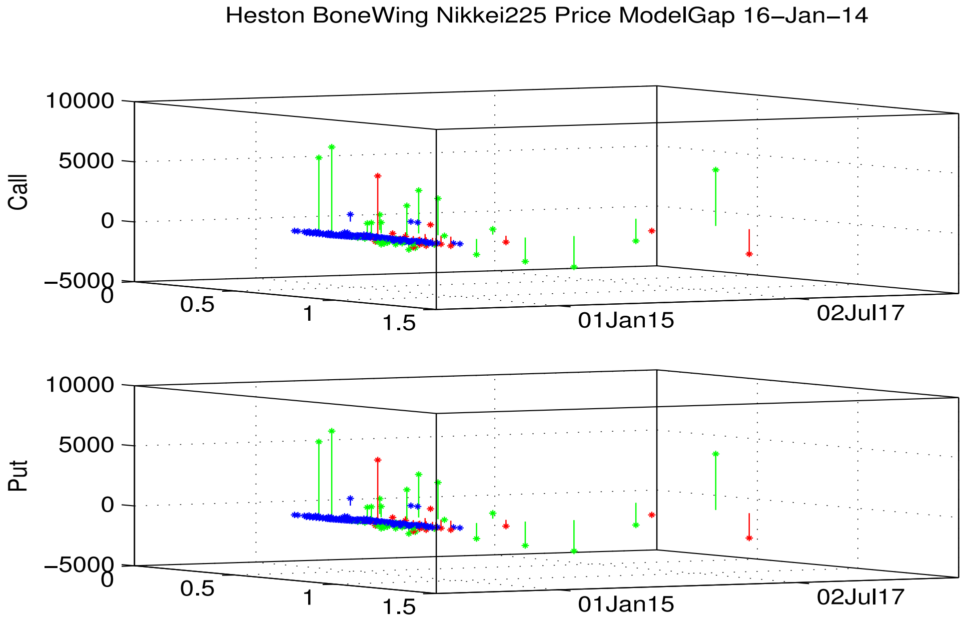


Figure B.99: Heston BoneWing Nikkei225 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



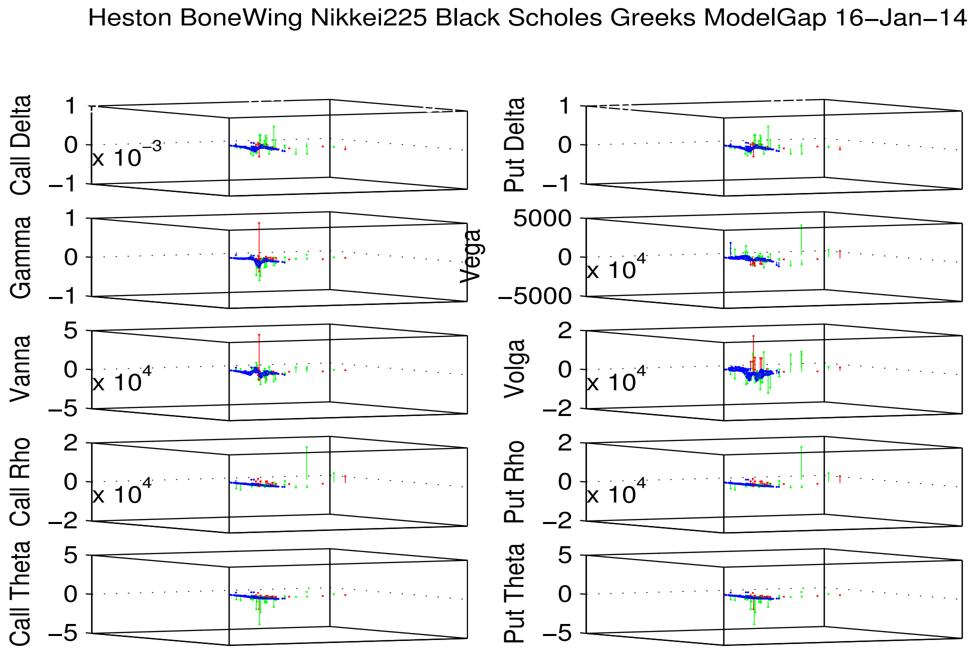
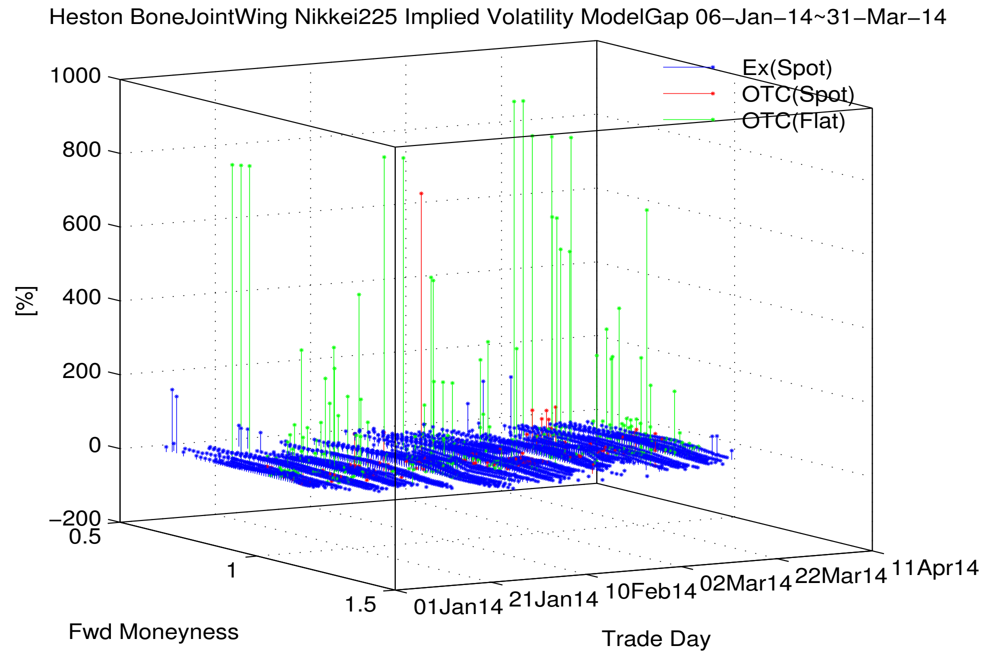


Figure B.100: Heston BoneWing Nikkei225 Price, Black Scholes Greeks ModelGap (16-Jan-14)



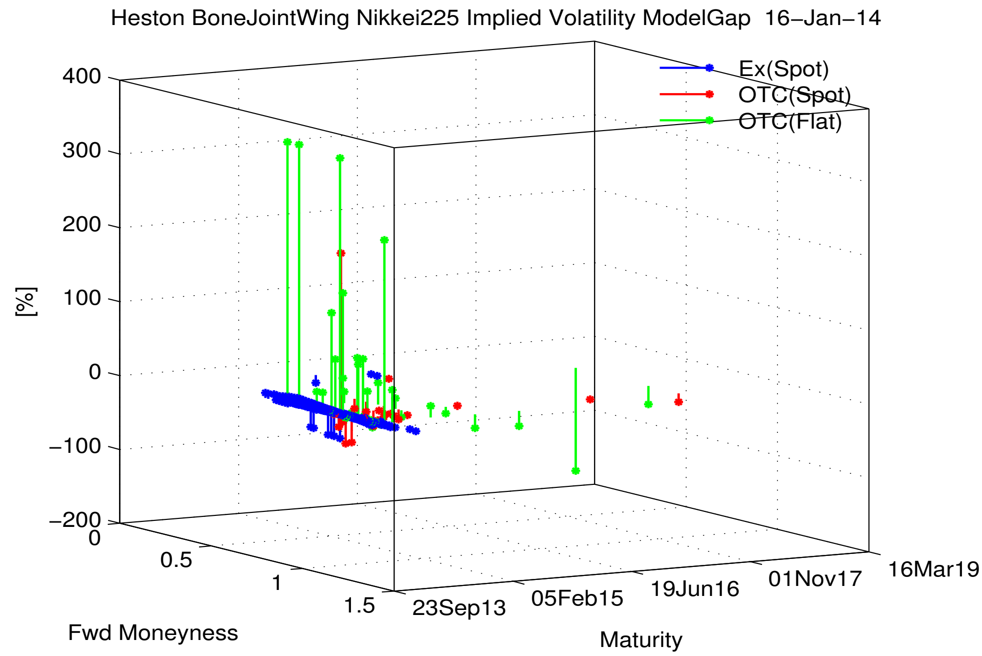


Figure B.101: Heston BoneWing Nikkei225 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)

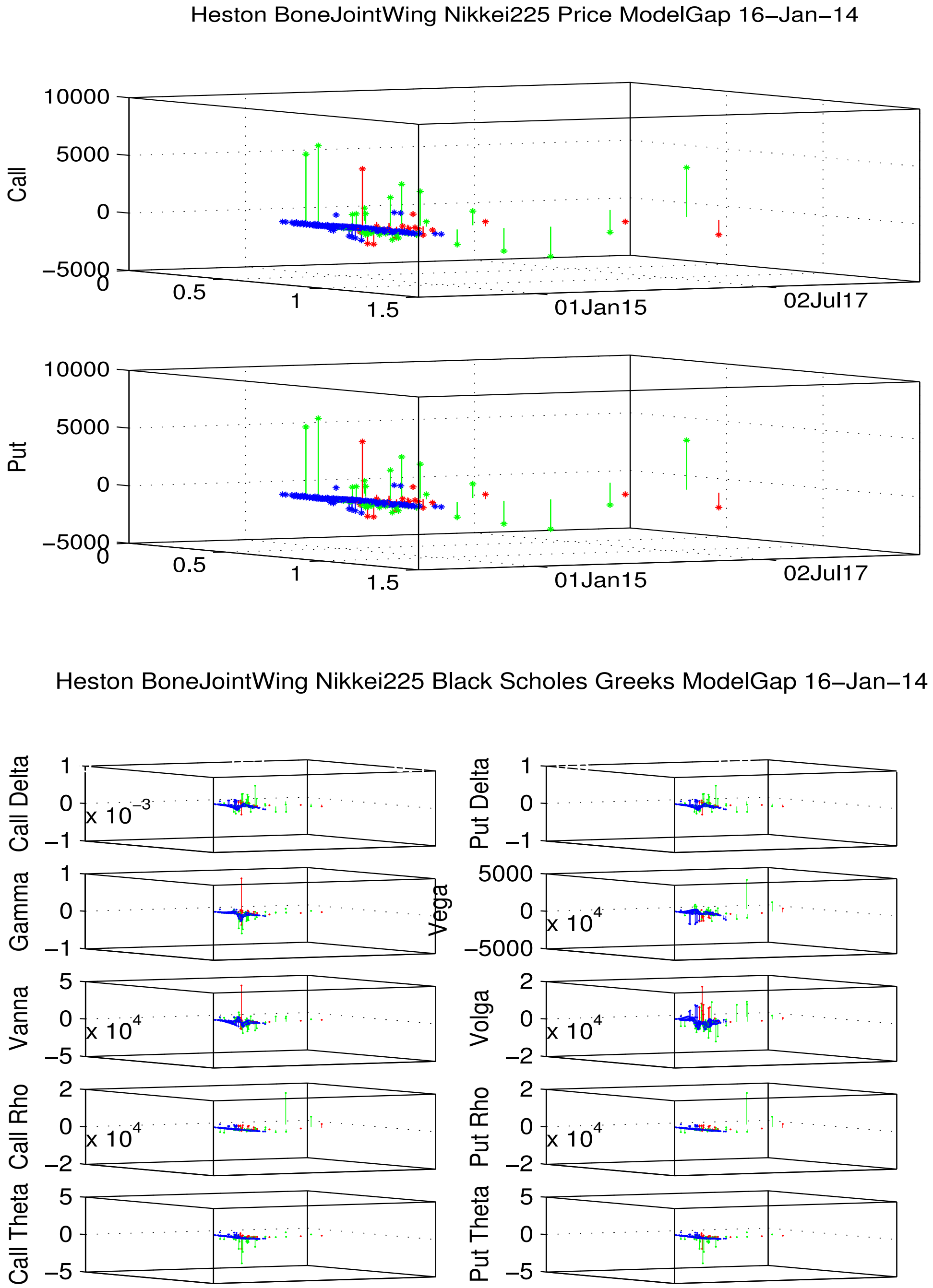
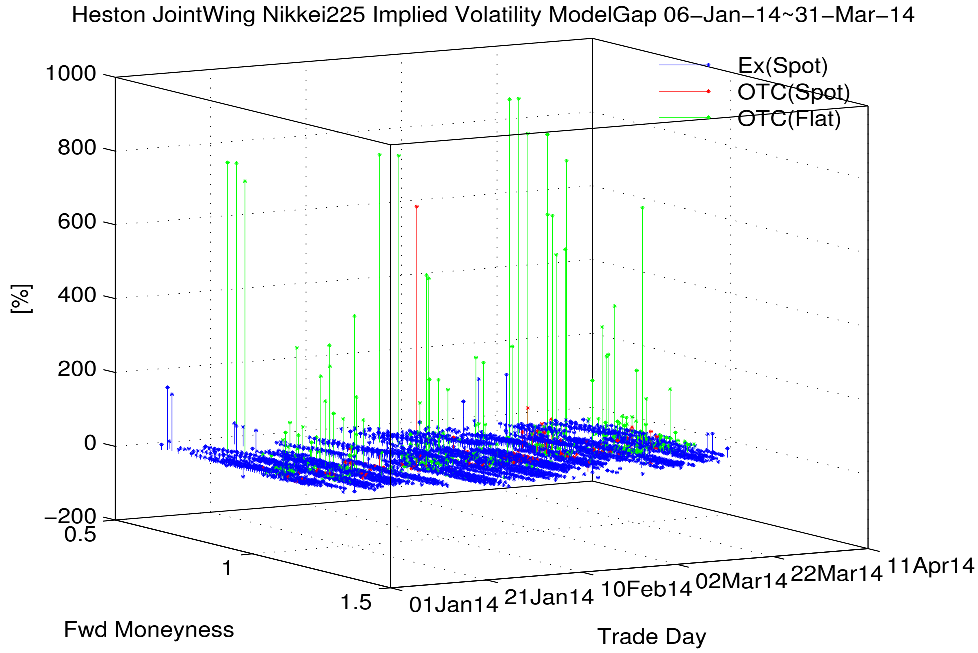


Figure B.102: Heston BoneWing Nikkei225 Price, Black Scholes Greeks ModelGap (16-Jan-14)



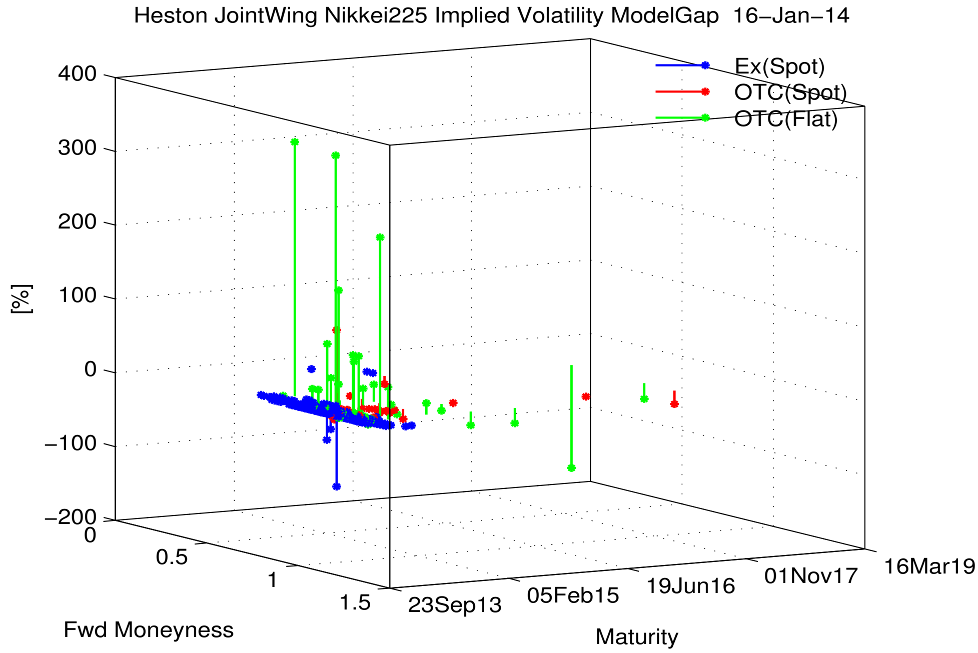
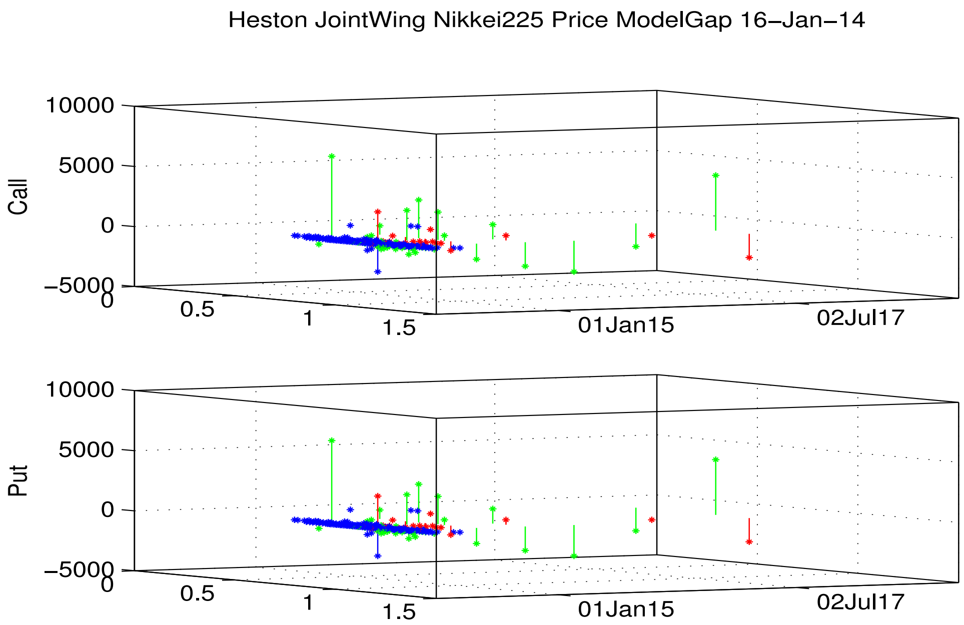


Figure B.103: Heston BoneWing Nikkei225 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



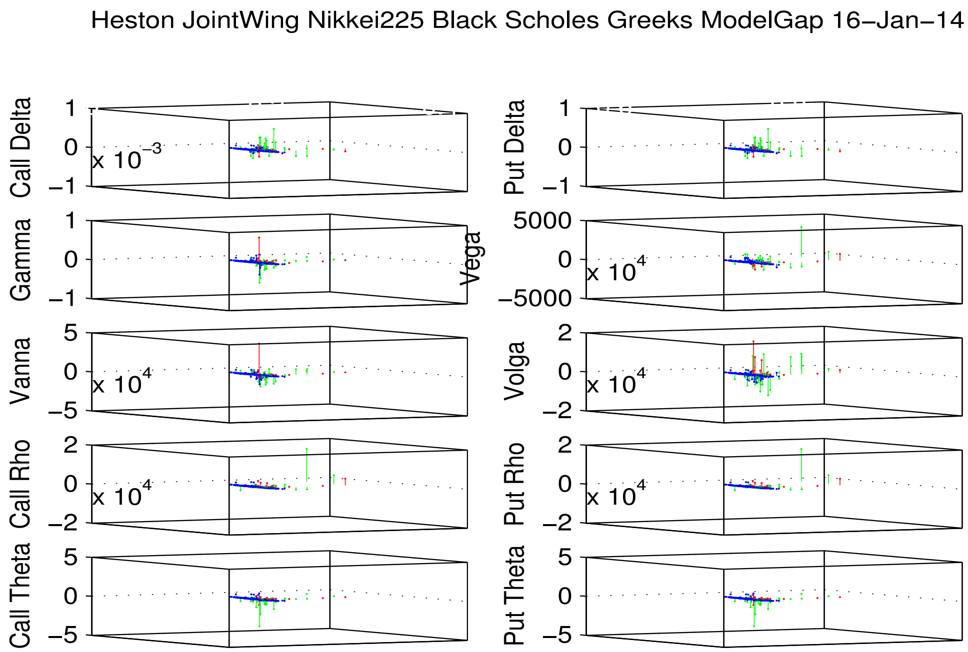


Figure B.104: Heston BoneWing Nikkei225 Price, Black Scholes Greeks ModelGap (16-Jan-14)

Table B.37: Root Mean Squared Nikkei225 Implied Volatility ModelGap (06-Jan-14∼31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
4.992E-01	3.962E-01	3.329E-01	4.154E-01	4.128E-01	3.916E-01

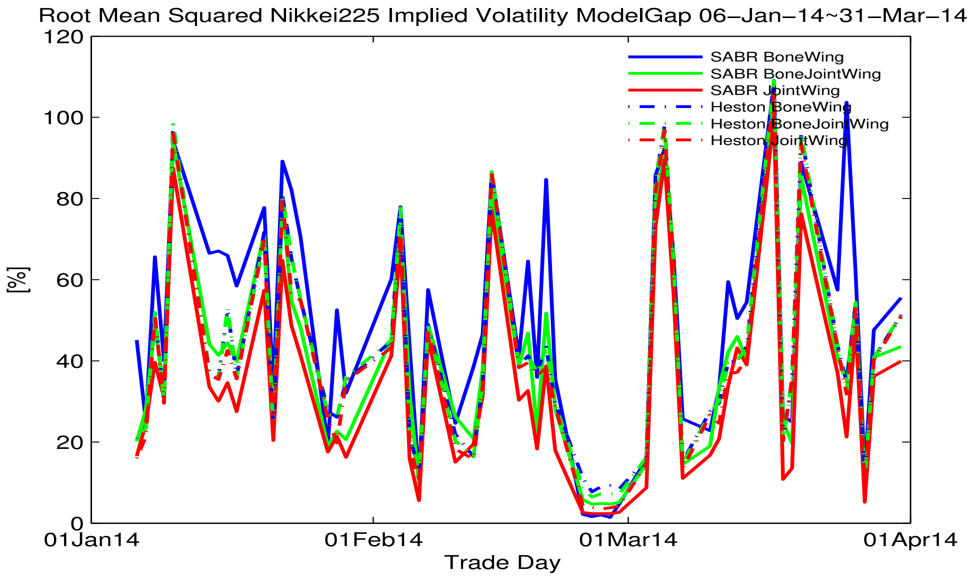


Figure B.105: Root Mean Squared Nikkei225 Implied Volatility ModelGap (02-Jan-14∼31-Mar-14)

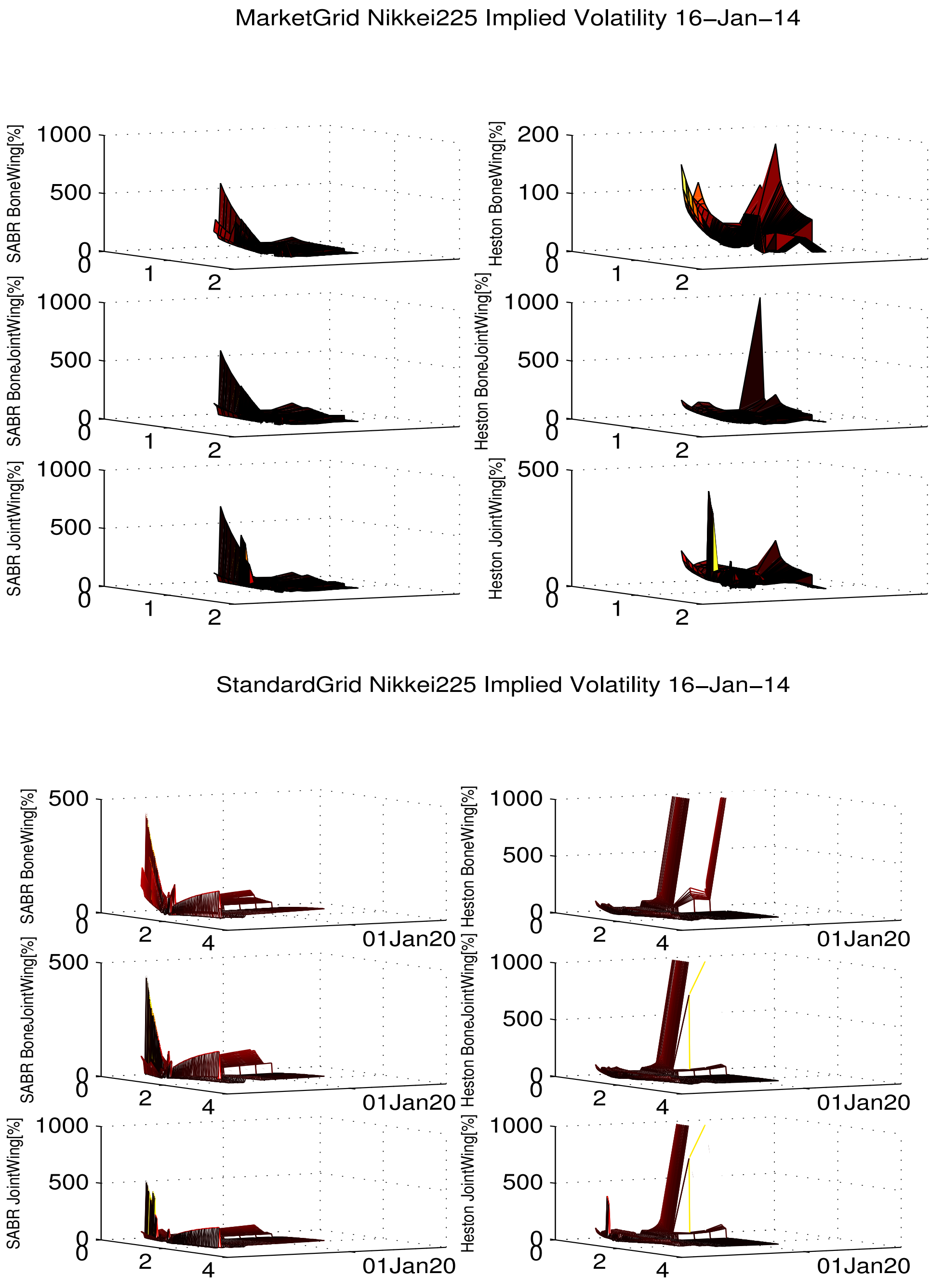


Figure B.106: Nikkei225 Market, Standard Grid Implied Volatility (16-Jan-14)

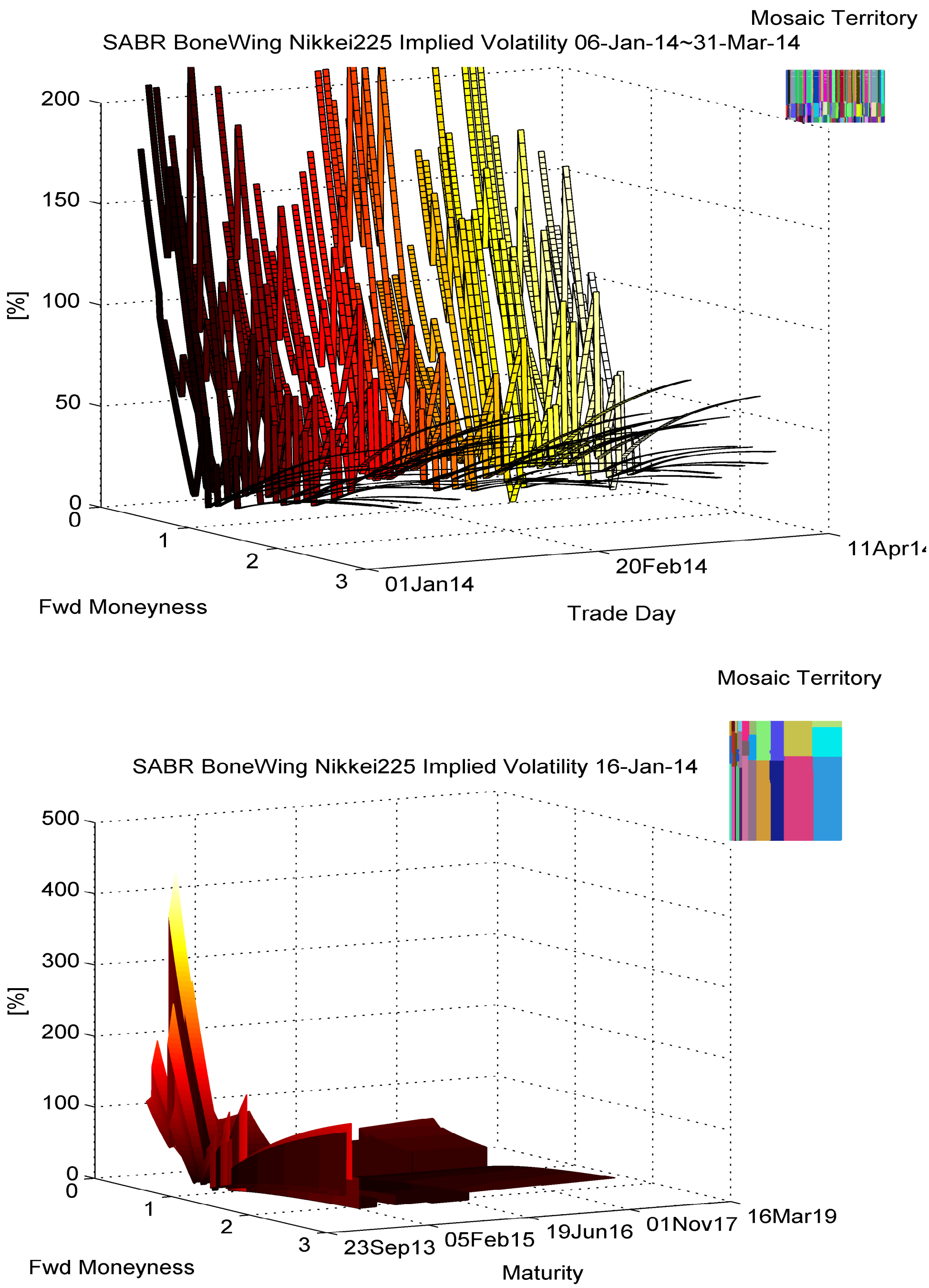
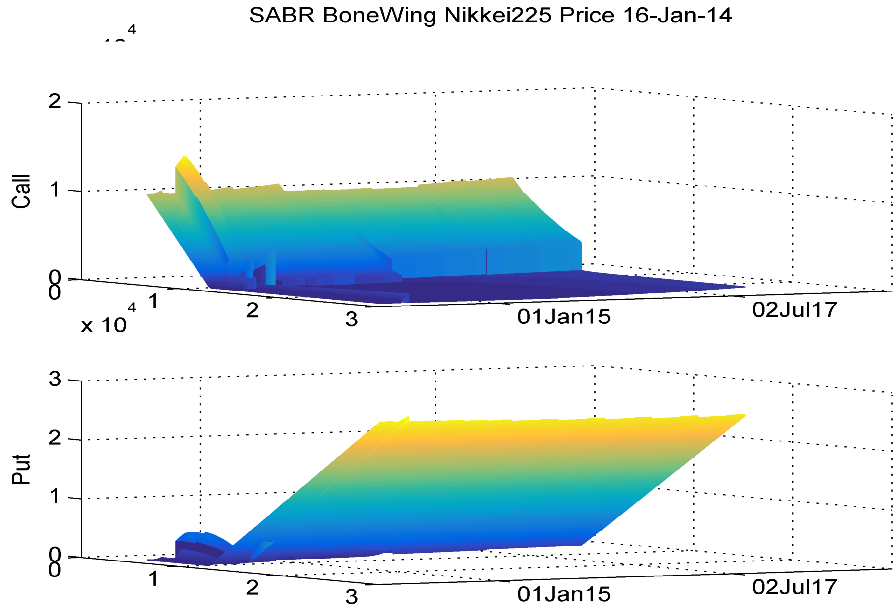


Figure B.107: SABR BoneWing Nikkei225 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



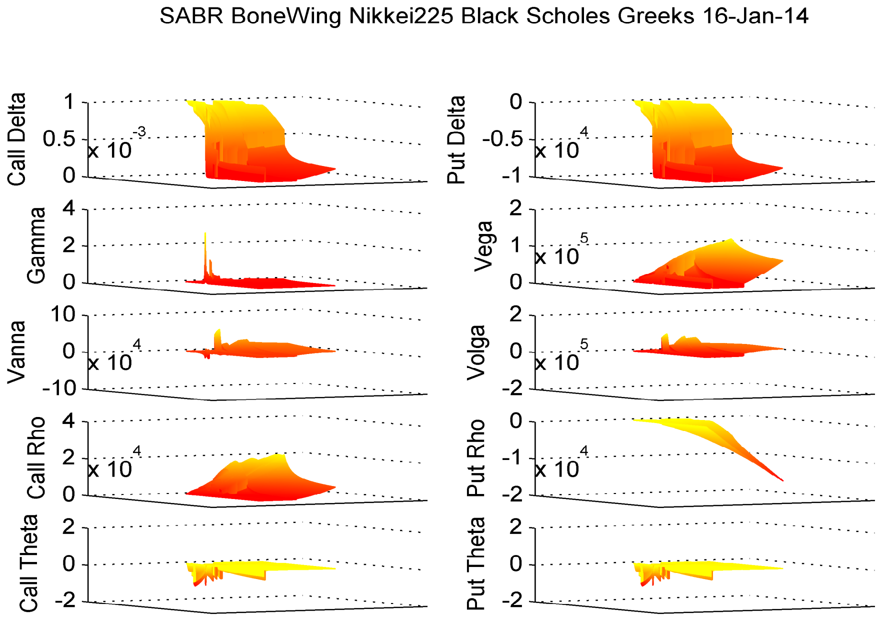
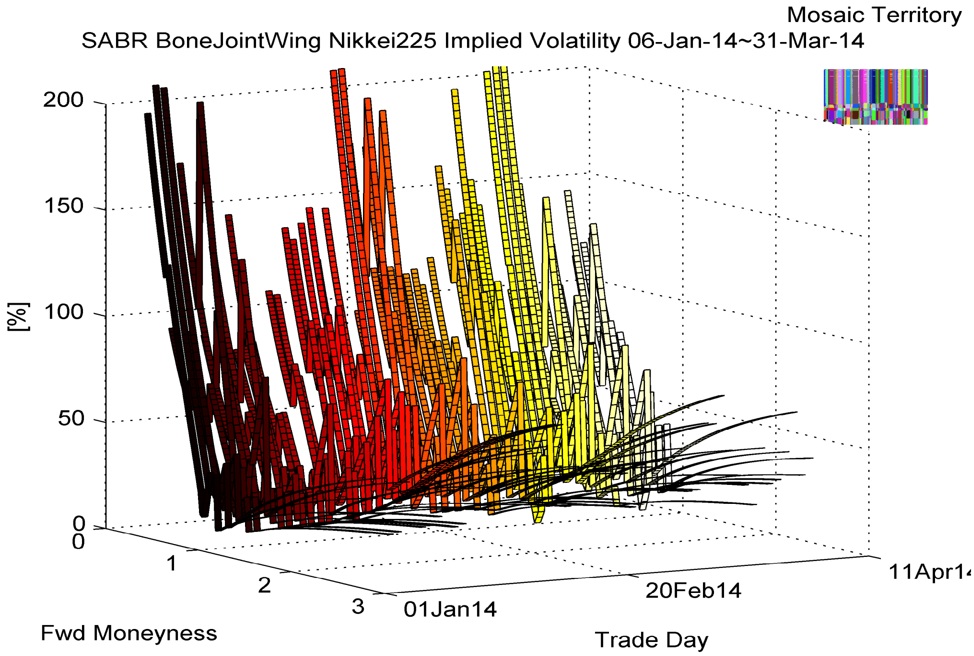


Figure B.108: SABR BoneWing Nikkei225 Price, Black Scholes Greeks (16-Jan-14)



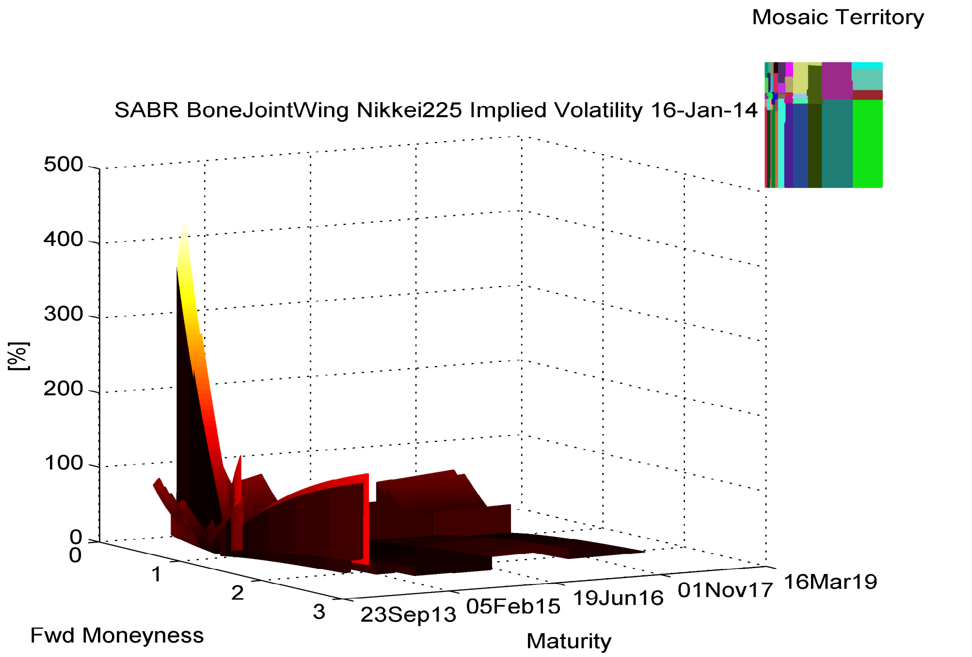
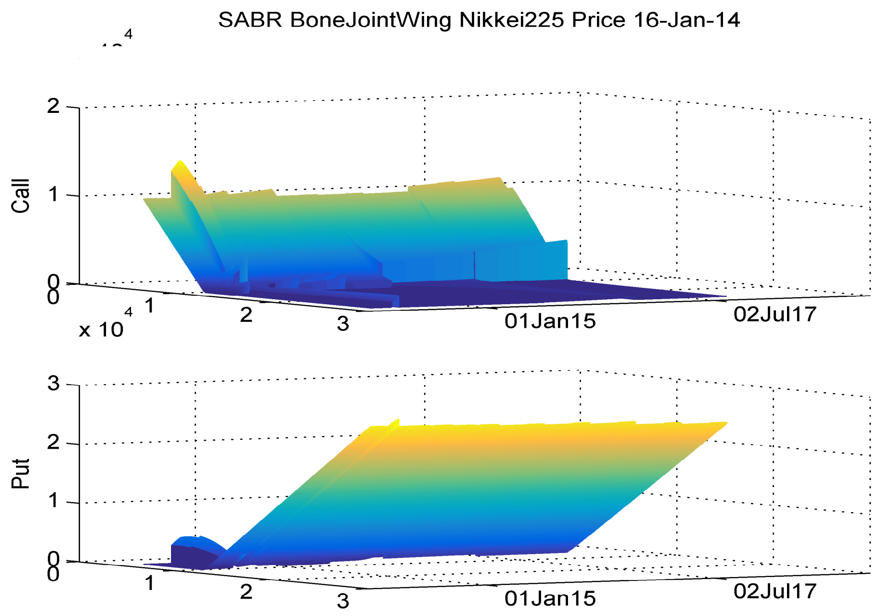


Figure B.109: SABR BoneWing Nikkei225 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



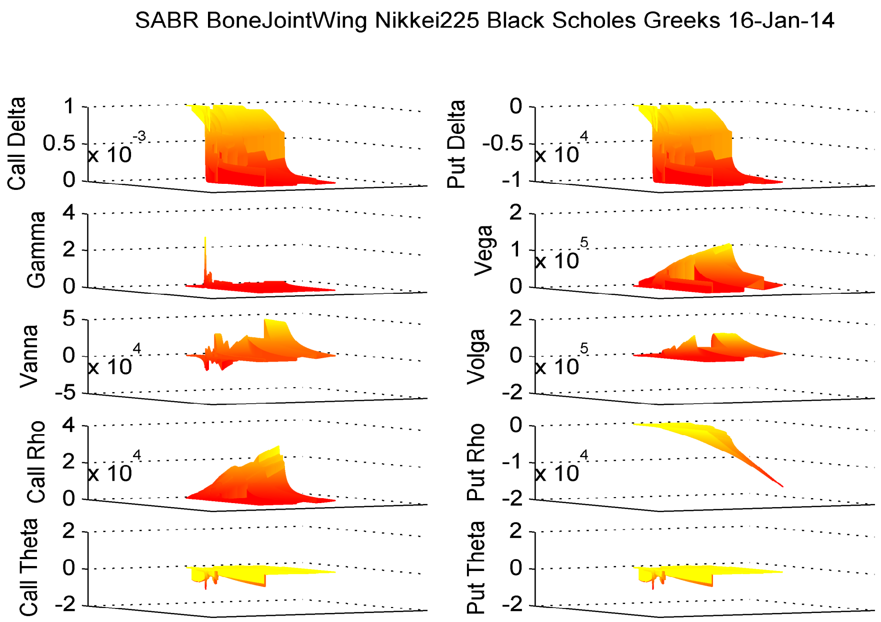
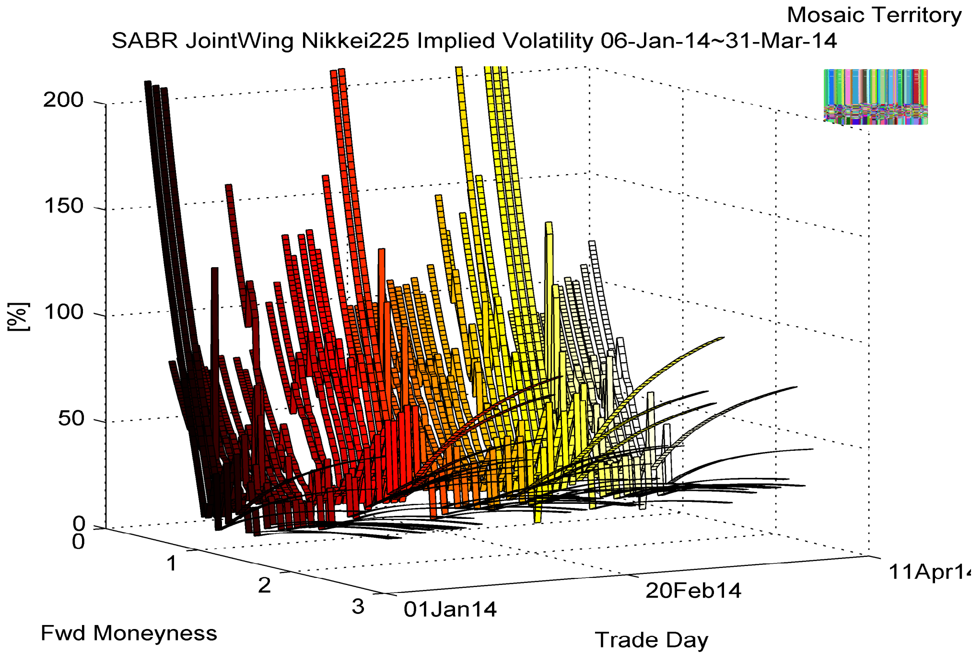


Figure B.110: SABR BoneWing Nikkei225 Price, Black Scholes Greeks (16-Jan-14)



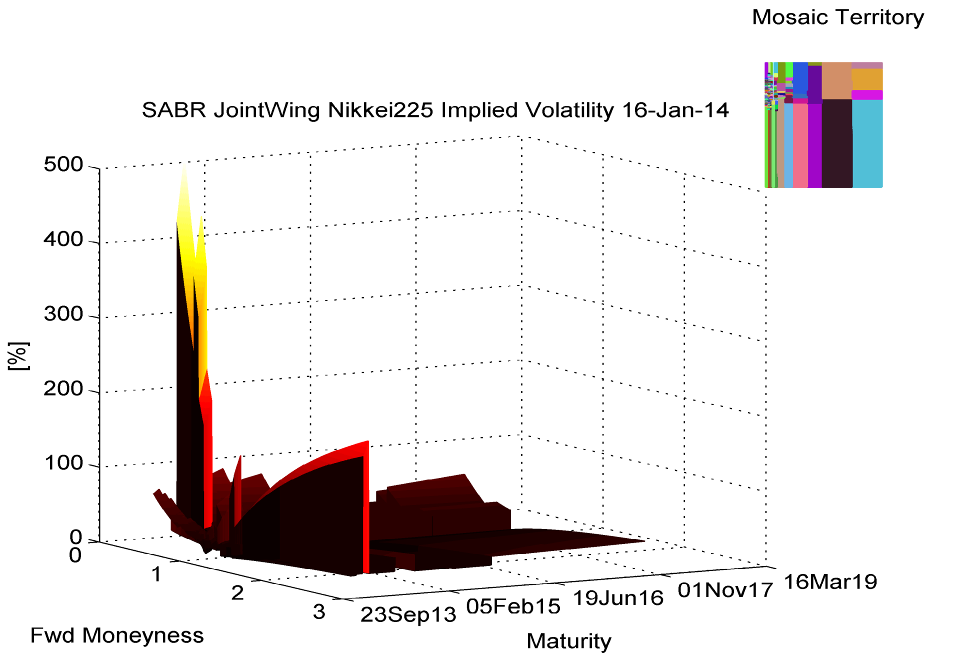
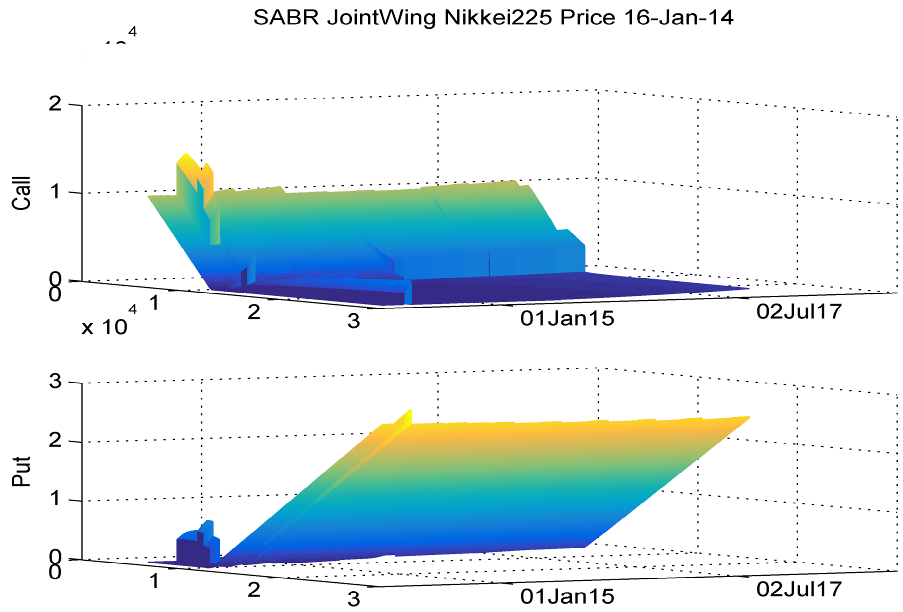


Figure B.111: SABR BoneWing Nikkei225 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



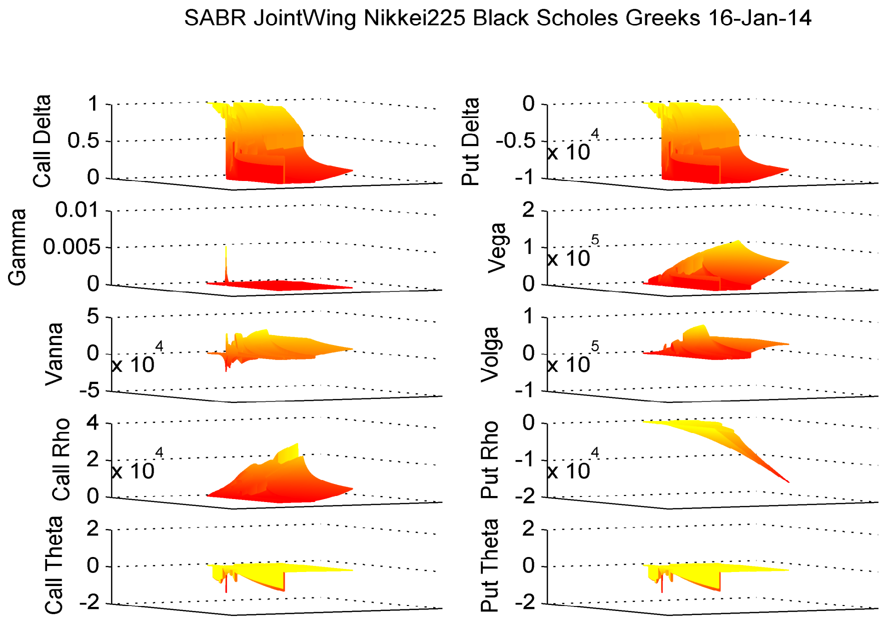
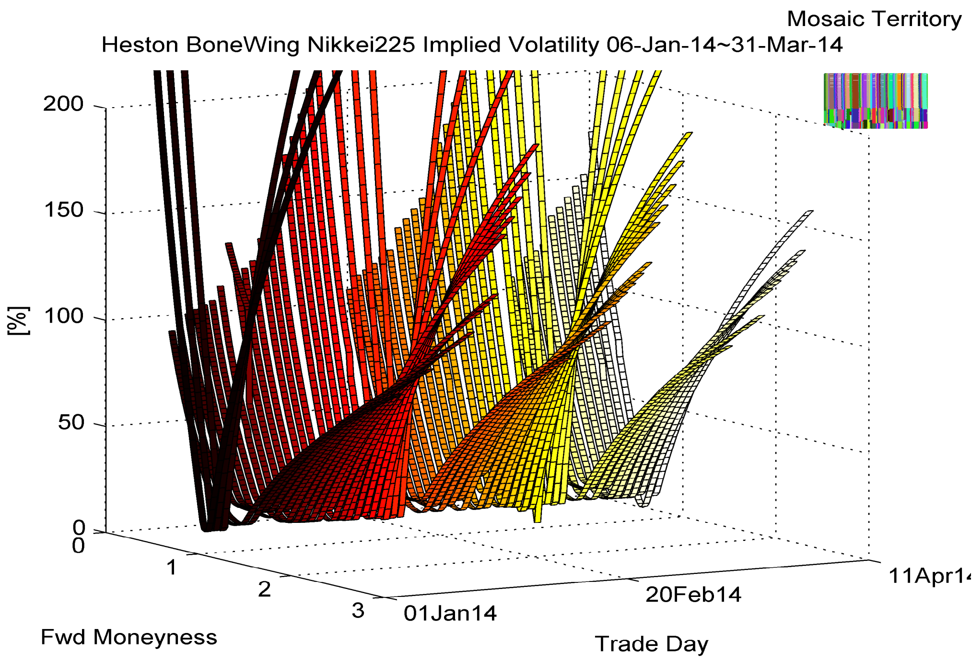


Figure B.112: SABR BoneWing Nikkei225 Price, Black Scholes Greeks (16-Jan-14)



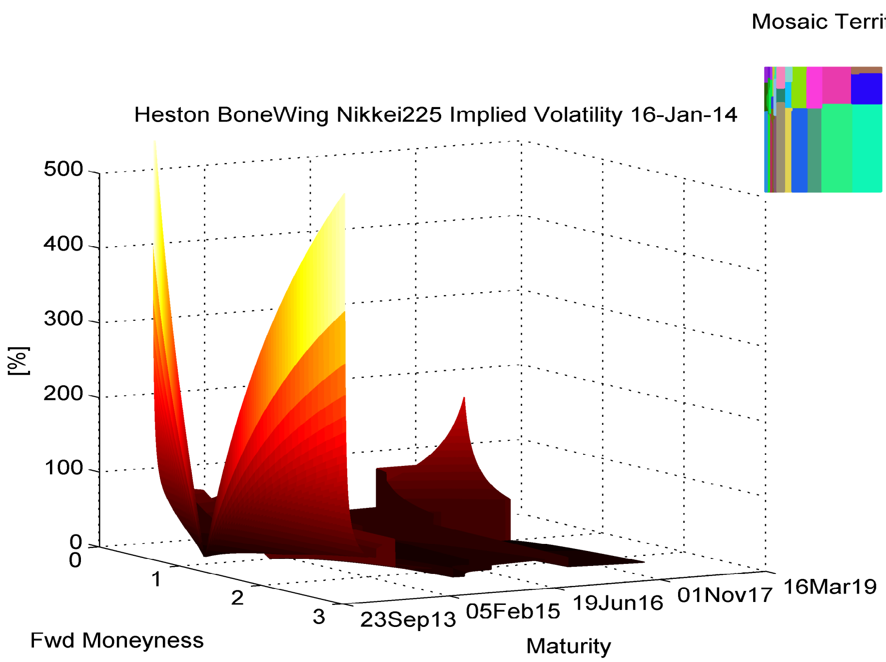
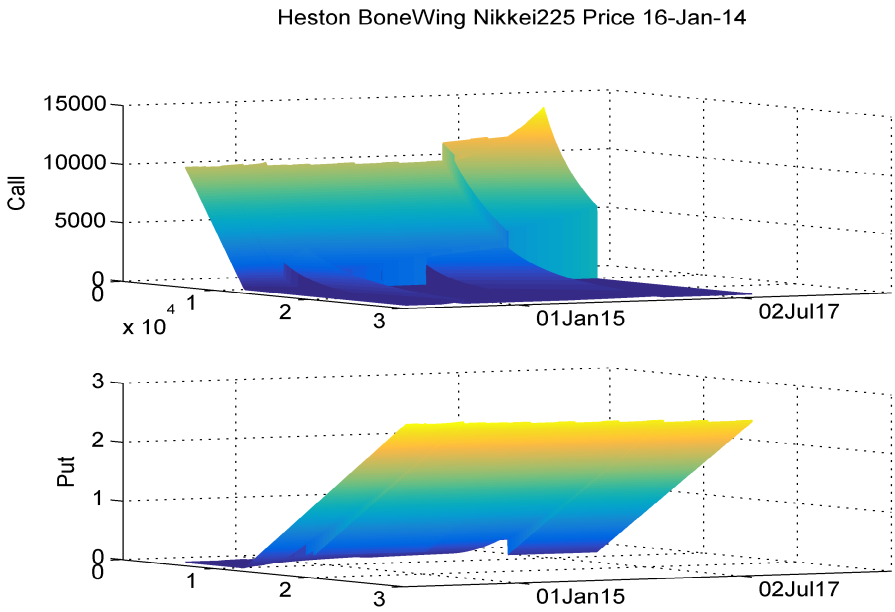


Figure B.113: Heston BoneWing Nikkei225 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



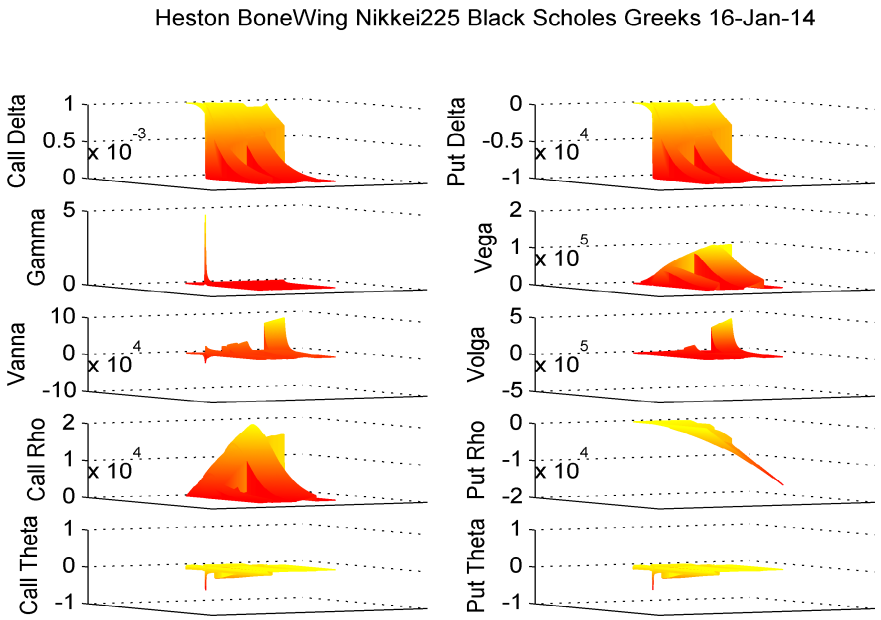
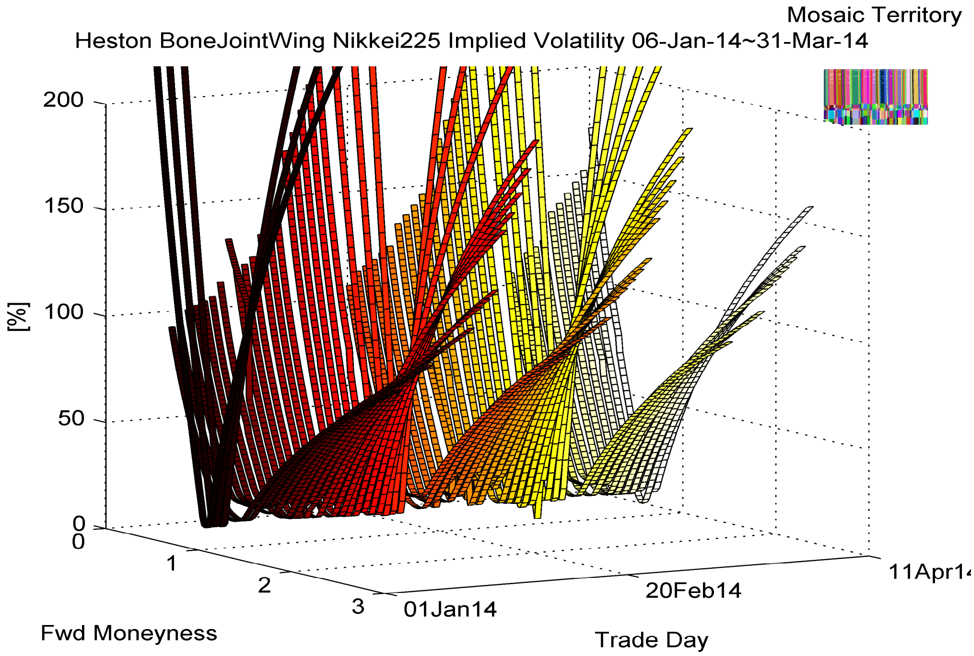


Figure B.114: Heston BoneWing Nikkei225 Price, Black Scholes Greeks (16-Jan-14)



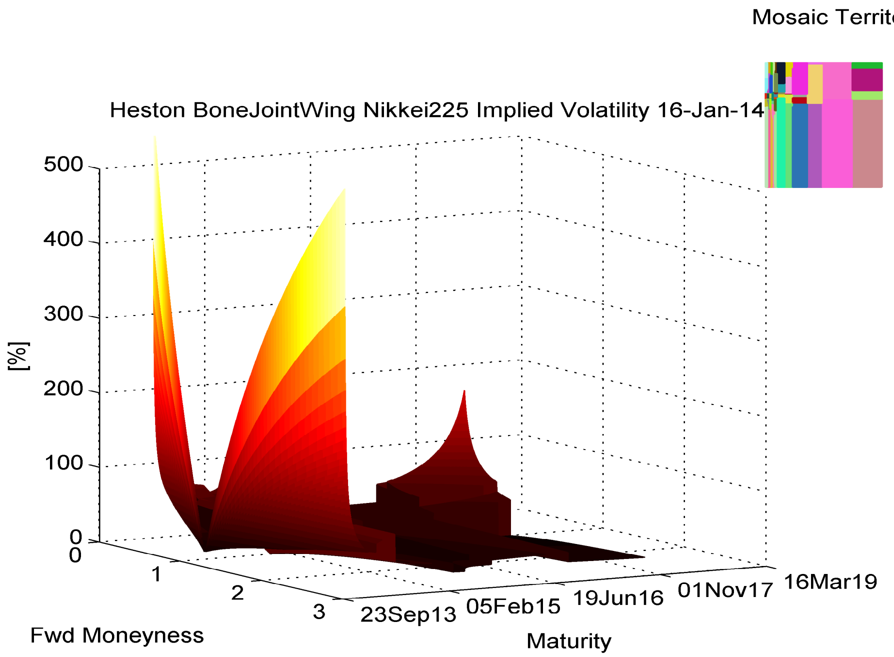
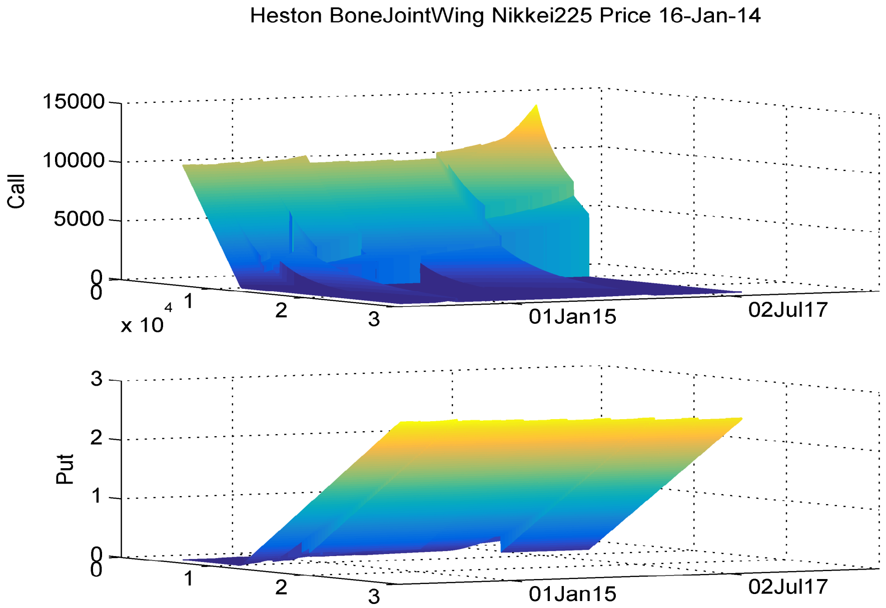


Figure B.115: Heston BoneWing Nikkei225 Implied Volatility (Nearest) (03-Jan-14∼31-Mar-14), (16-Jan-14)



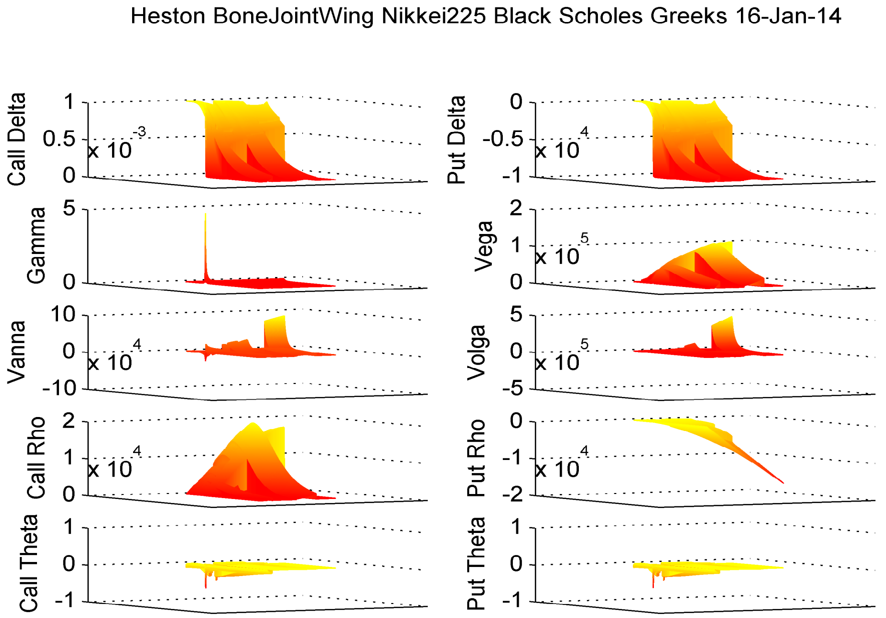
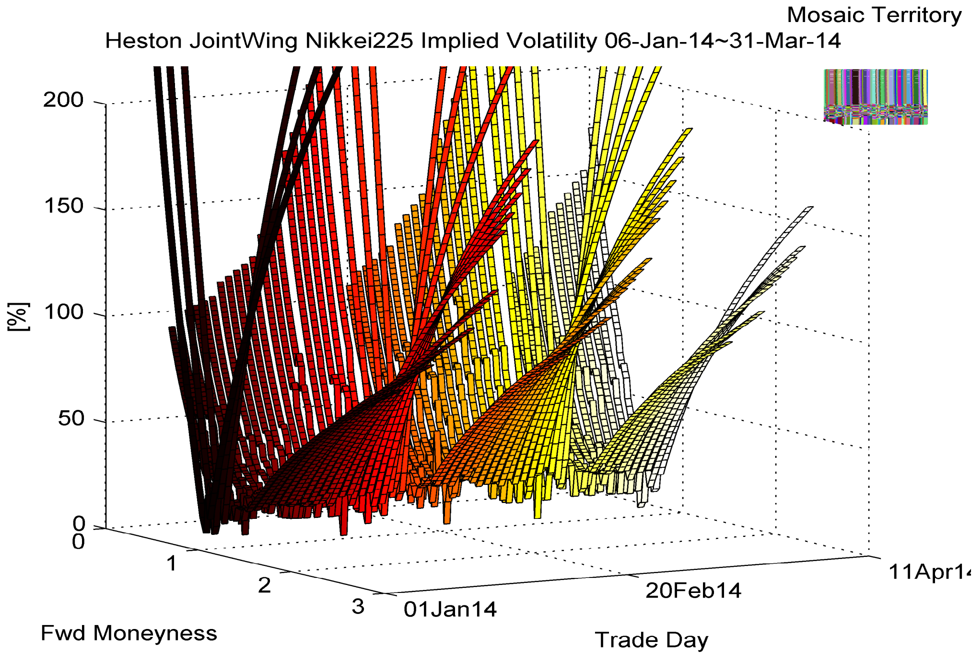


Figure B.116: Heston BoneWing Nikkei225 Price, Black Scholes Greeks (16-Jan-14)



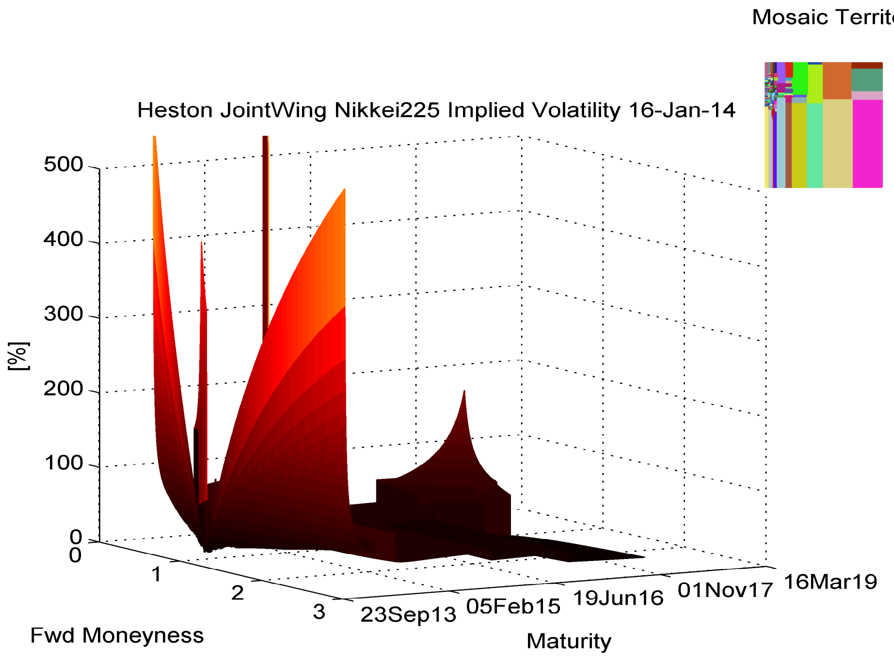
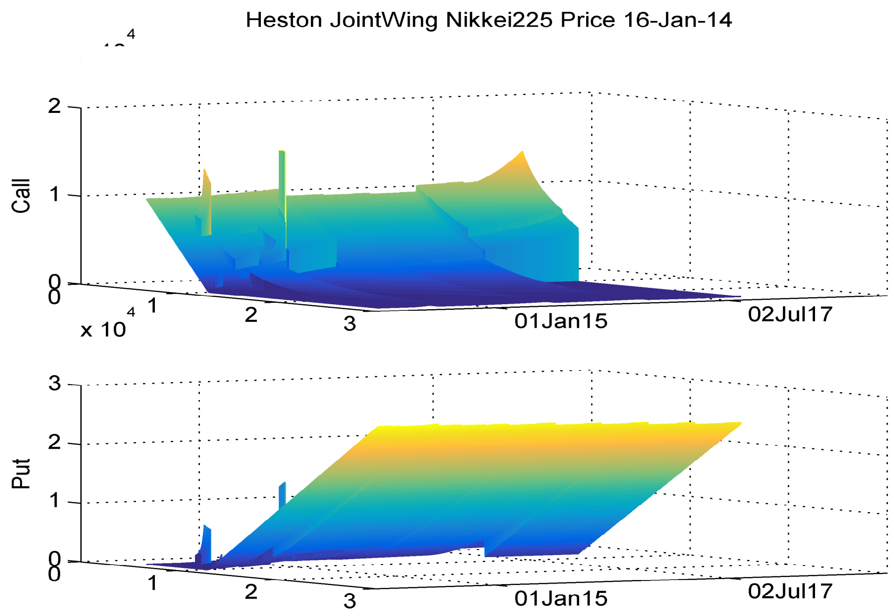


Figure B.117: Heston BoneWing Nikkei225 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



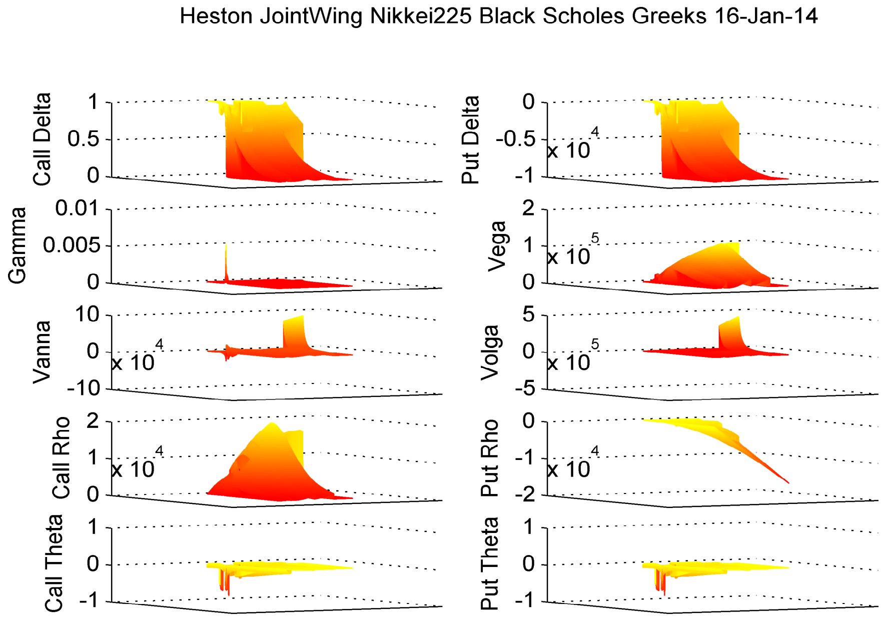
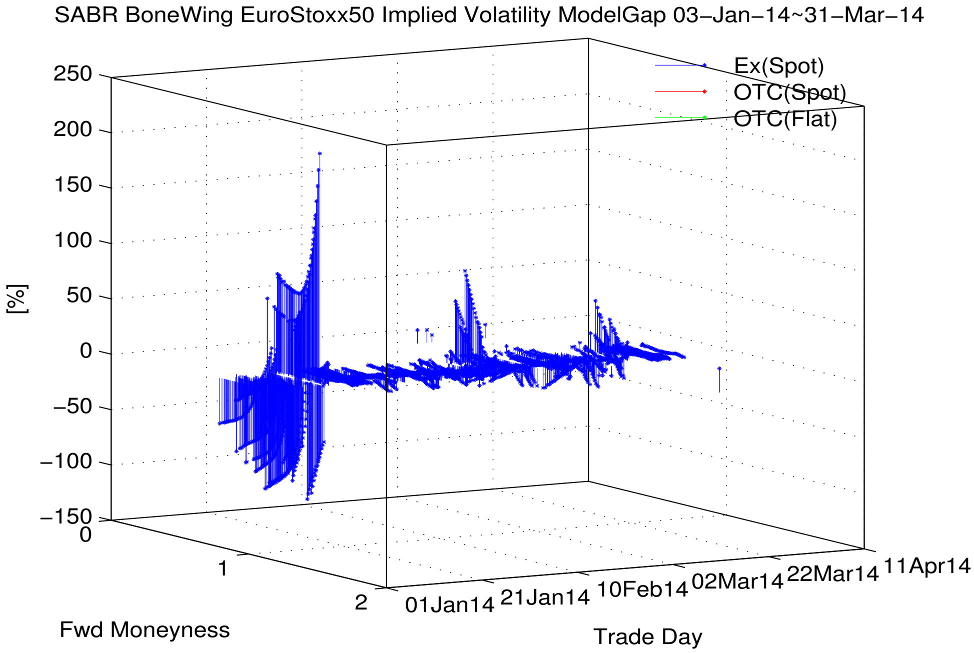


Figure B.118: Heston BoneWing Nikkei225 Price, Black Scholes Greeks (16-Jan-14)



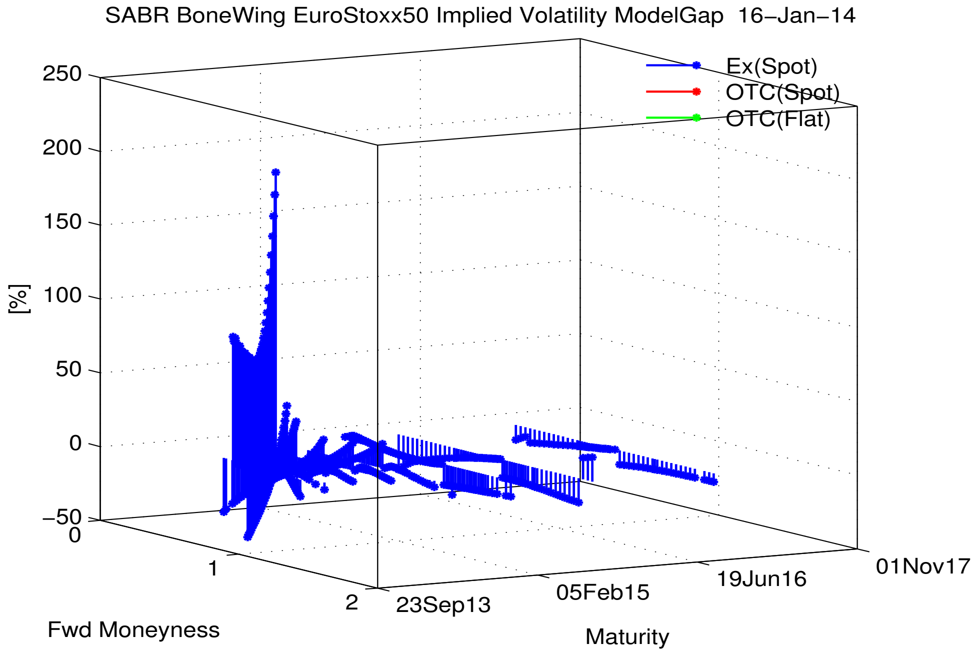


Figure B.119: SABR BoneWing EuroStoxx50 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)

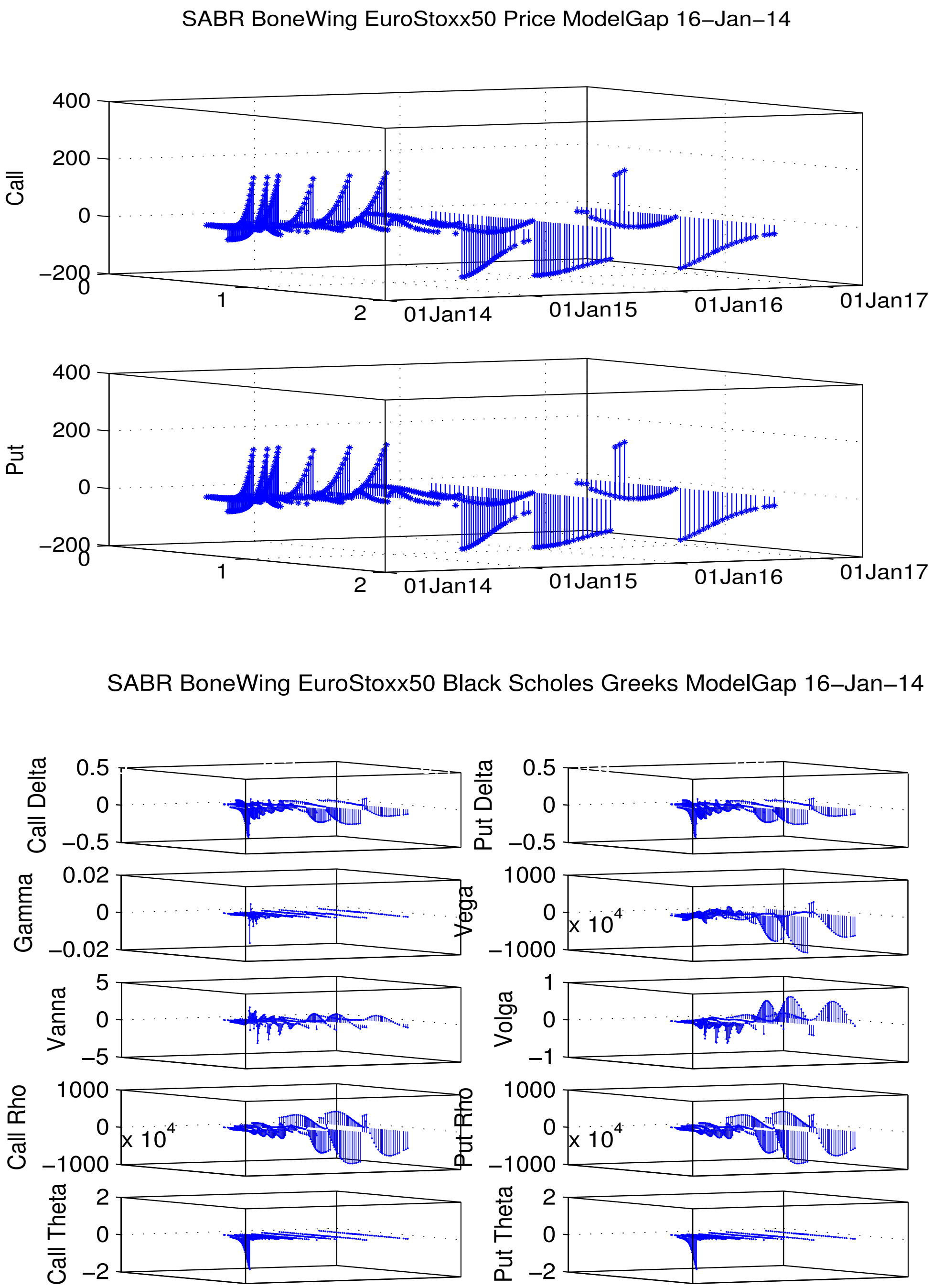
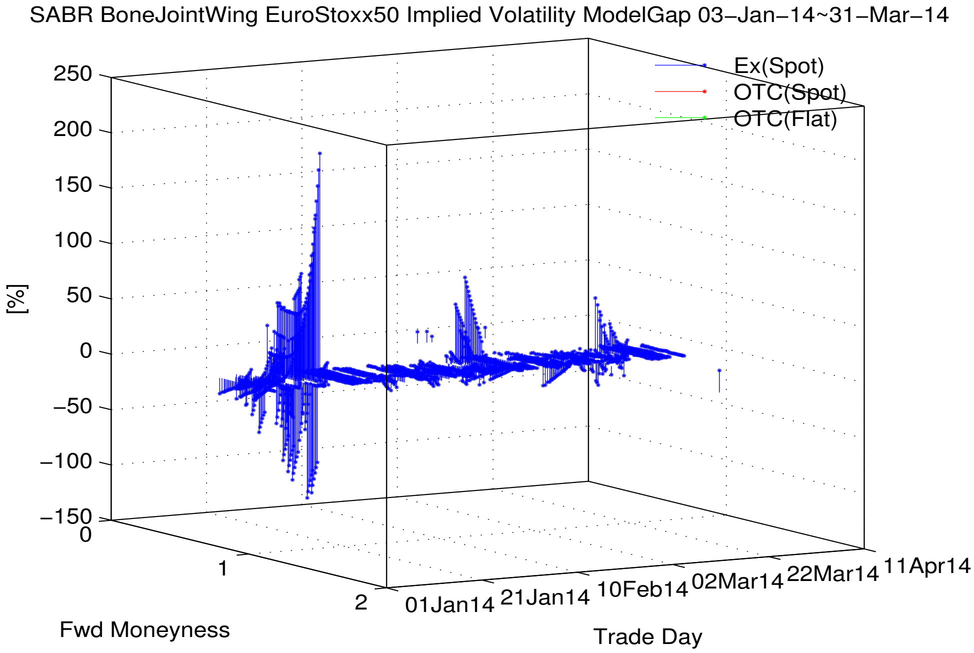


Figure B.120: SABR BoneWing EuroStoxx50 Price, Black Scholes Greeks ModelGap (16-Jan-14)



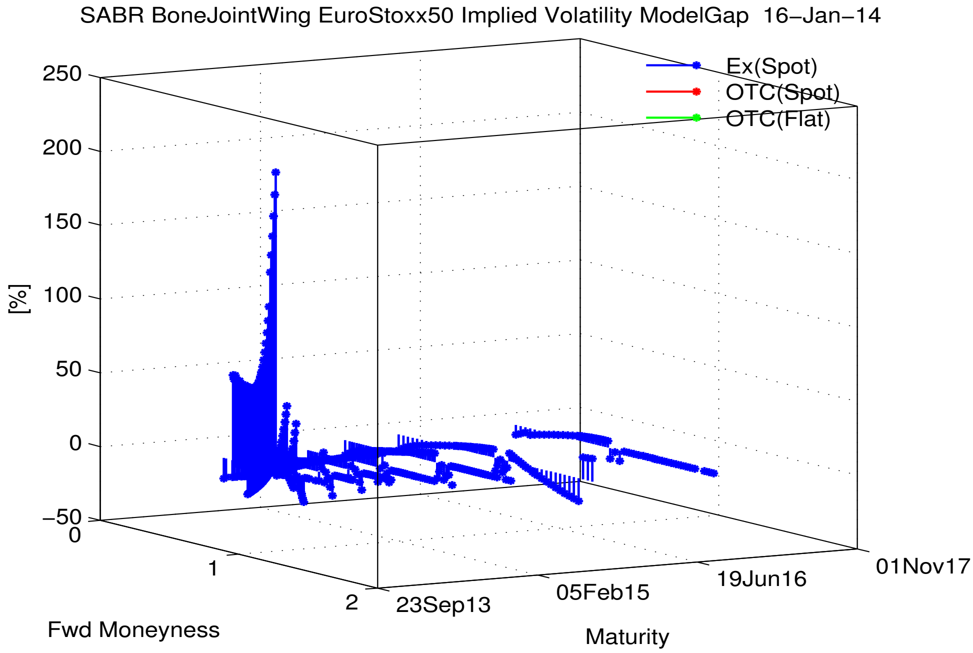
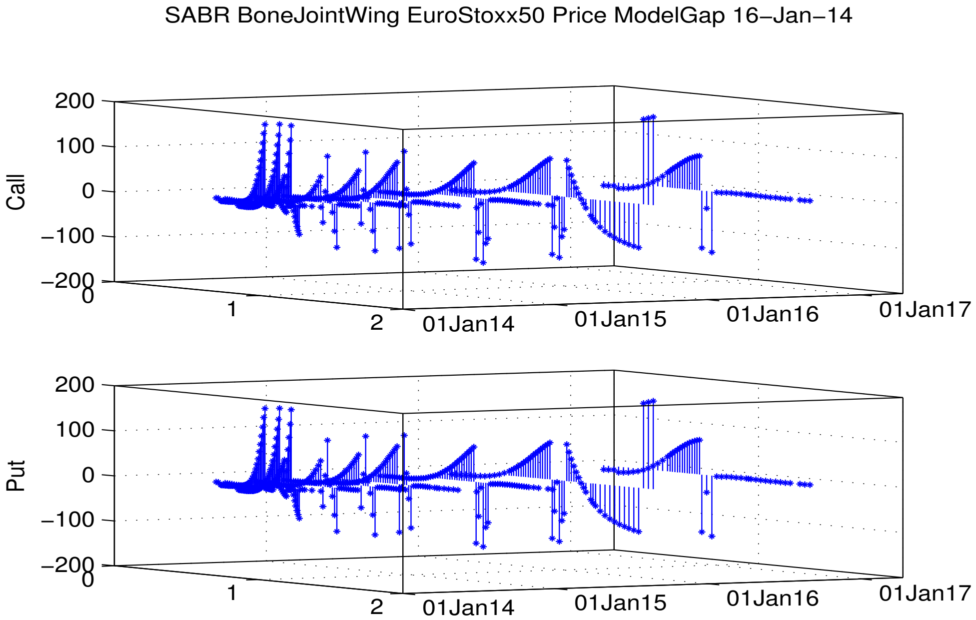


Figure B.121: SABR BoneWing EuroStoxx50 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



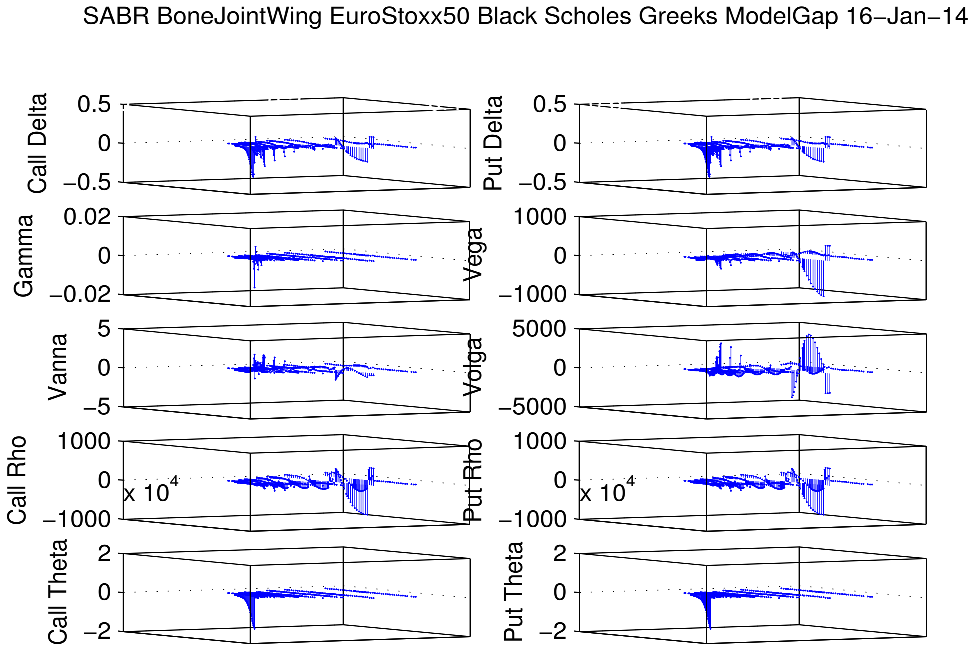
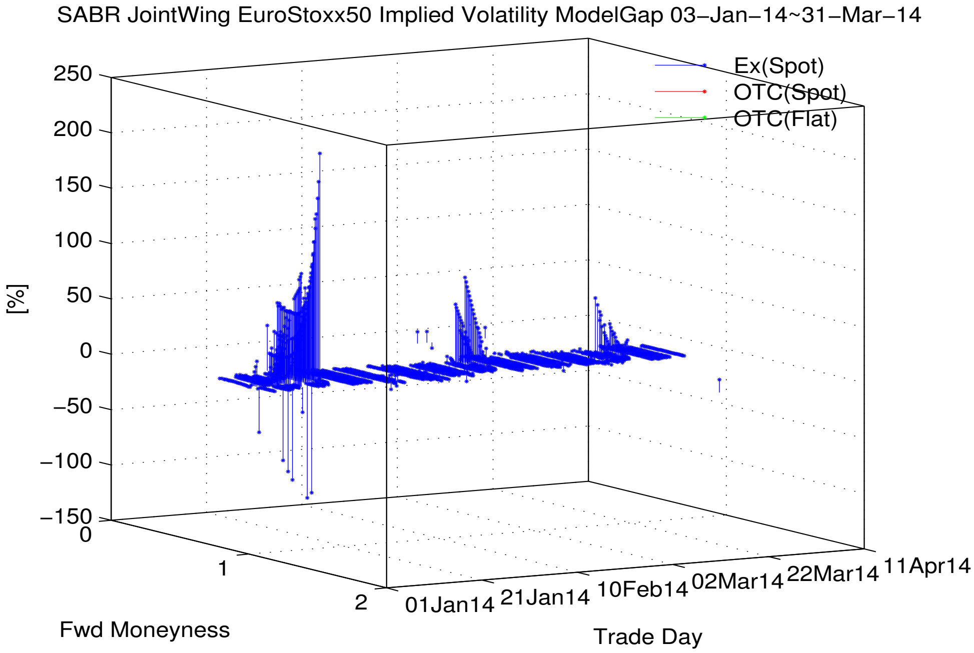


Figure B.122: SABR BoneWing EuroStoxx50 Price, Black Scholes Greeks ModelGap (16-Jan-14)



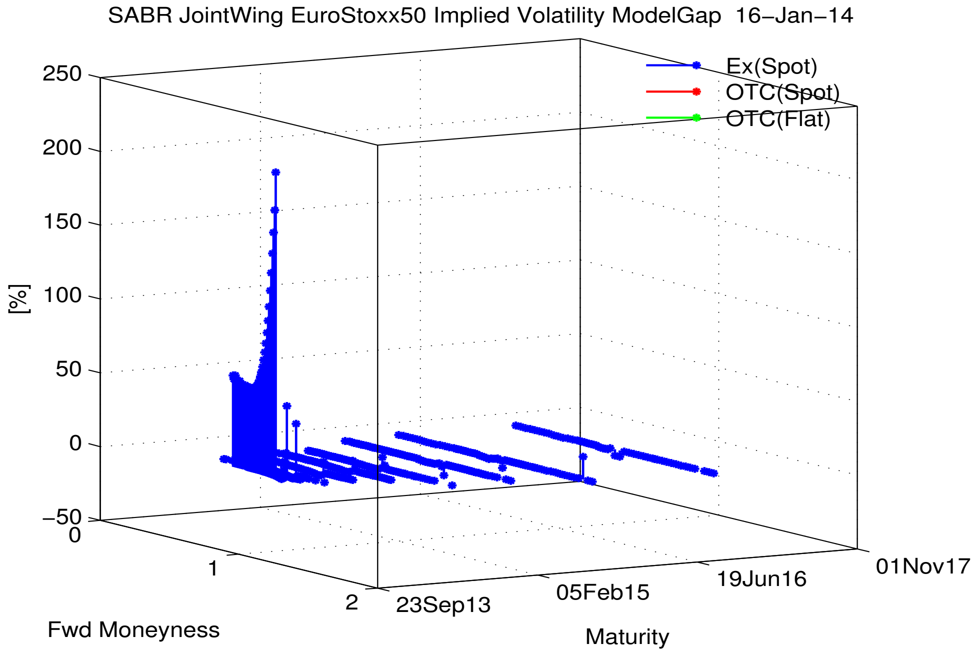
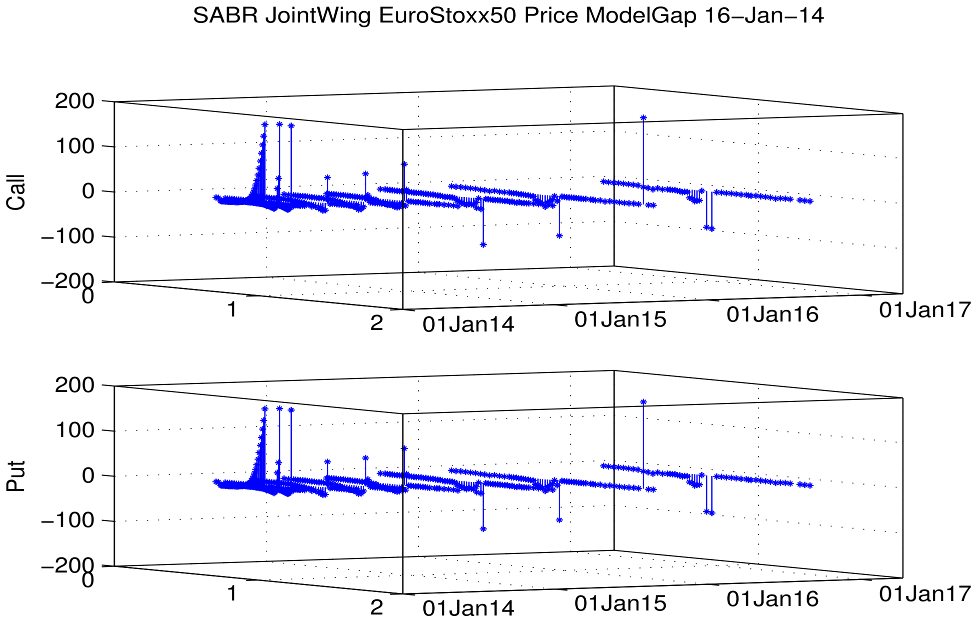


Figure B.123: SABR BoneWing EuroStoxx50 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



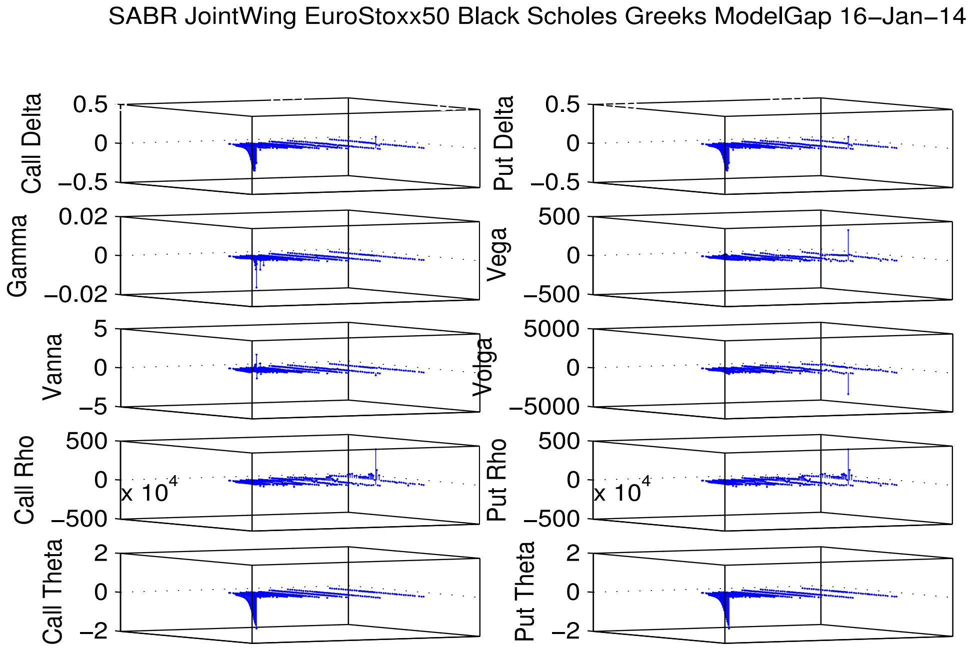
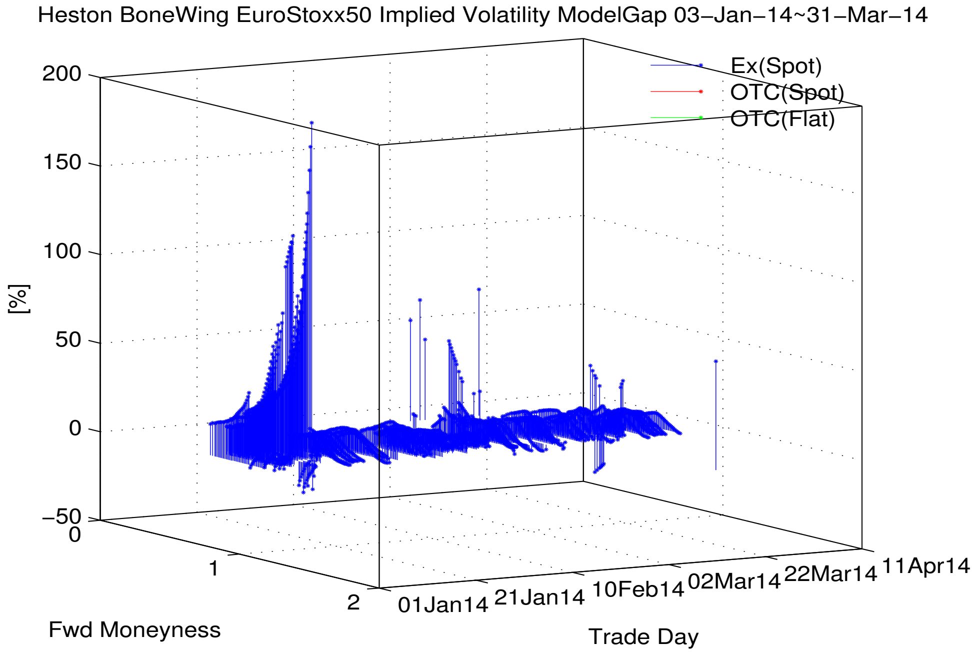


Figure B.124: SABR BoneWing EuroStoxx50 Price, Black Scholes Greeks ModelGap (16-Jan-14)



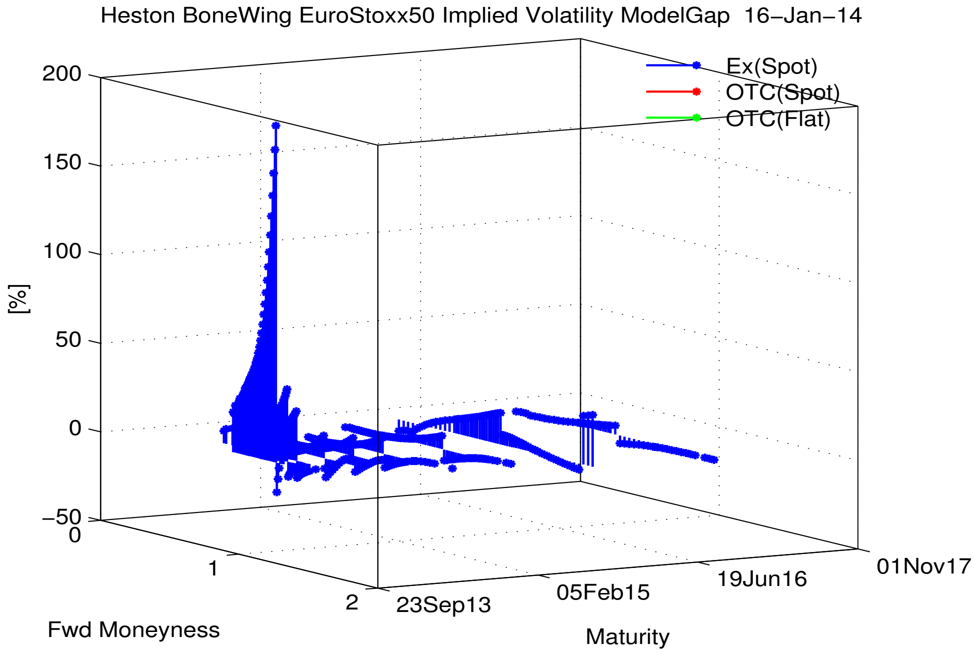
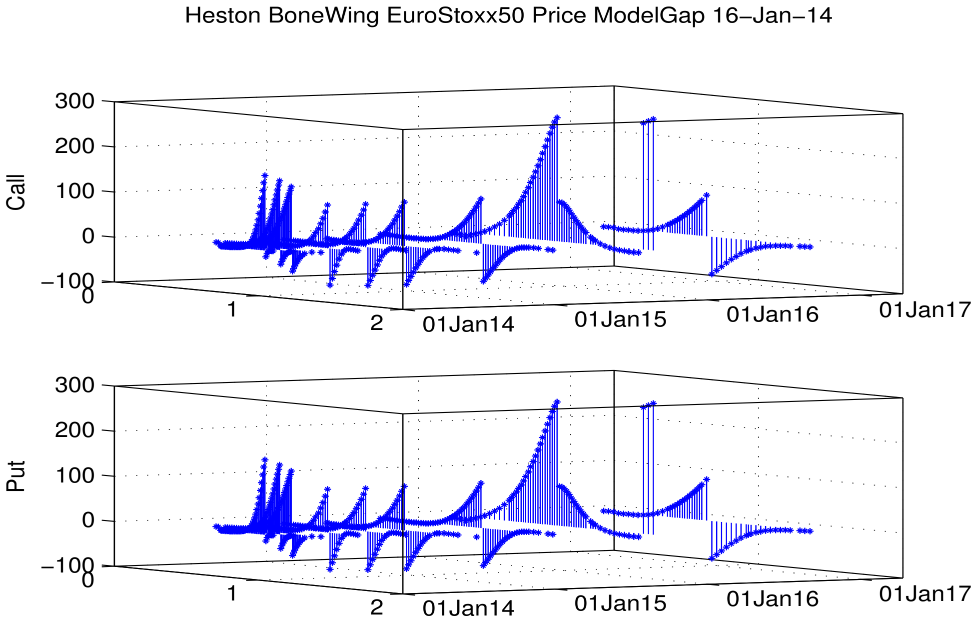


Figure B.125: Heston BoneWing EuroStoxx50 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



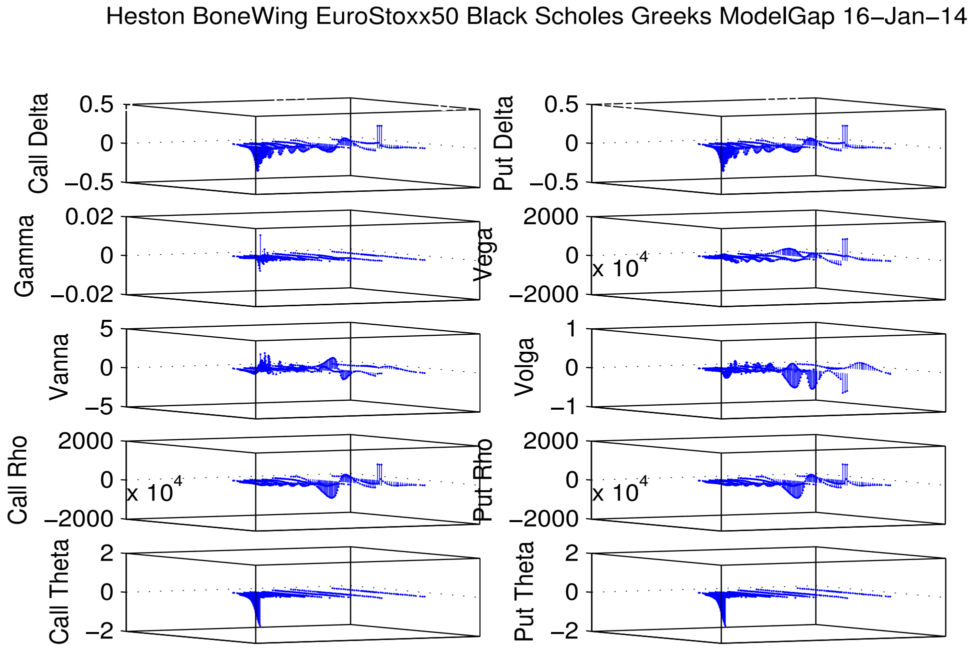
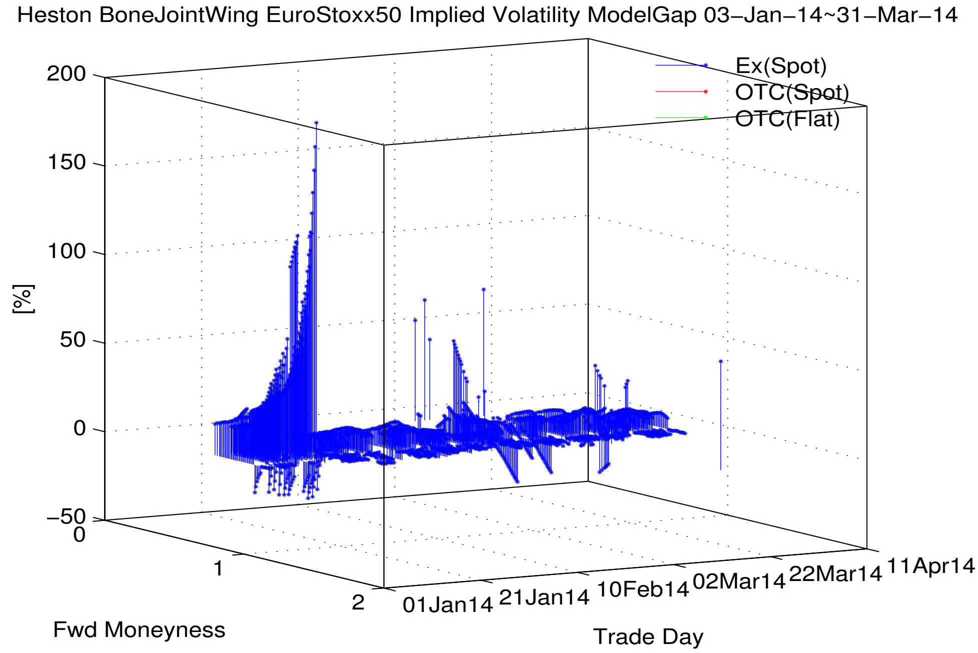


Figure B.126: Heston BoneWing EuroStoxx50 Price, Black Scholes Greeks ModelGap (16-Jan-14)



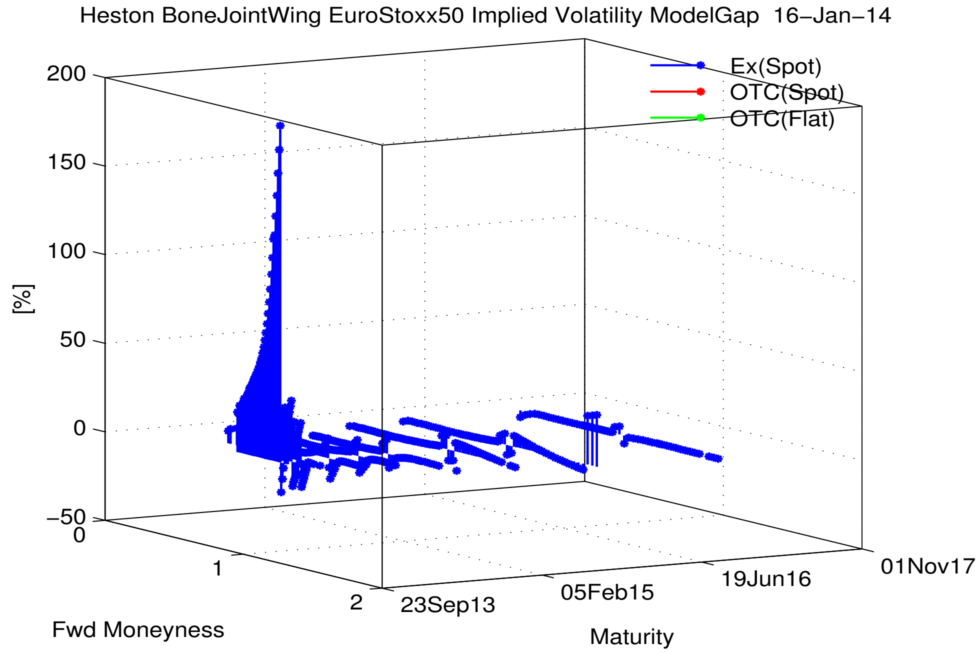
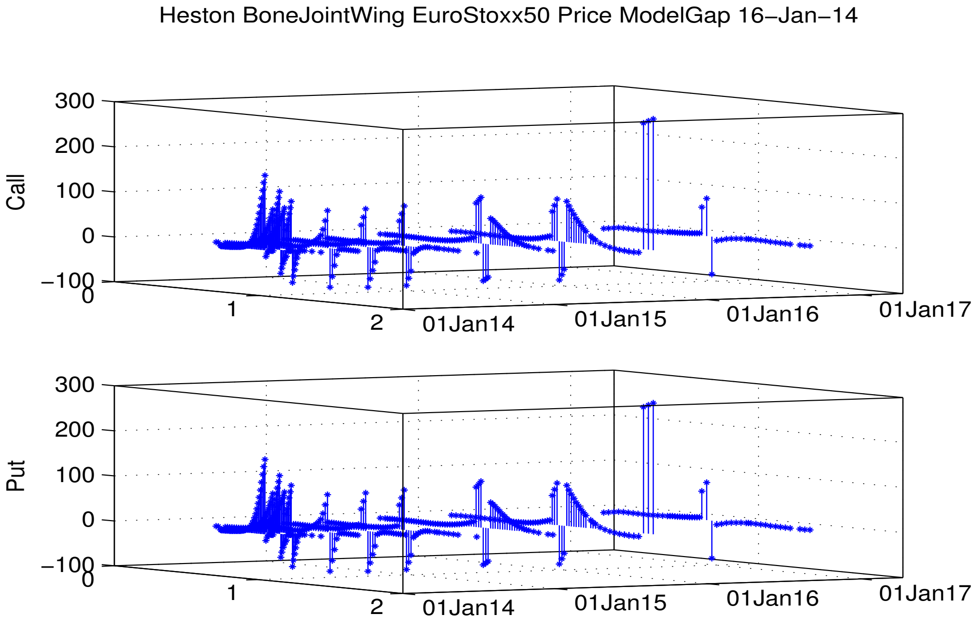


Figure B.127: Heston BoneWing EuroStoxx50 Implied Volatility (Nearest) ModelGap (03-Jan-14∼31-Mar-14), (16-Jan-14)



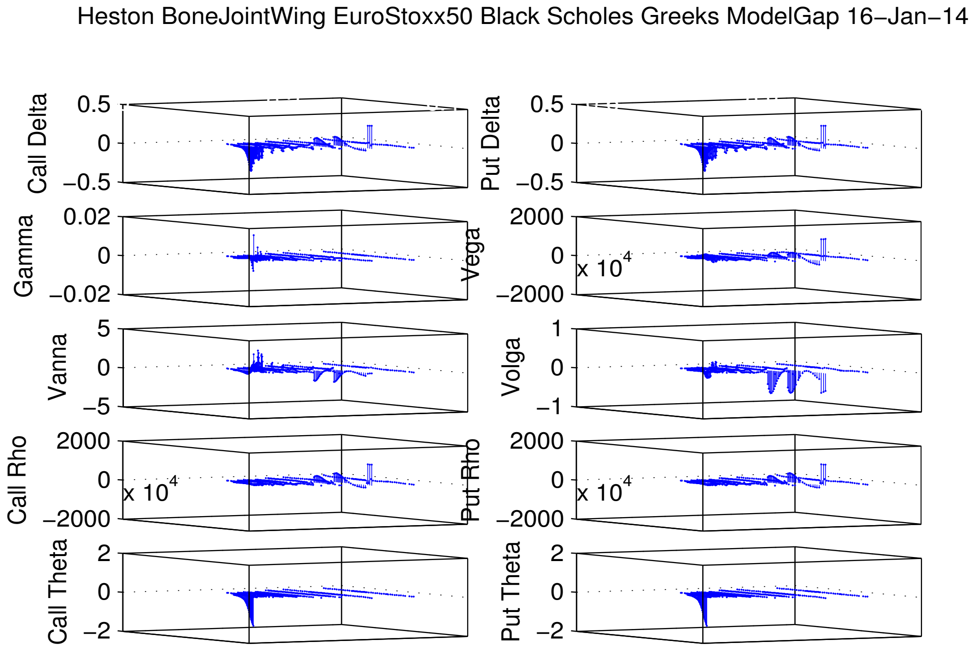


Figure B.128: Heston BoneWing EuroStoxx50 Price, Black Scholes Greeks ModelGap (16-Jan-14)

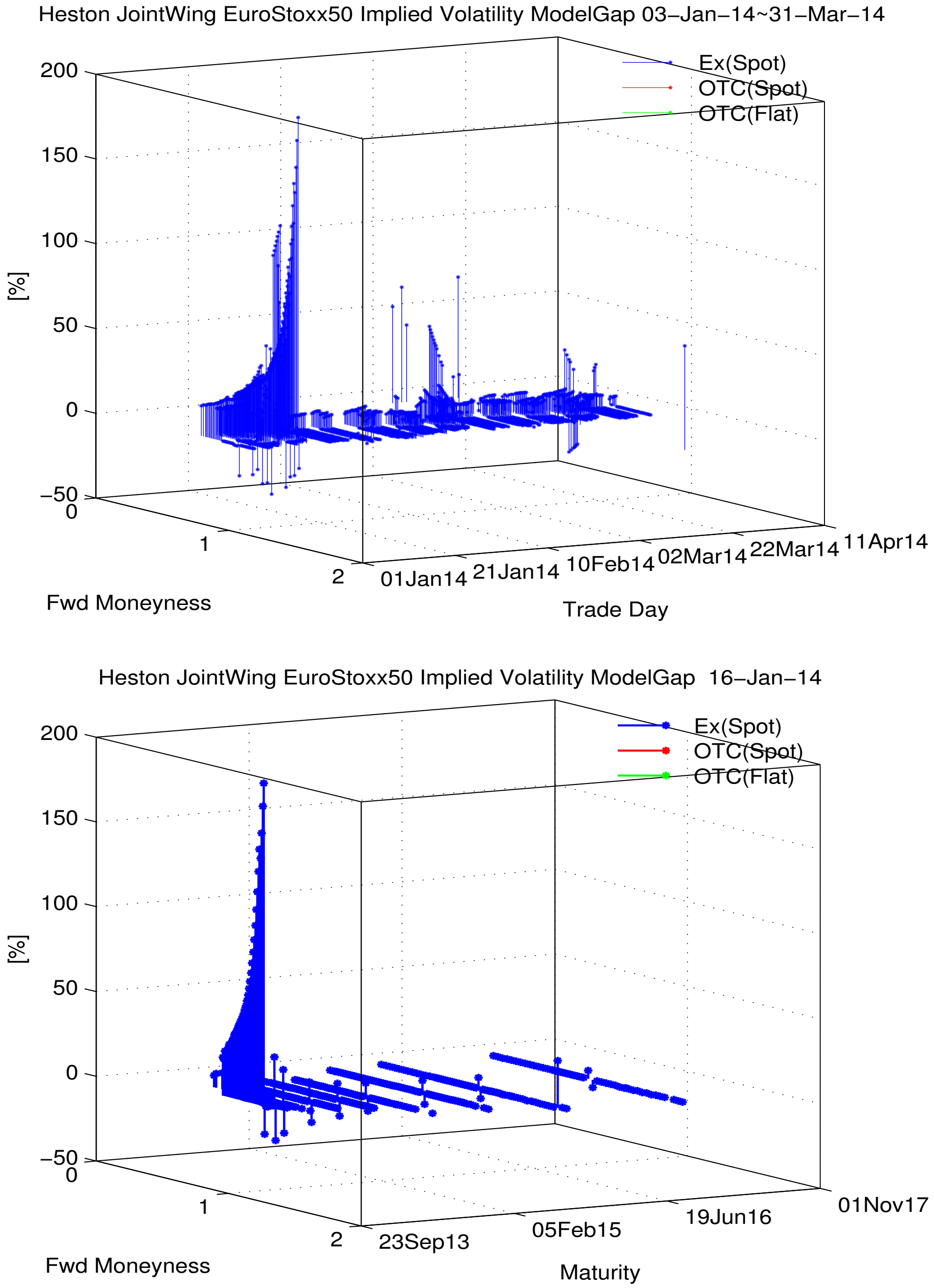
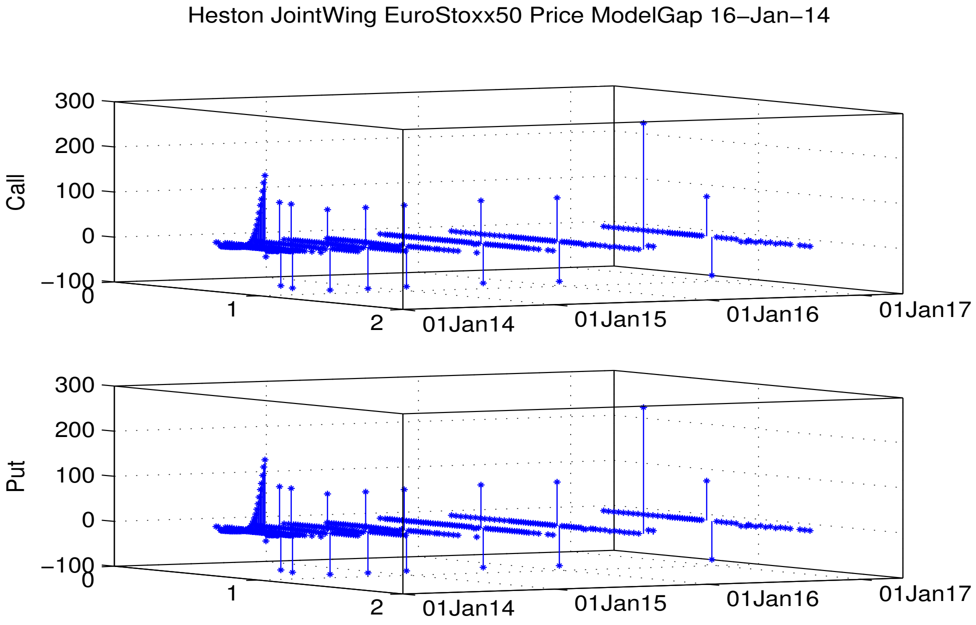


Figure B.129: Heston BoneWing EuroStoxx50 Implied Volatility (Nearest) ModelGap (03-Jan-14~31-Mar-14), (16-Jan-14)



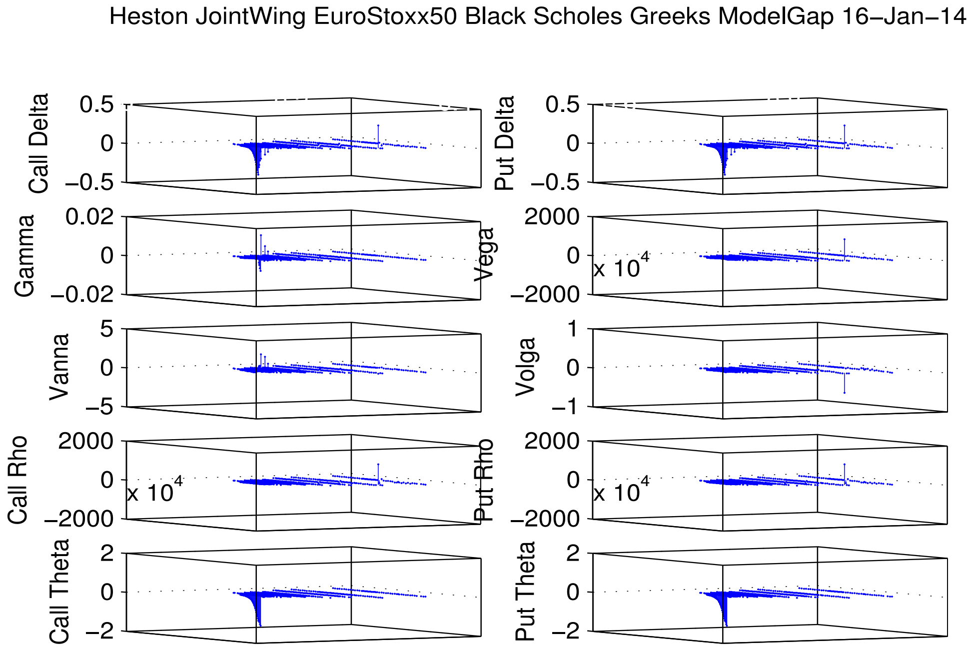


Figure B.130: Heston BoneWing EuroStoxx50 Price, Black Scholes Greeks ModelGap (16-Jan-14)

Table B.38: Root Mean Squared EuroStoxx50 Implied Volatility ModelGap (03-Jan-14∼31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
5.275E-02	4.085E-02	2.422E-02	6.255E-02	5.396E-02	3.628E-02

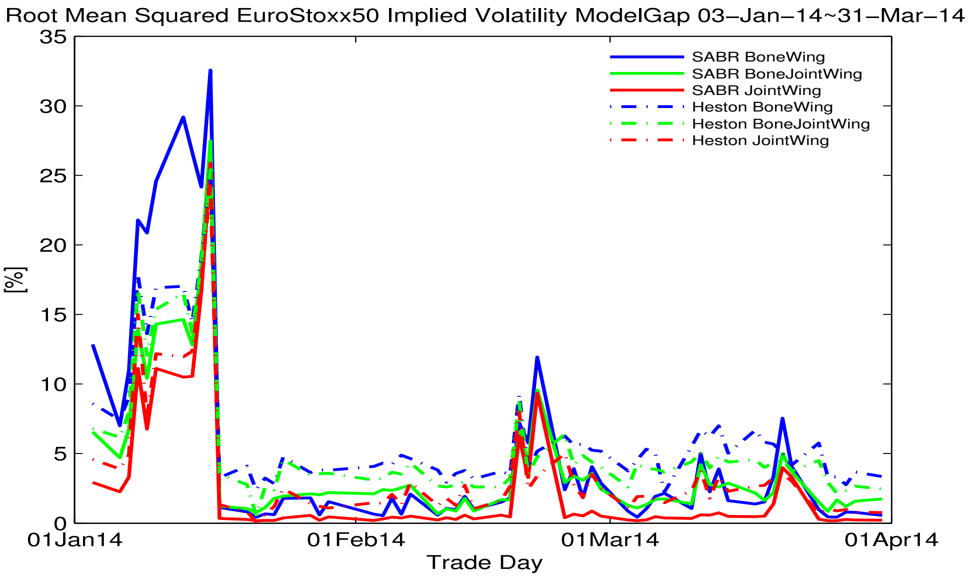


Figure B.131: Root Mean Squared EuroStoxx50 Implied Volatility ModelGap (02-Jan-14∼31-Mar-14)

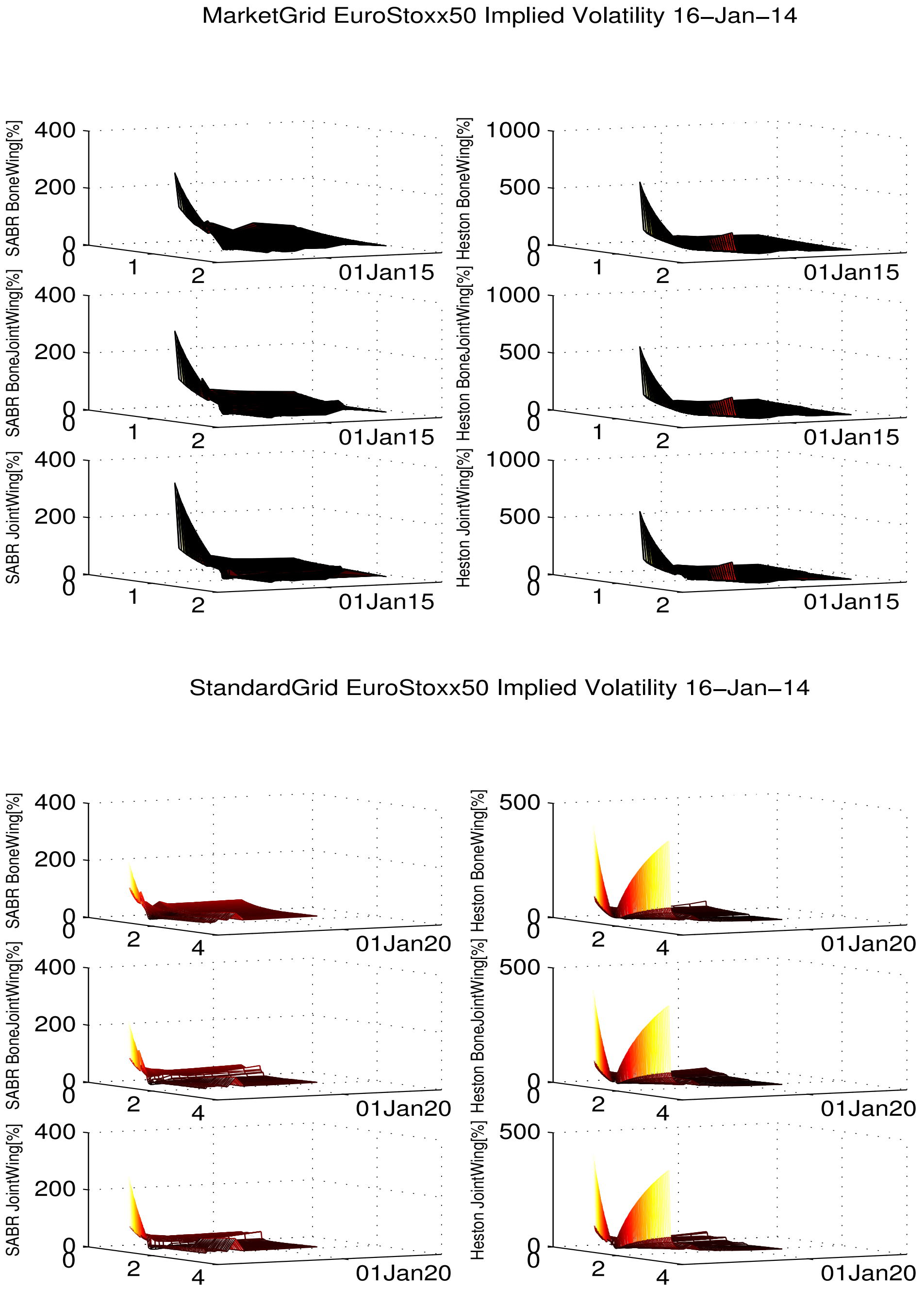


Figure B.132: EuroStoxx50 Market, Standard Grid Implied Volatility (16-Jan-14)

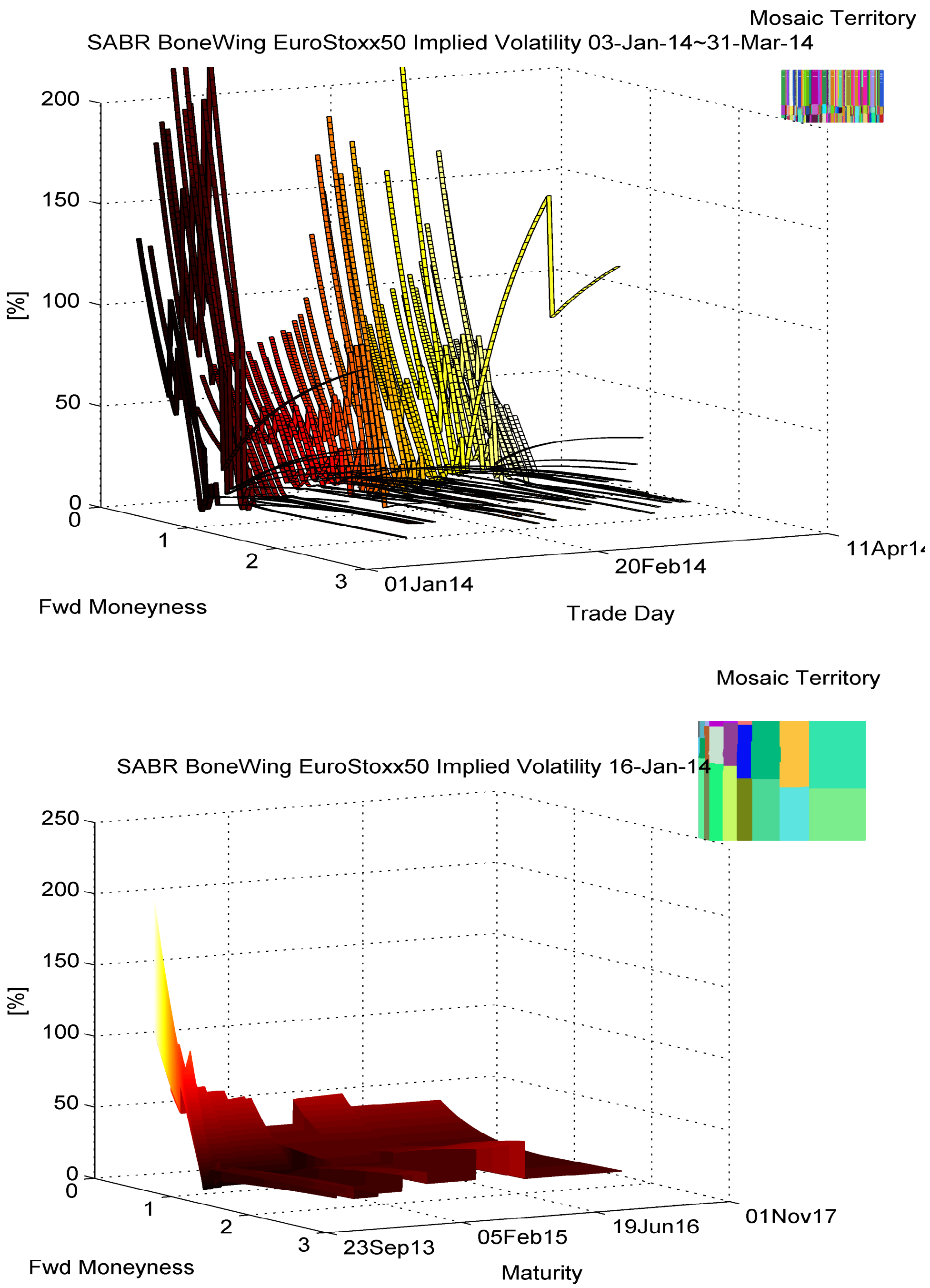
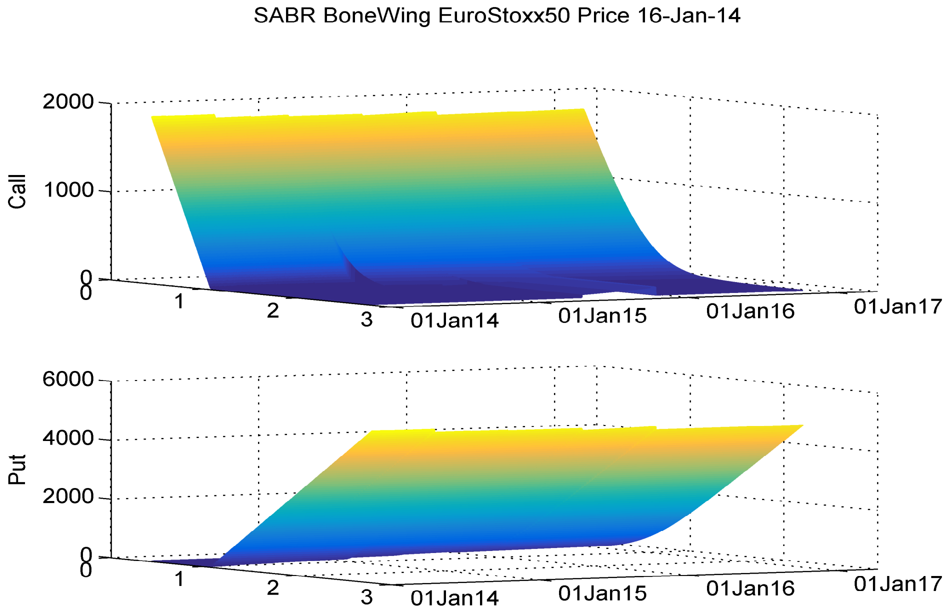


Figure B.133: SABR BoneWing EuroStoxx50 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



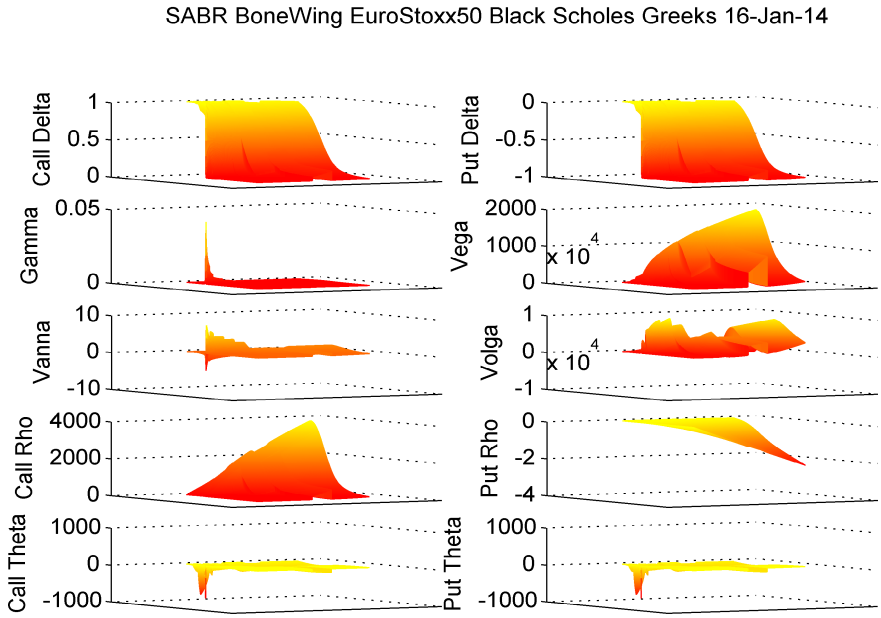
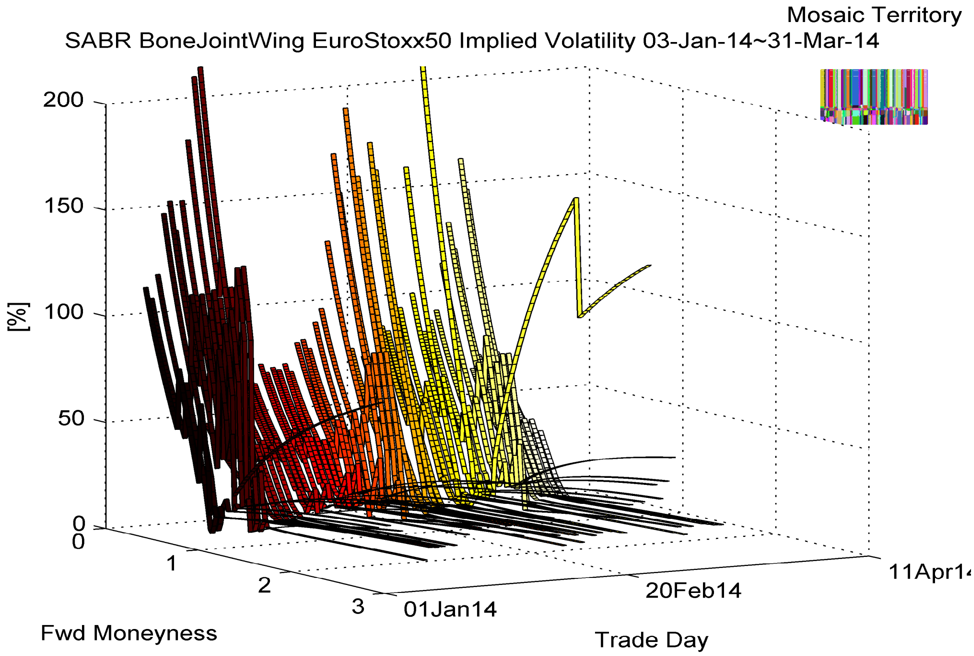


Figure B.134: SABR BoneWing EuroStoxx50 Price, Black Scholes Greeks (16-Jan-14)



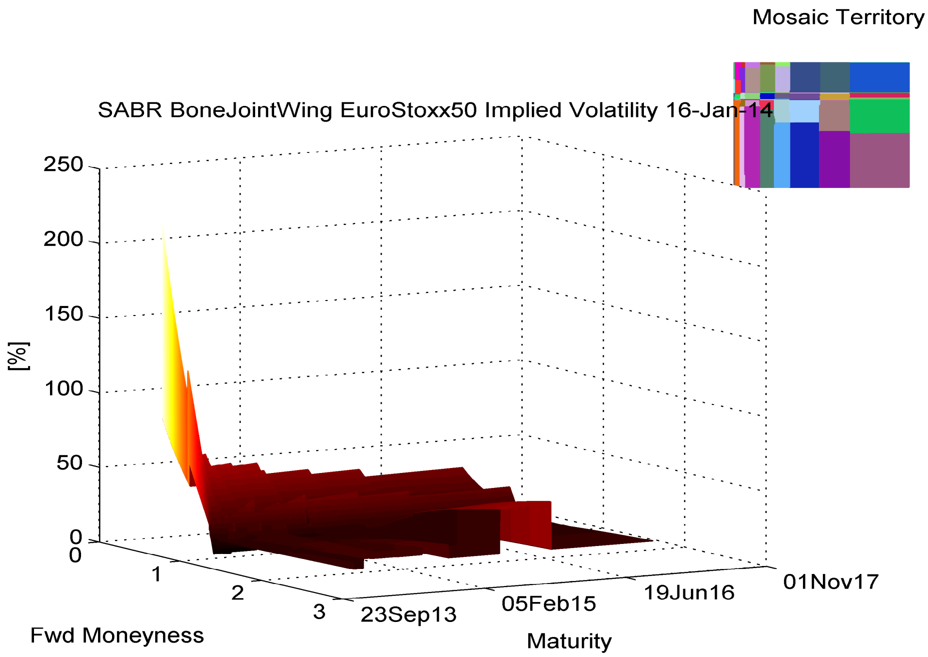
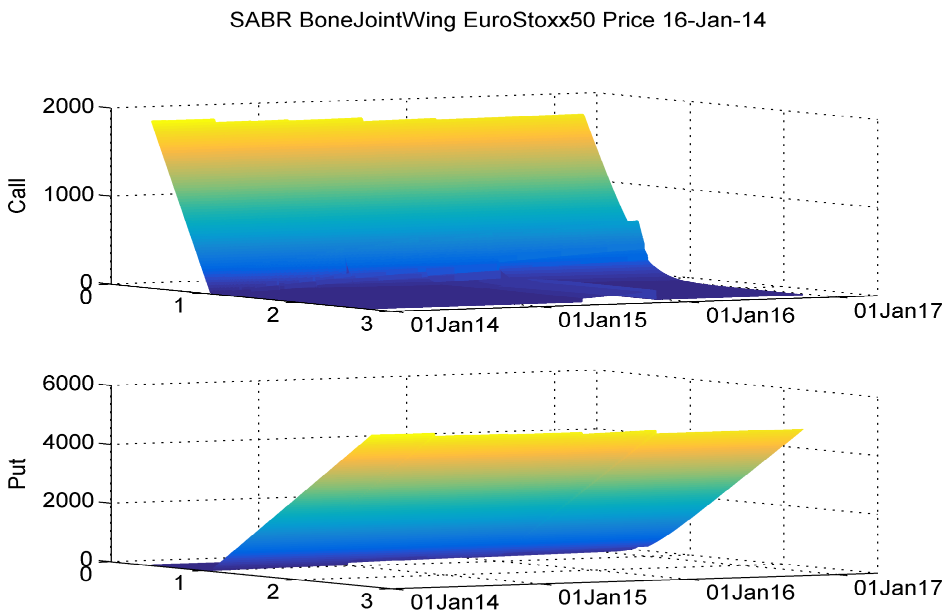


Figure B.135: SABR BoneWing EuroStoxx50 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



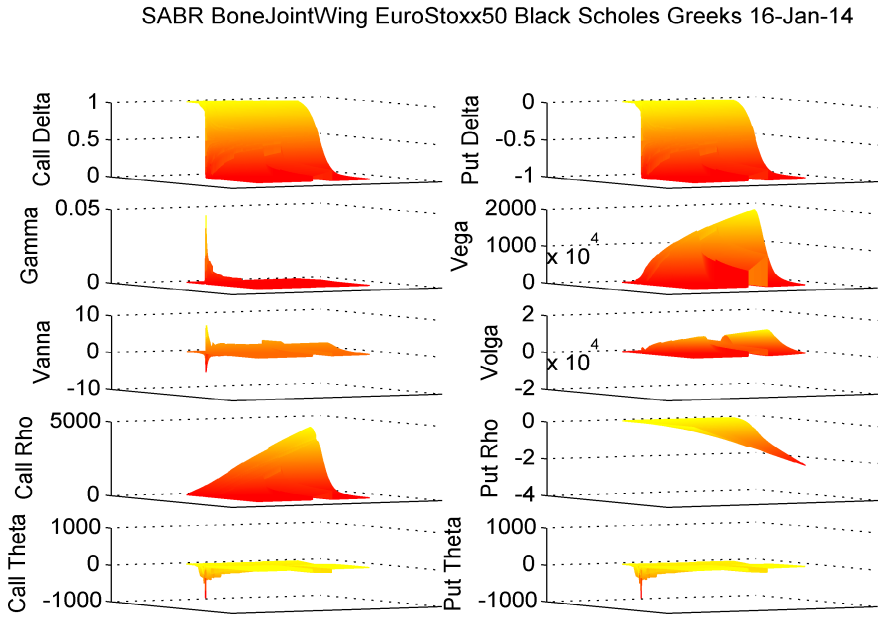
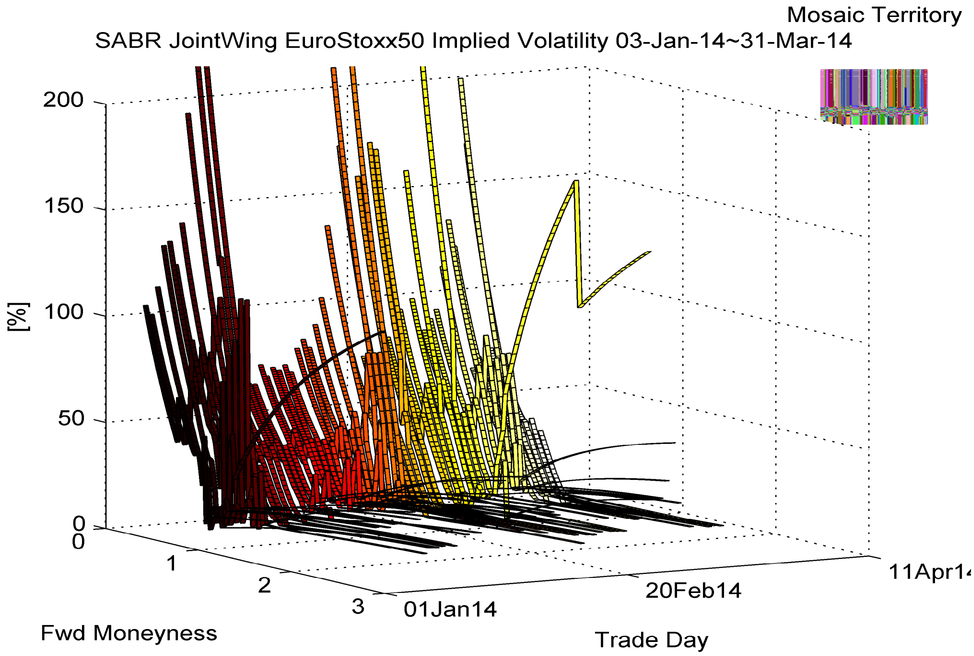


Figure B.136: SABR BoneWing EuroStoxx50 Price, Black Scholes Greeks (16-Jan-14)



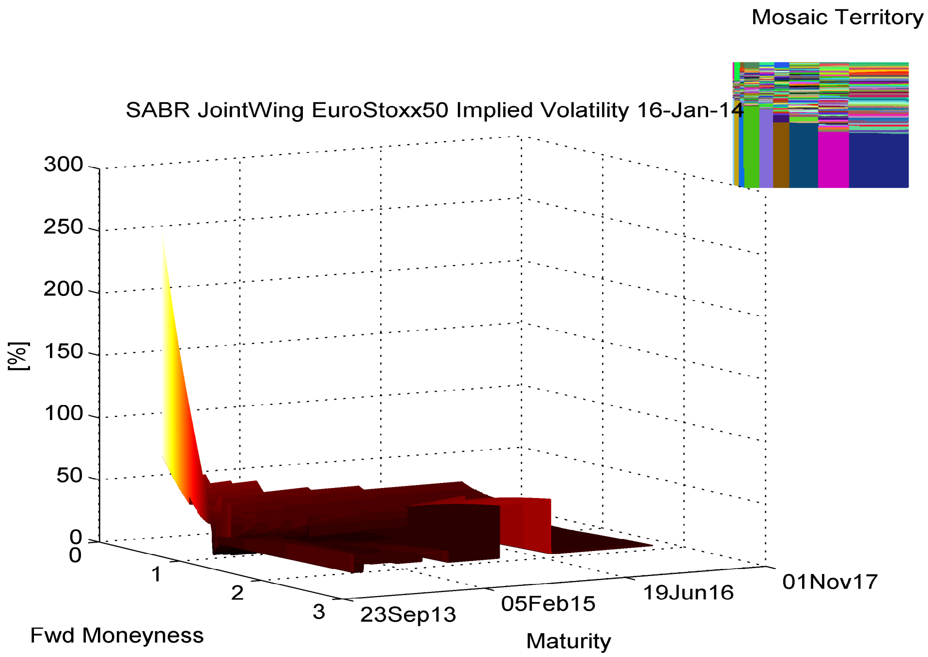
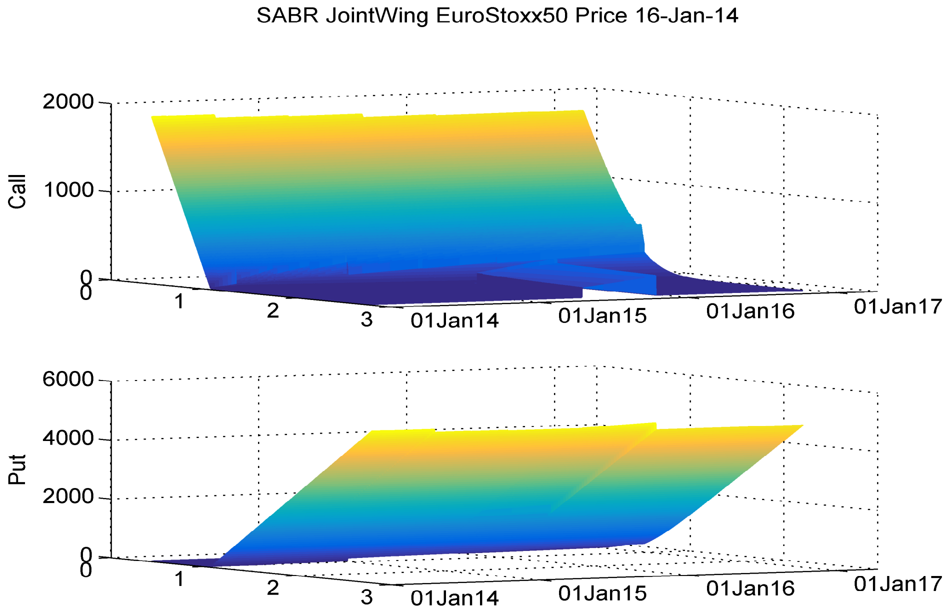


Figure B.137: SABR BoneWing EuroStoxx50 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



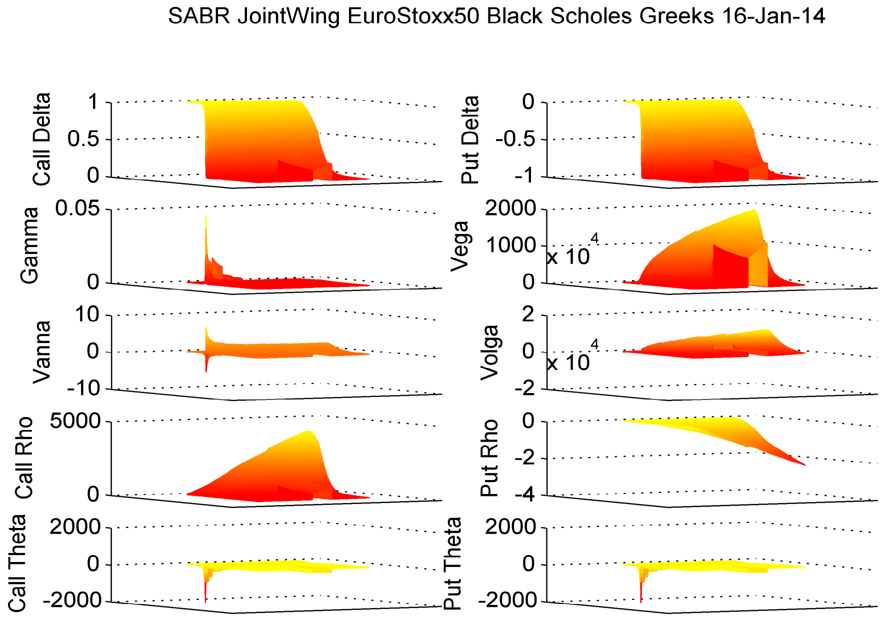
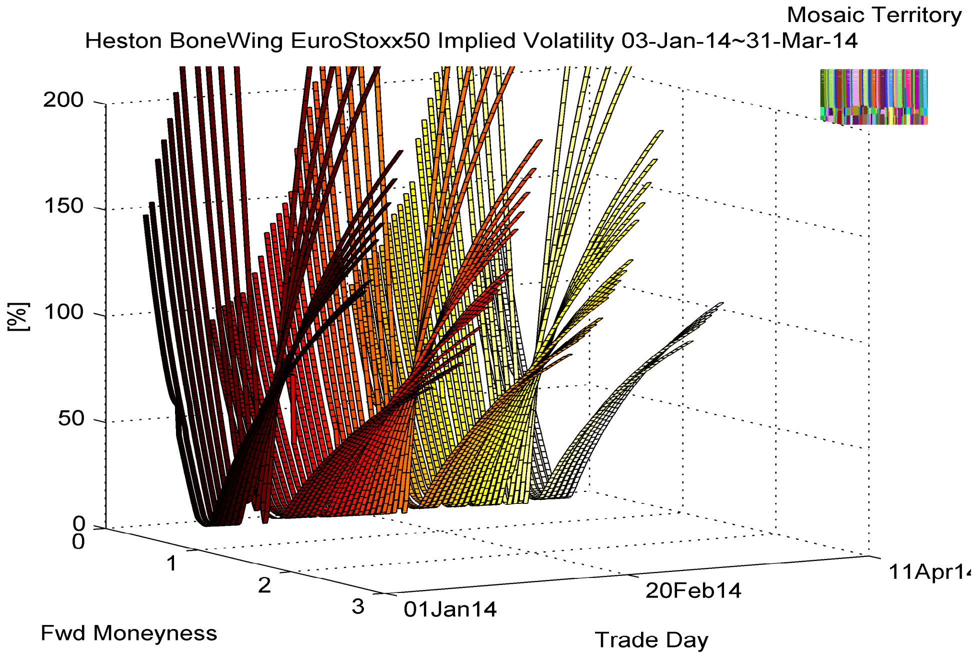


Figure B.138: SABR BoneWing EuroStoxx50 Price, Black Scholes Greeks (16-Jan-14)



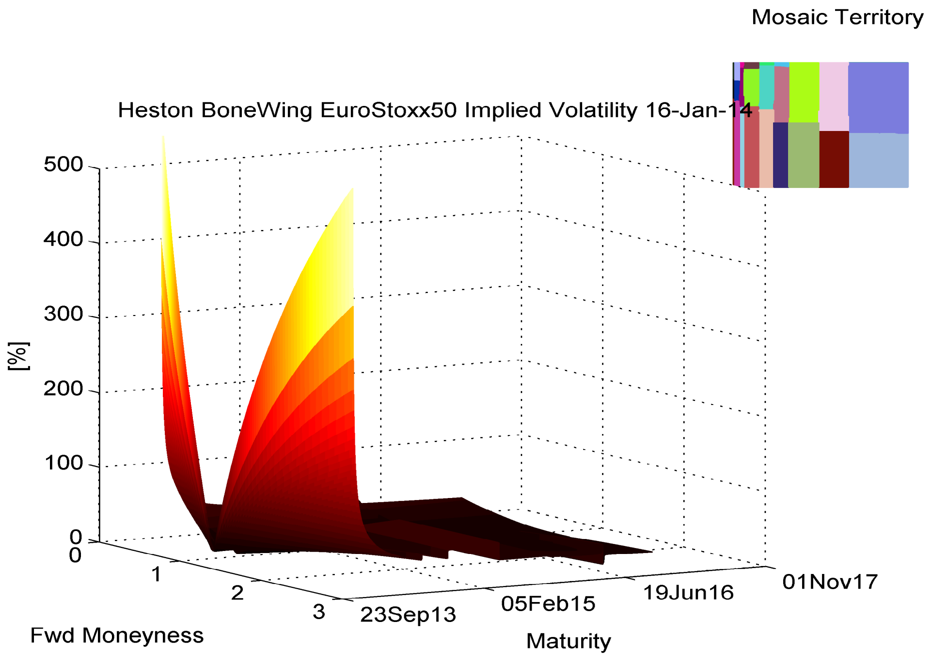
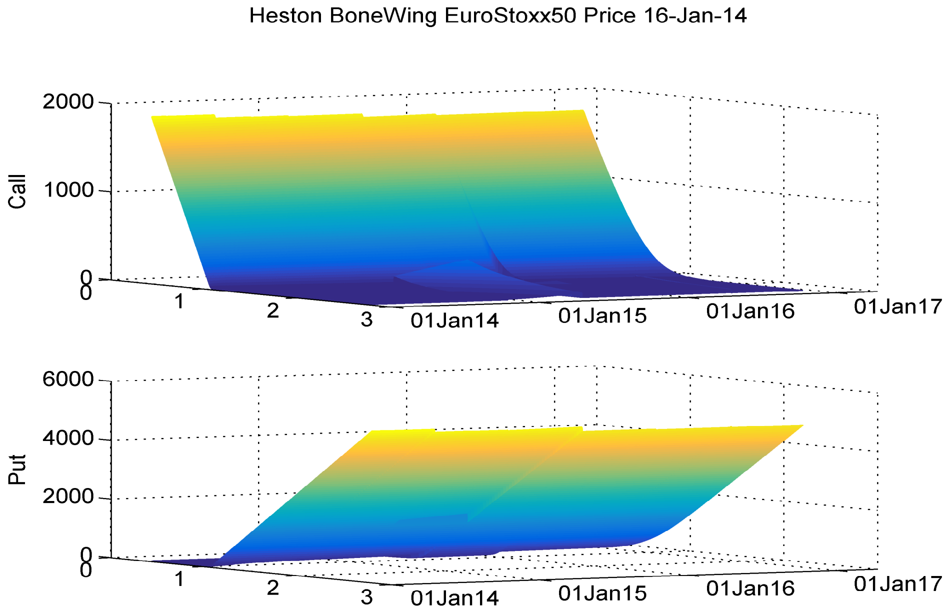


Figure B.139: Heston BoneWing EuroStoxx50 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



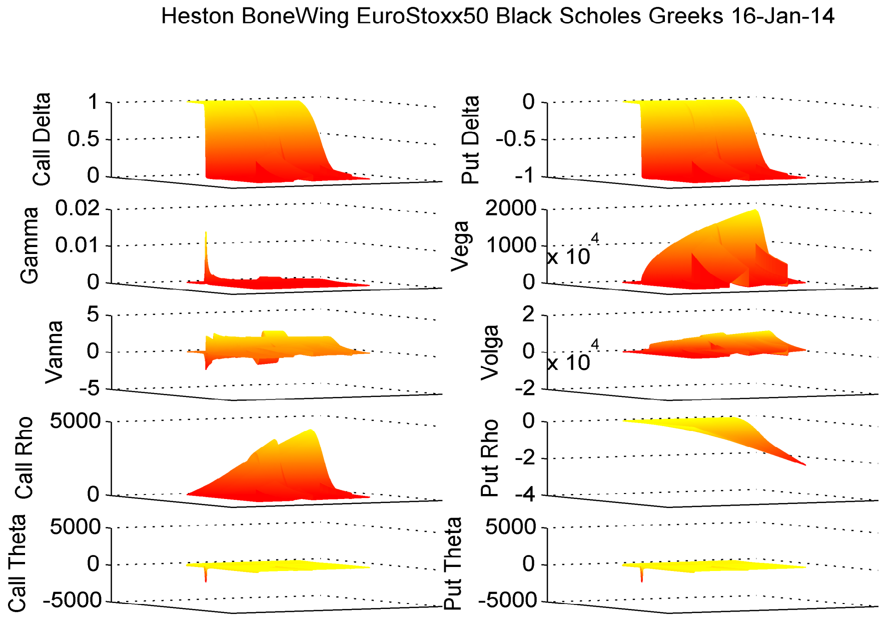
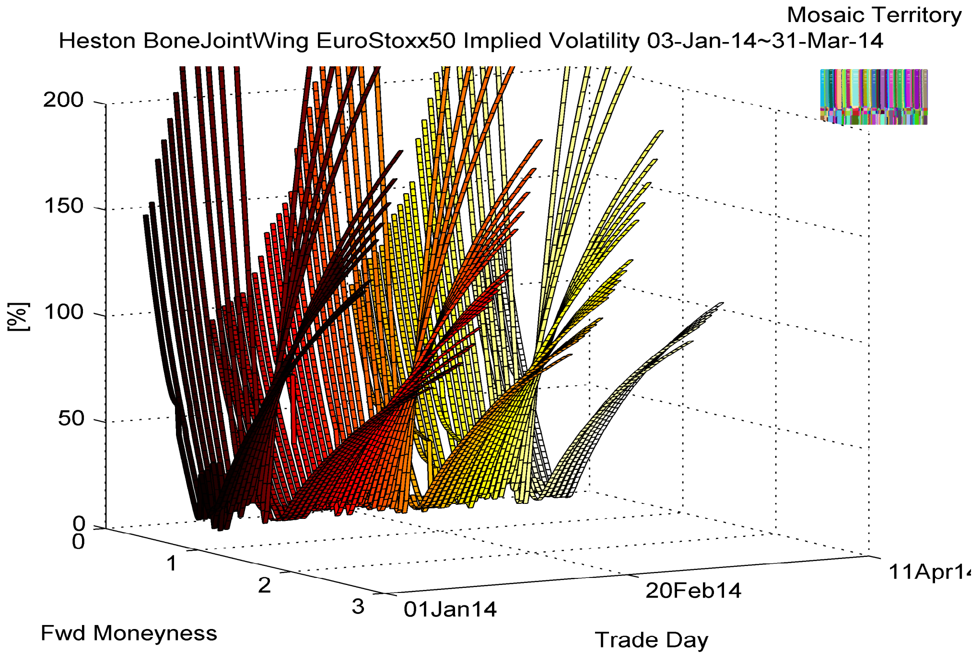


Figure B.140: Heston BoneWing EuroStoxx50 Price, Black Scholes Greeks (16-Jan-14)



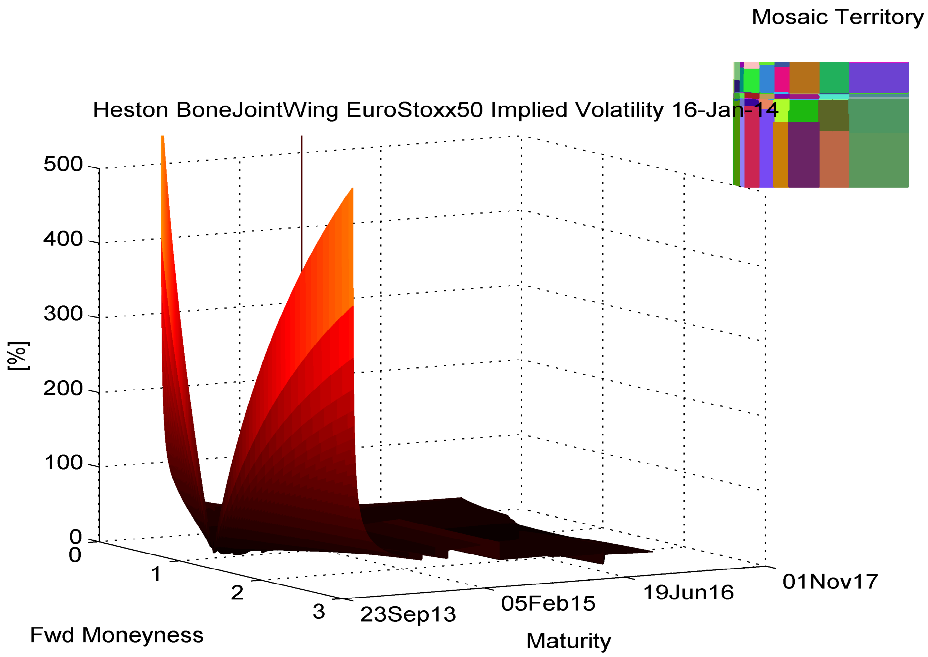
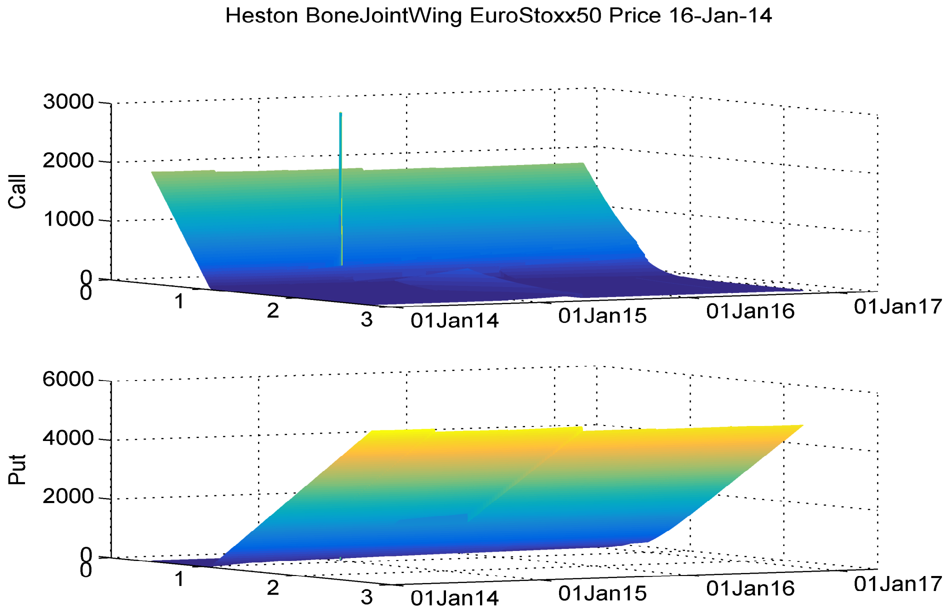


Figure B.141: Heston BoneWing EuroStoxx50 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)



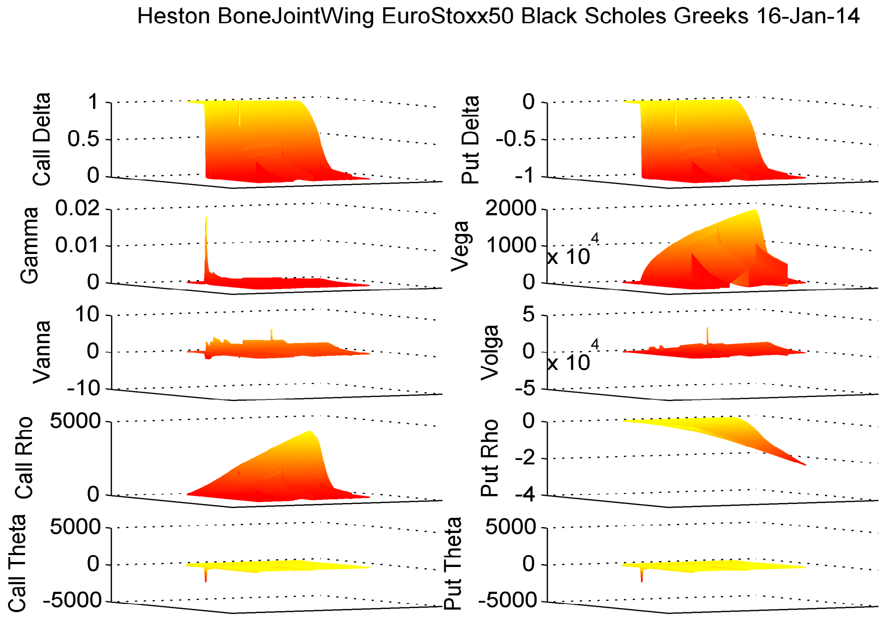
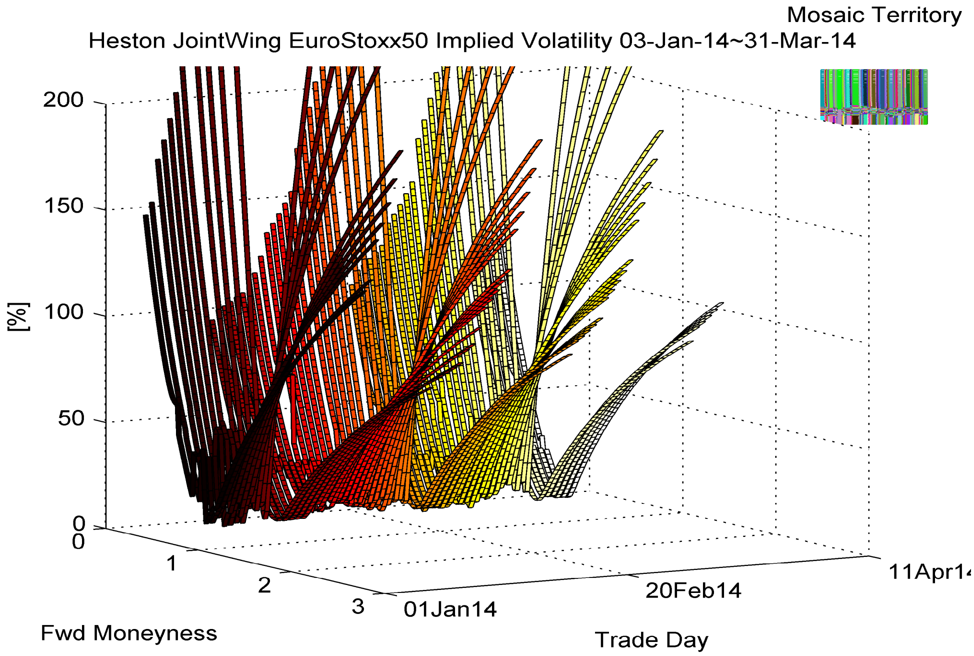


Figure B.142: Heston BoneWing EuroStoxx50 Price, Black Scholes Greeks (16-Jan-14)



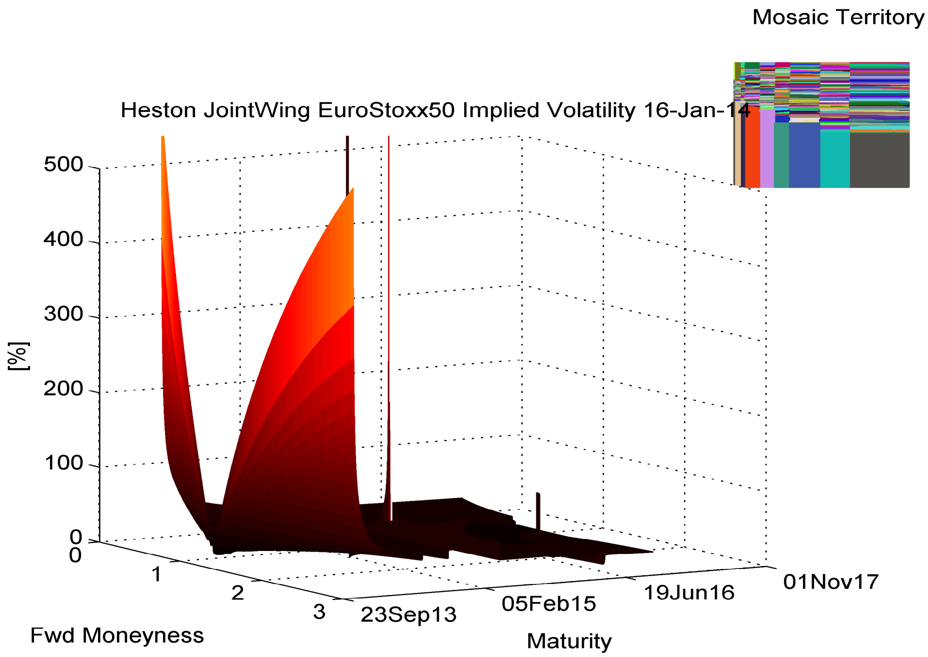


Figure B.143: Heston BoneWing EuroStoxx50 Implied Volatility (Nearest) (03-Jan-14~31-Mar-14), (16-Jan-14)

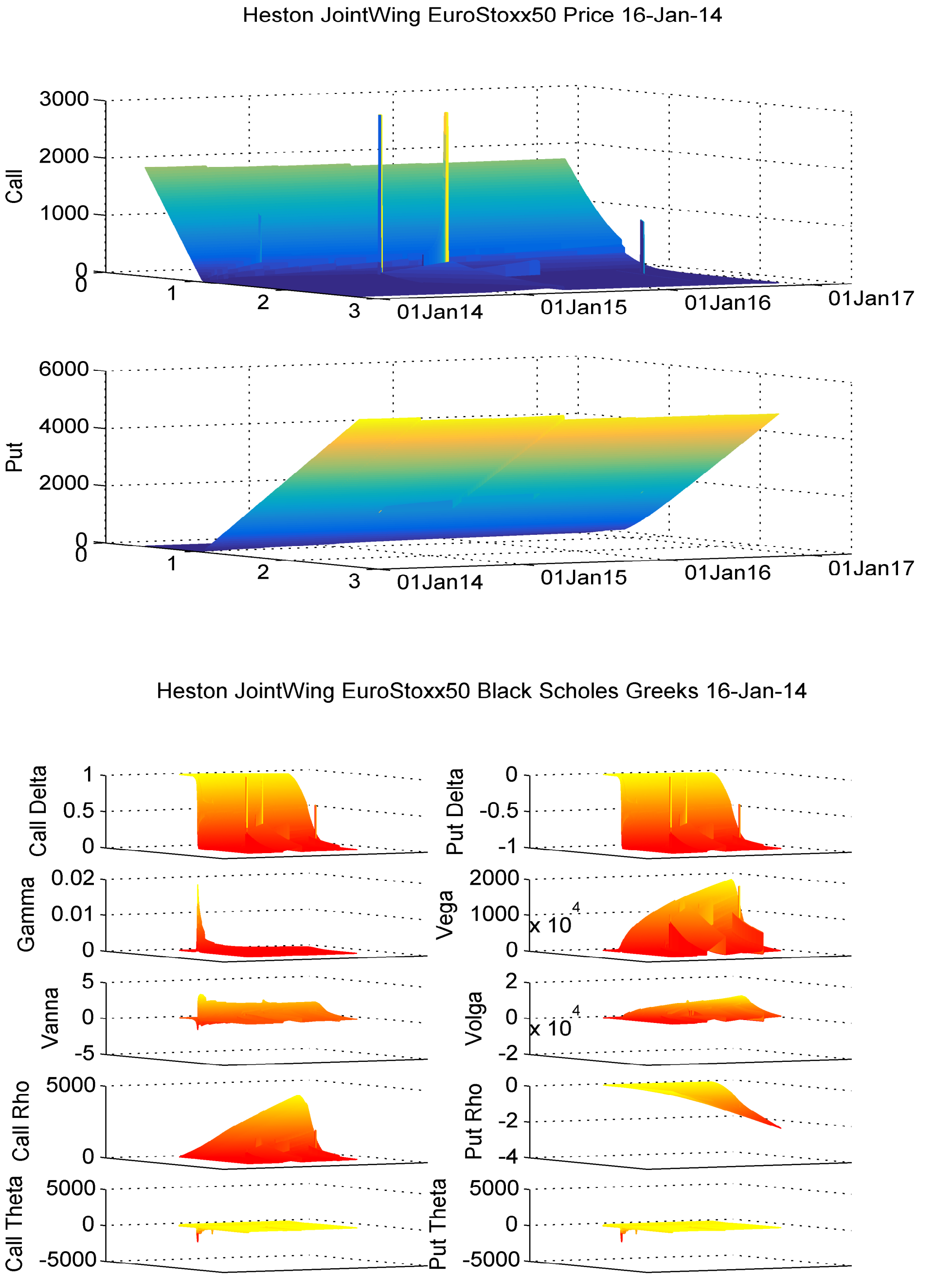
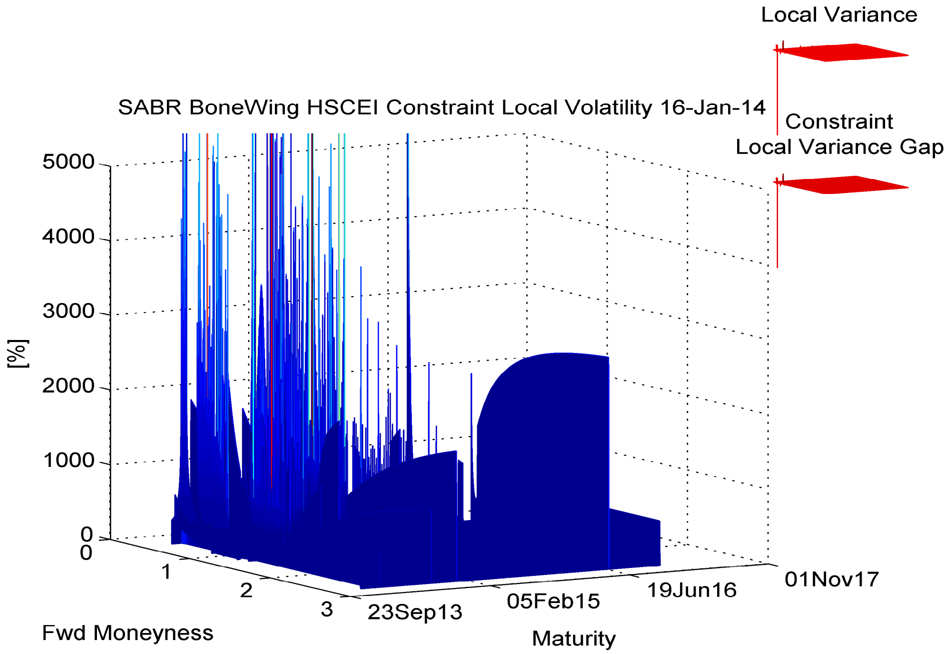


Figure B.144: Heston BoneWing EuroStoxx50 Price, Black Scholes Greeks (16-Jan-14)



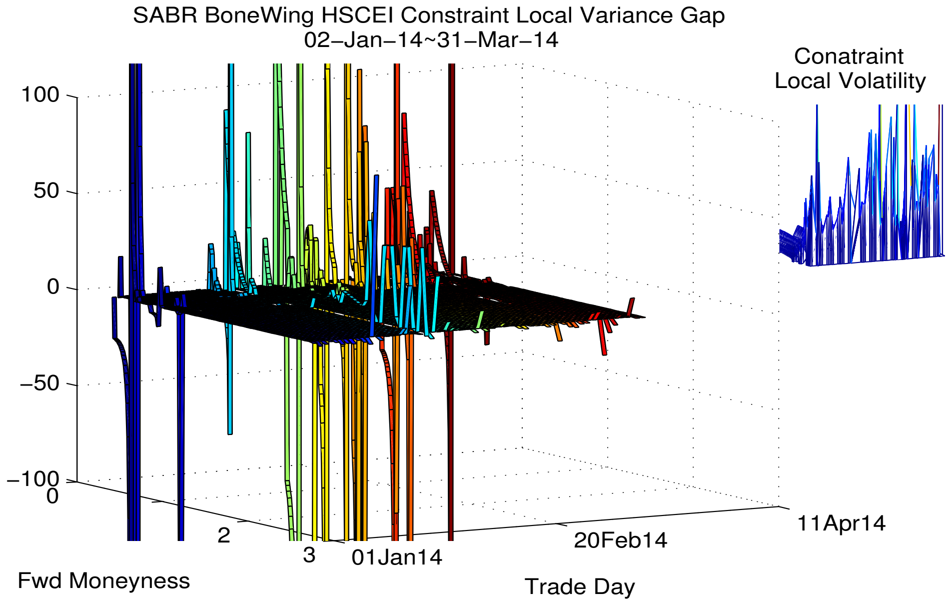
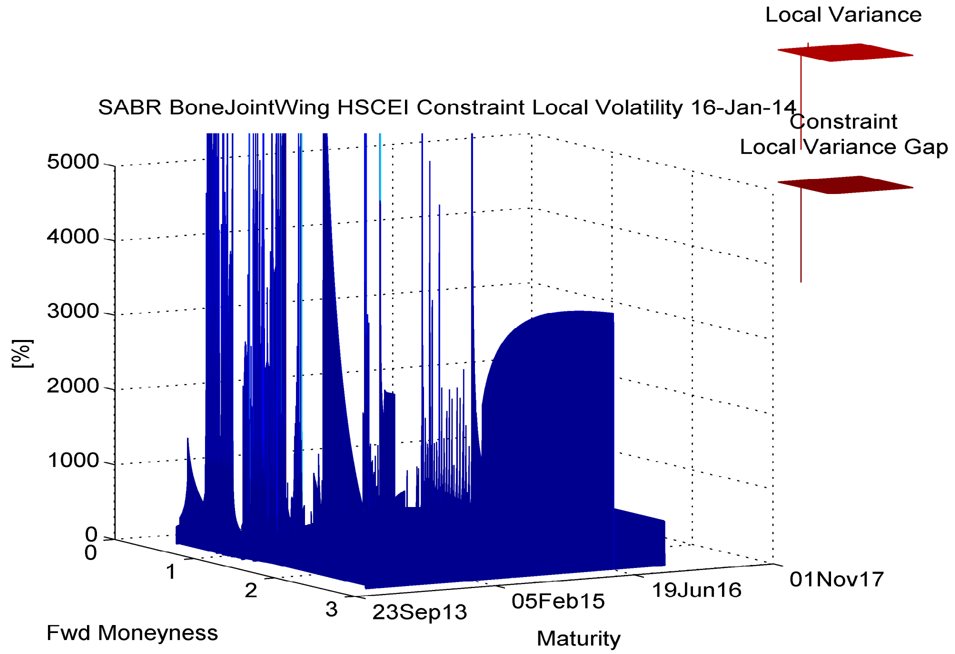


Figure B.145: SABR BoneWing HSCEI Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)



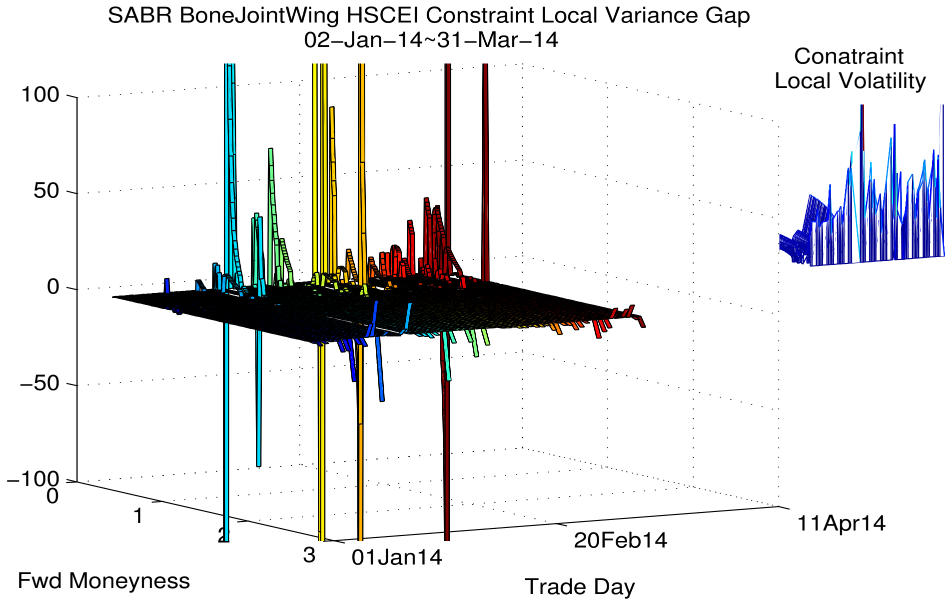
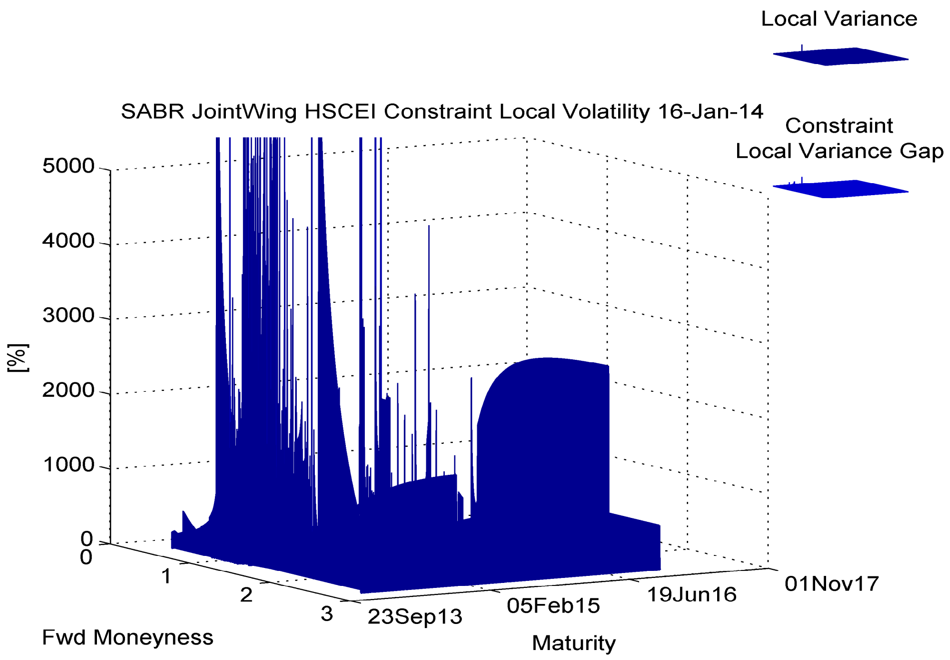


Figure B.146: SABR BoneJointWing HSCEI Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



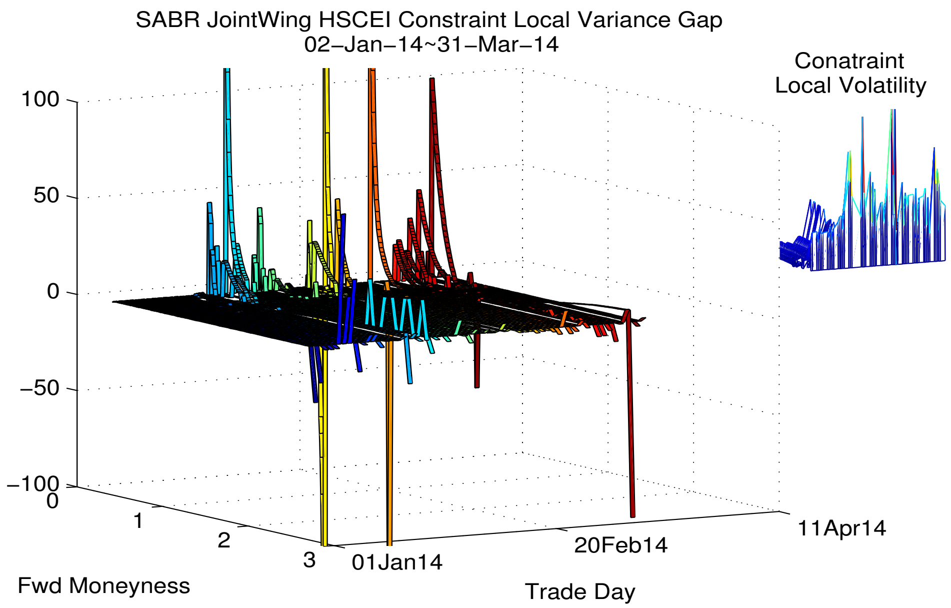
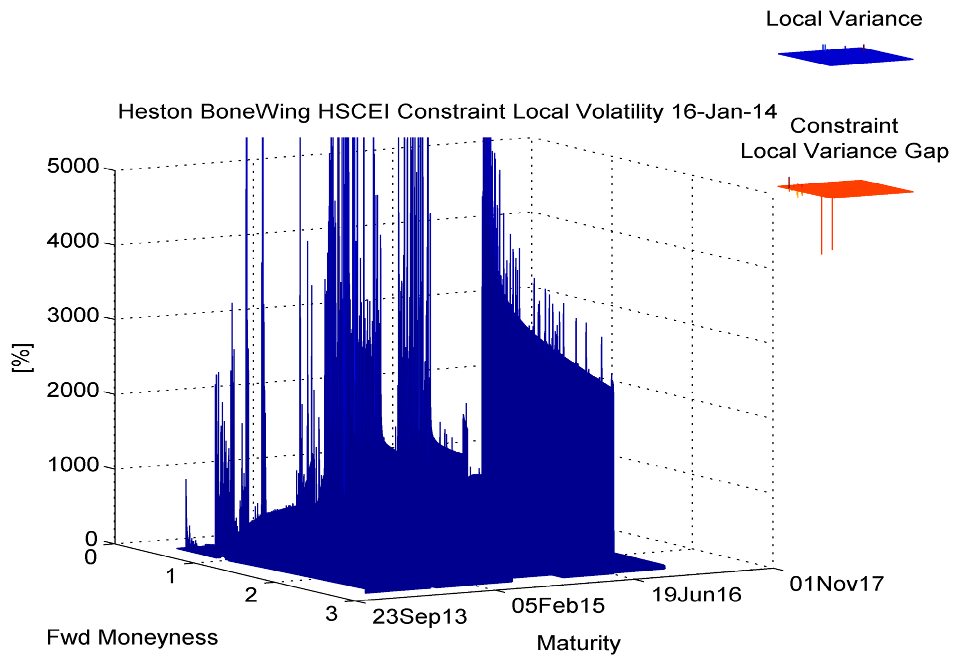


Figure B.147: SABR JointWing HSCEI Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



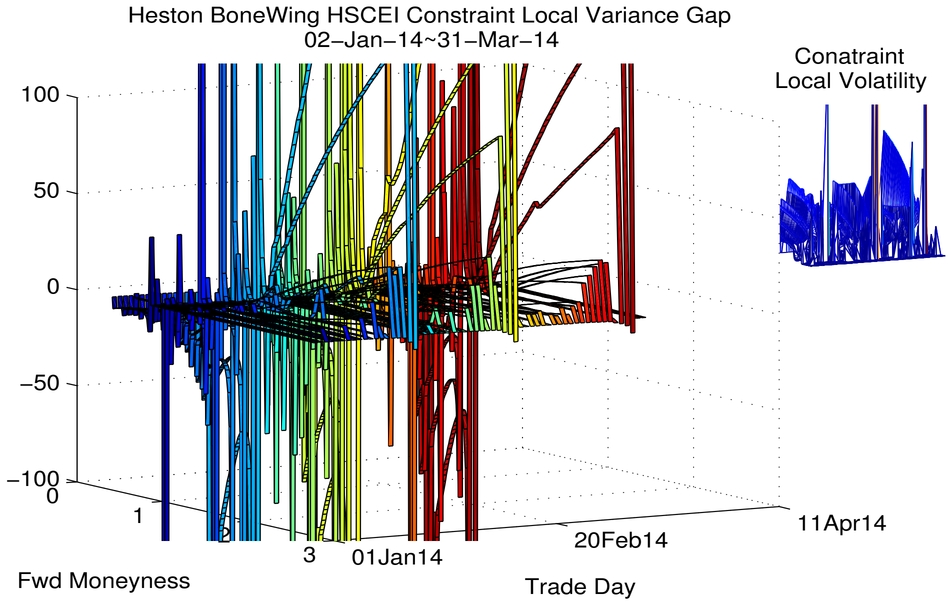
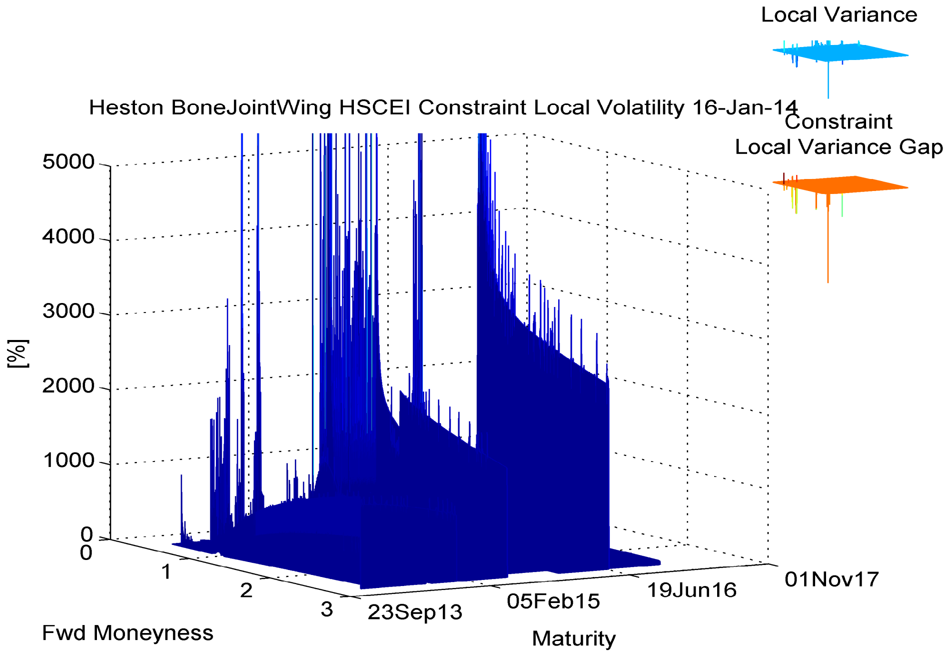


Figure B.148: Heston BoneWing HSCEI Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)



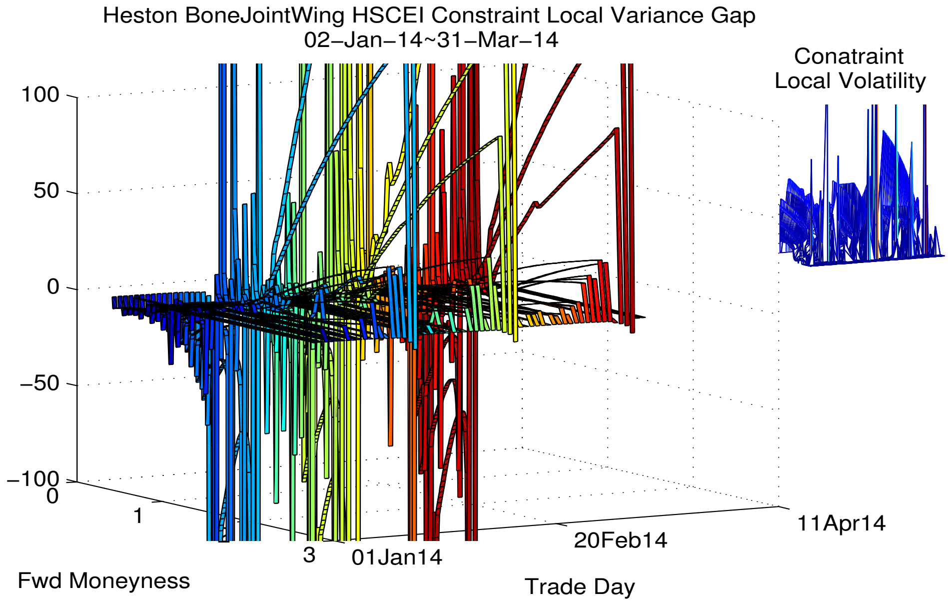
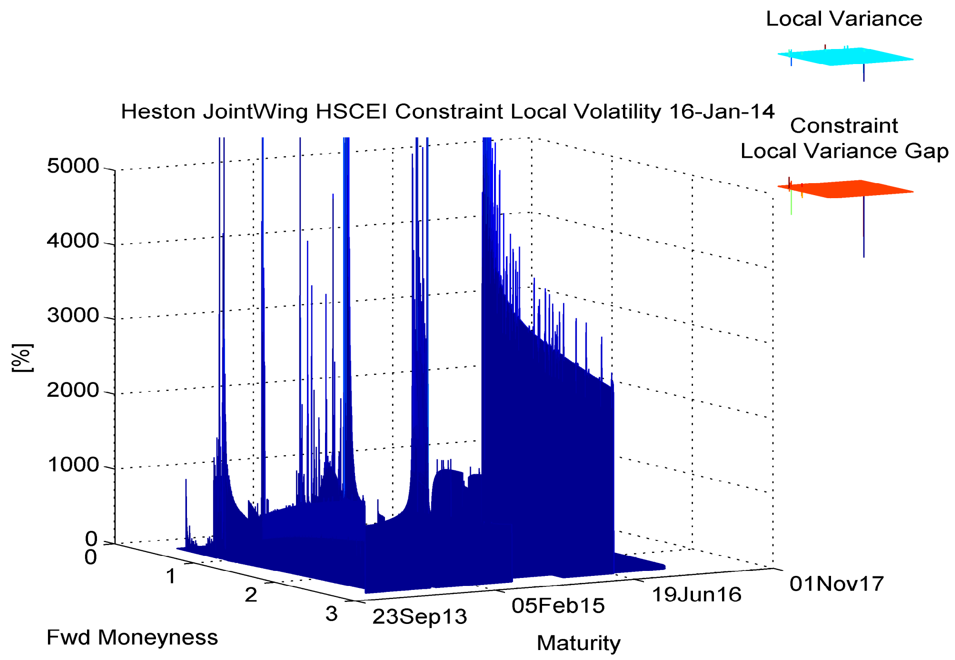


Figure B.149: Heston BoneJointWing HSCEI Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



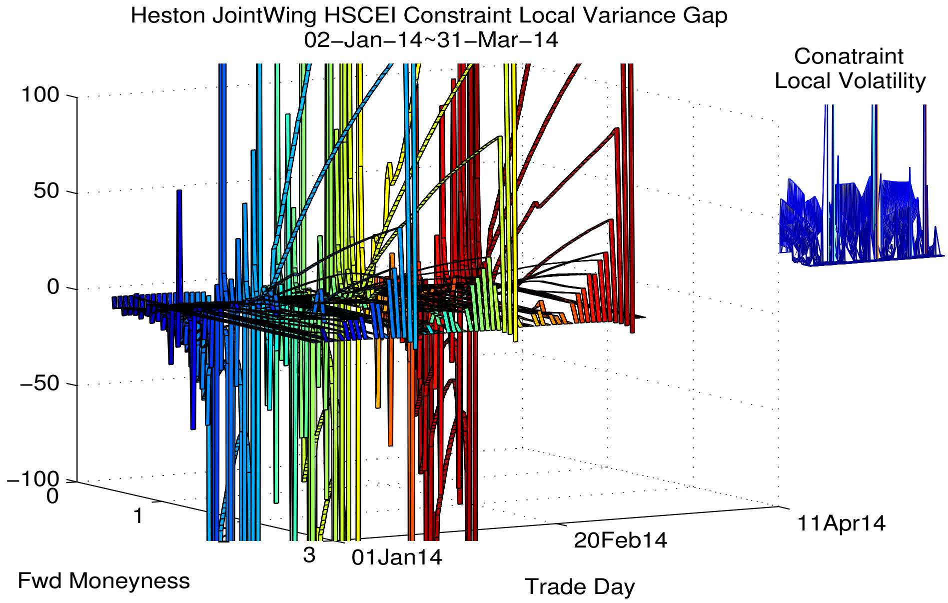


Figure B.150: Heston JointWing HSCEI Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)

Table B.39: Root Mean Absolute HSCEI Constraint Local Variance Gap (02-Jan-14~31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
1.726E+0	1.456E+0	1.261E+0	1.352E+0	1.290E+0	1.132E+0

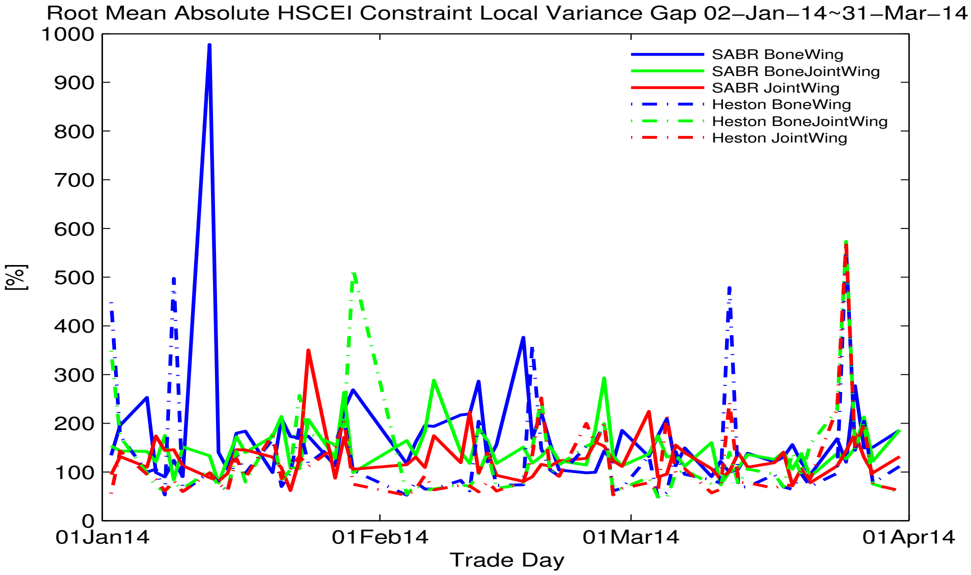
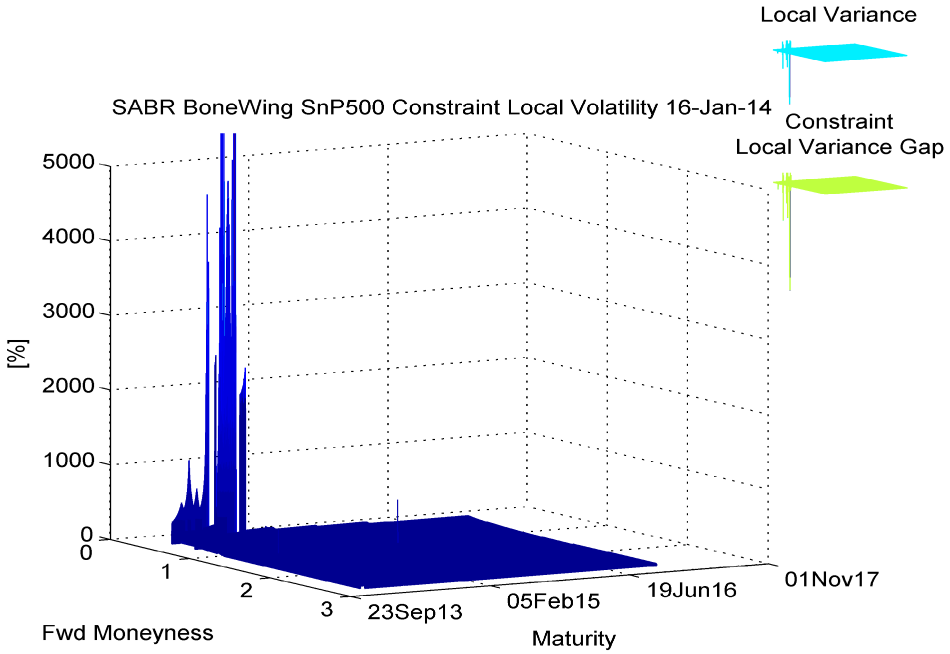


Figure B.151: Root Mean Absolute HSCEI Local Variance Gap (02-Jan-14~31-Mar-14)



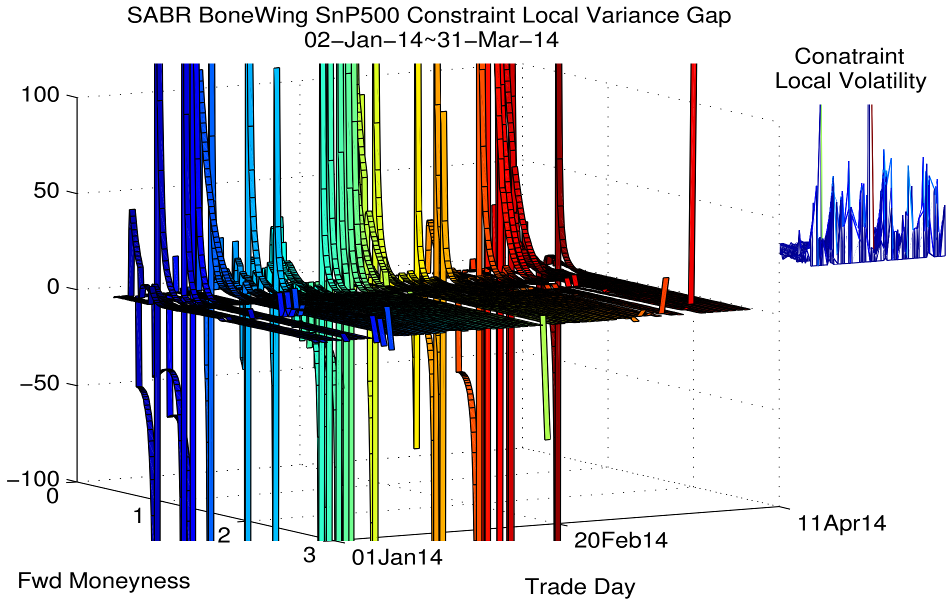
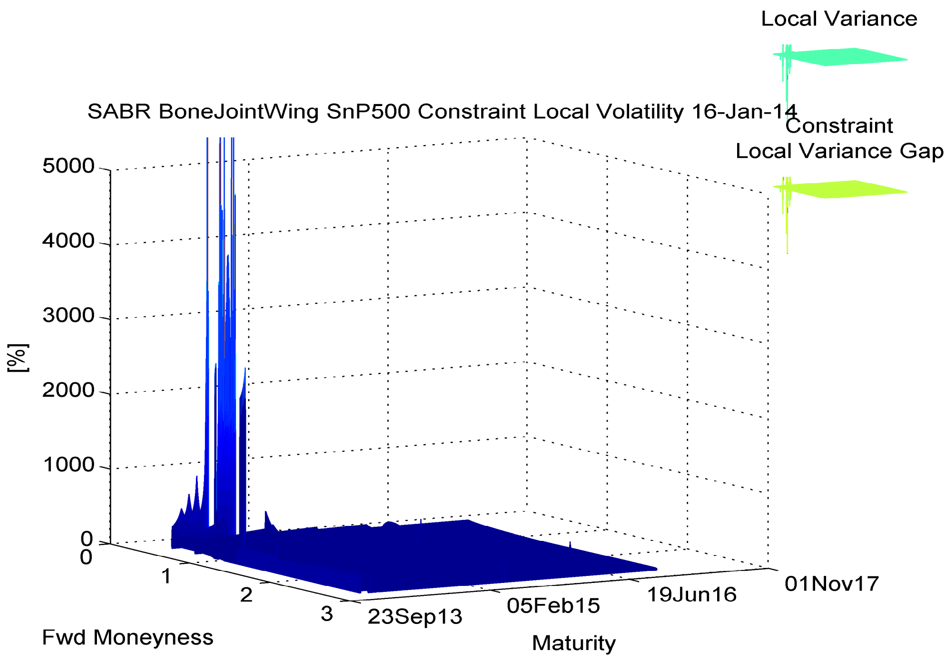


Figure B.152: SABR BoneWing SnP500 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)



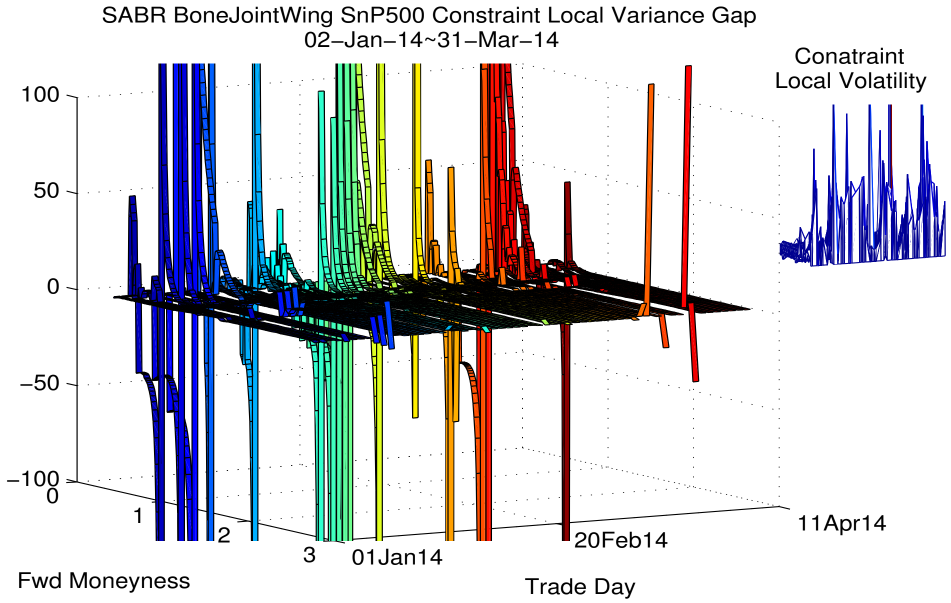
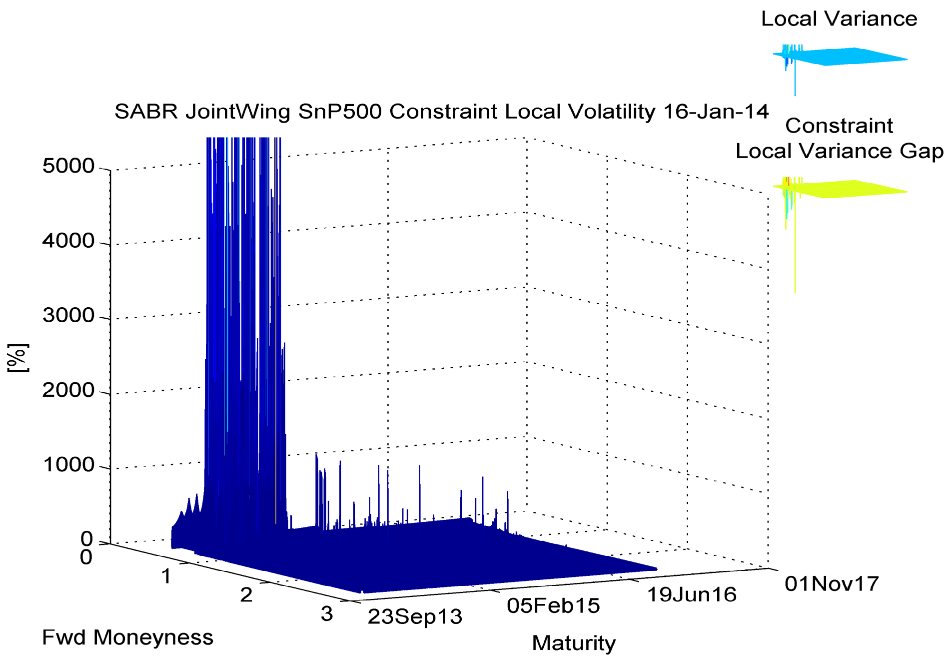


Figure B.153: SABR BoneJointWing SnP500 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



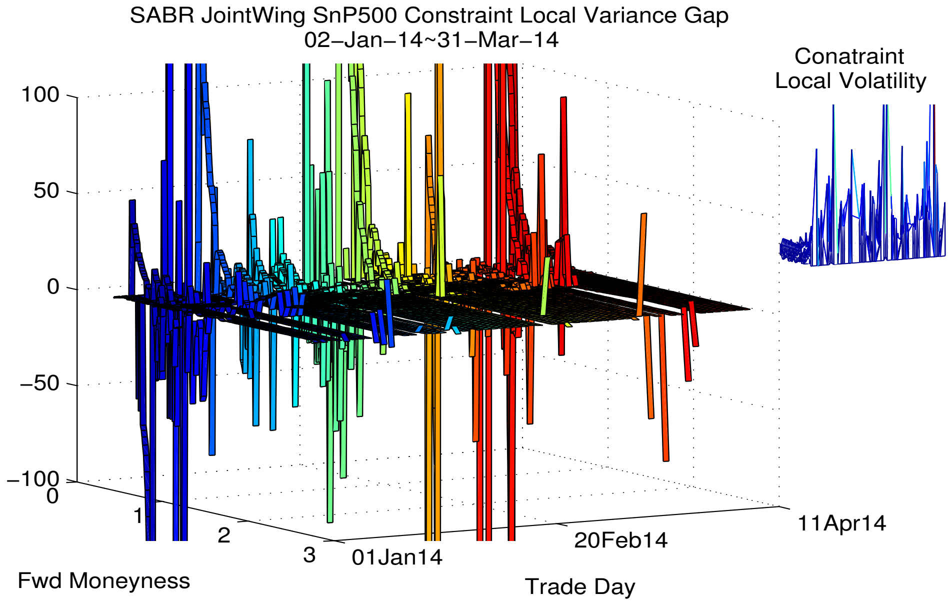
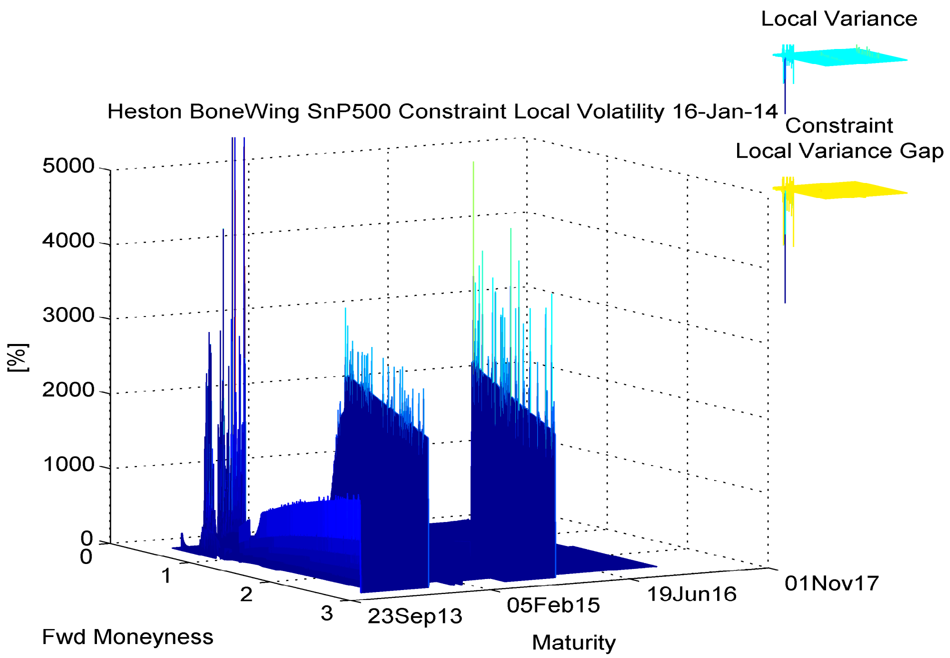


Figure B.154: SABR JointWing SnP500 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)



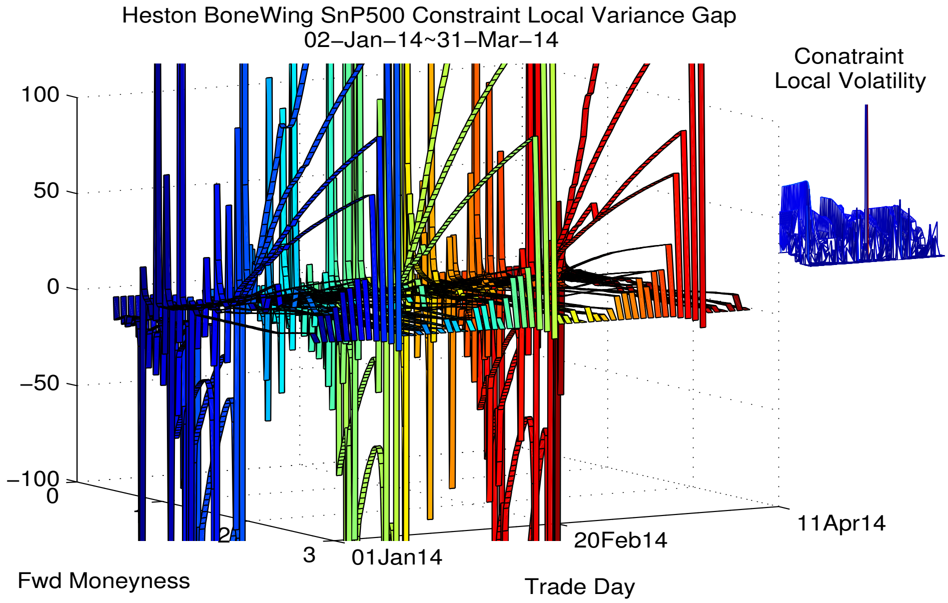
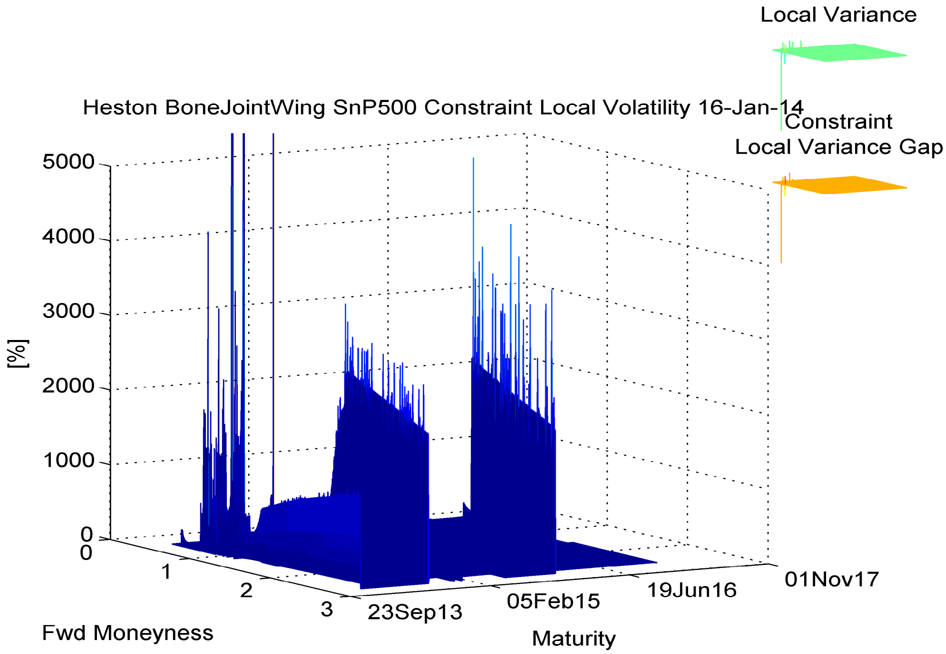


Figure B.155: Heston BoneWing SnP500 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



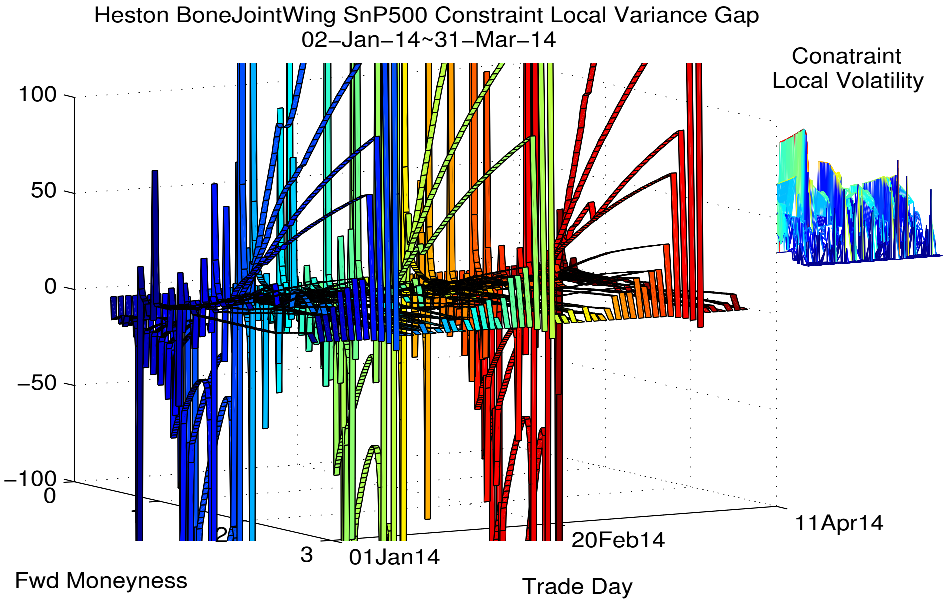
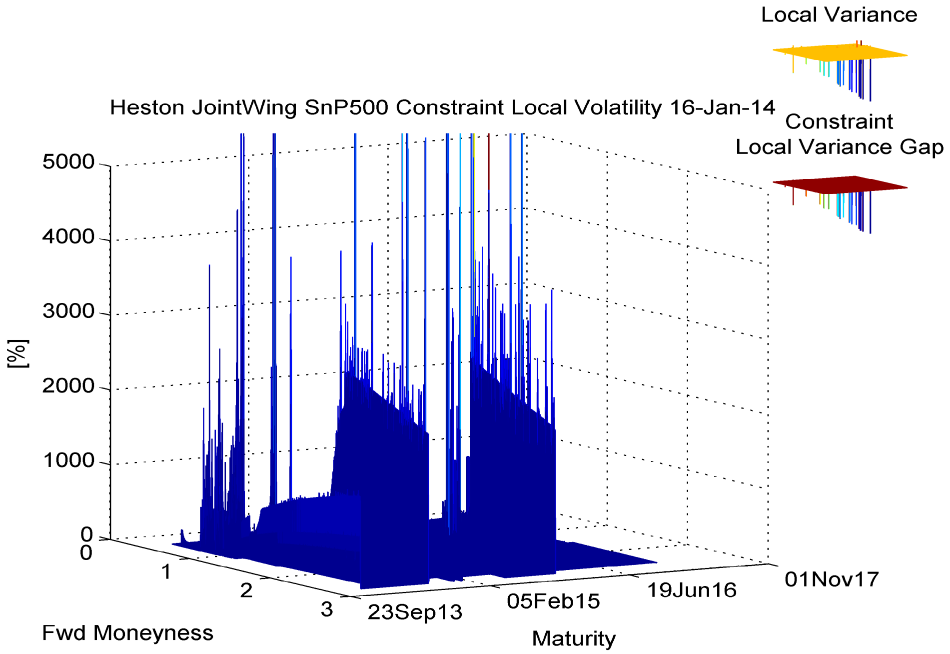


Figure B.156: Heston BoneJointWing SnP500 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



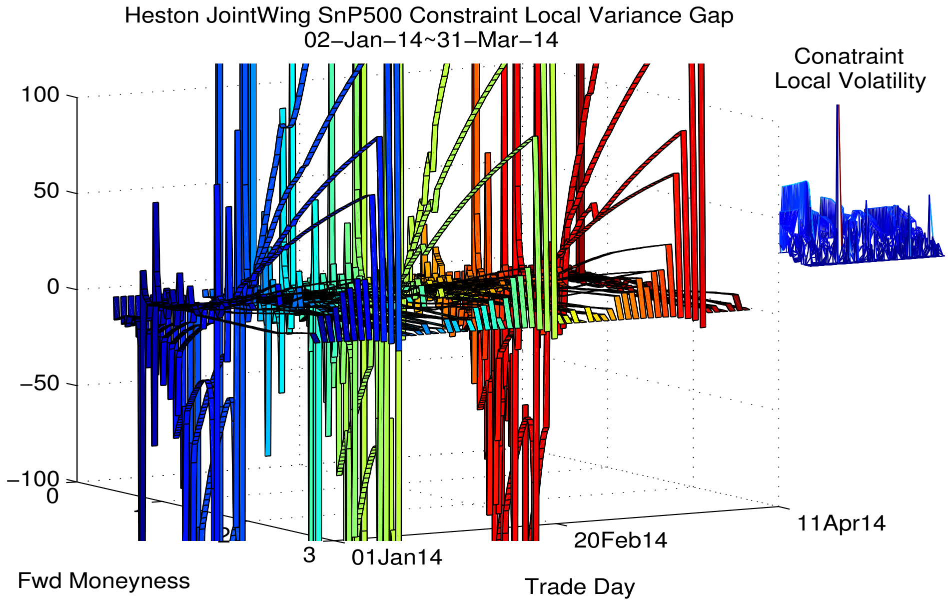


Figure B.157: Heston JointWing SnP500 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)

Table B.40: Root Mean Absolute SnP500 Constraint Local Variance Gap (02-Jan-14~31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
1.278E+0	1.362E+0	1.363E+0	1.189E+0	1.133E+0	1.171E+0

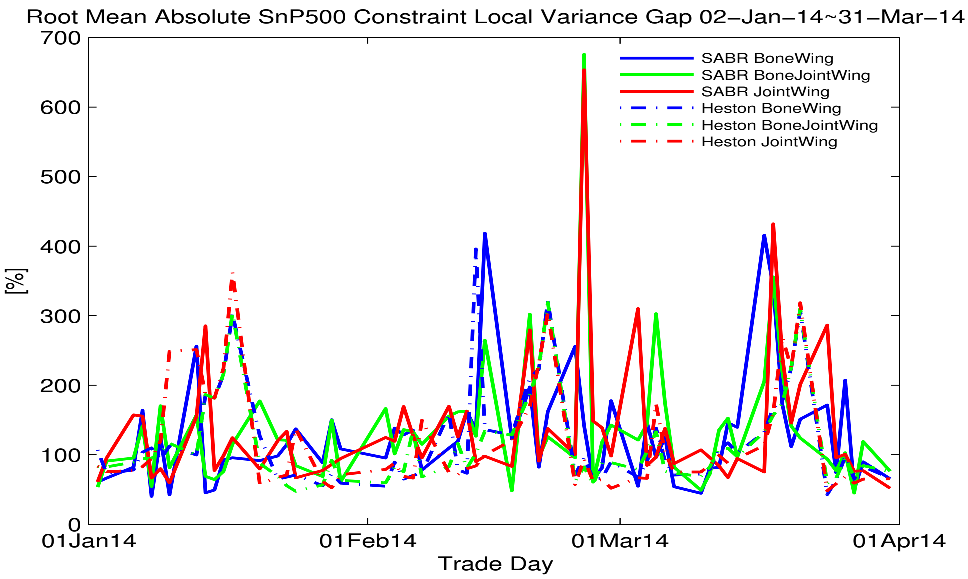
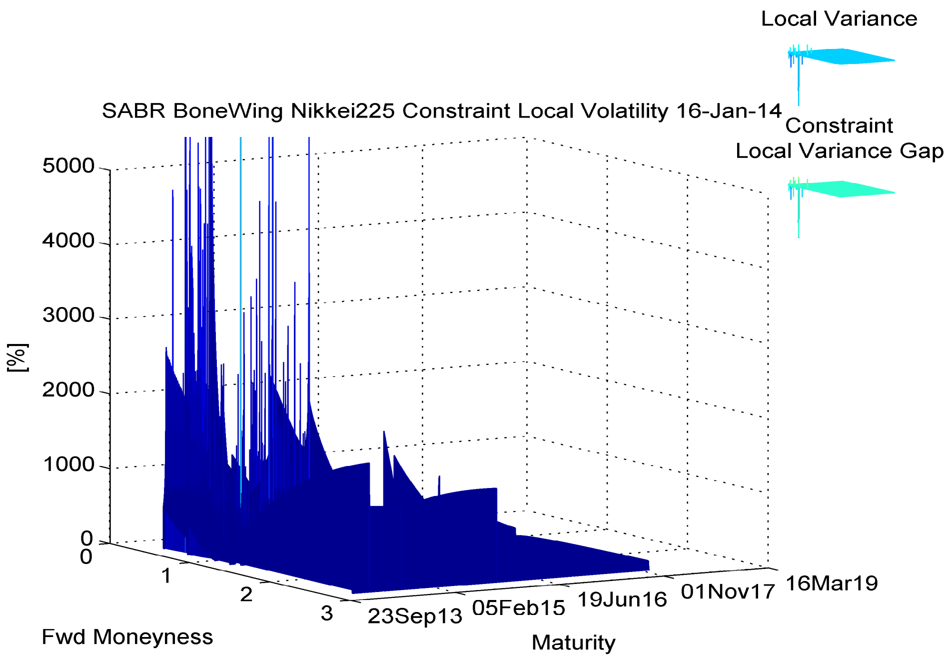


Figure B.158: Root Mean Absolute SnP500 Local Variance Gap (02-Jan-14~31-Mar-14)



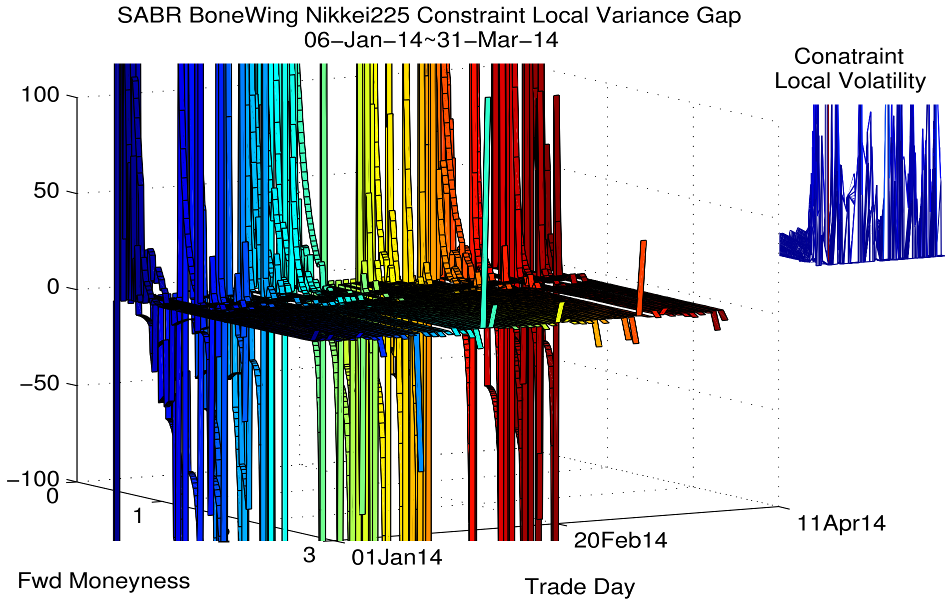
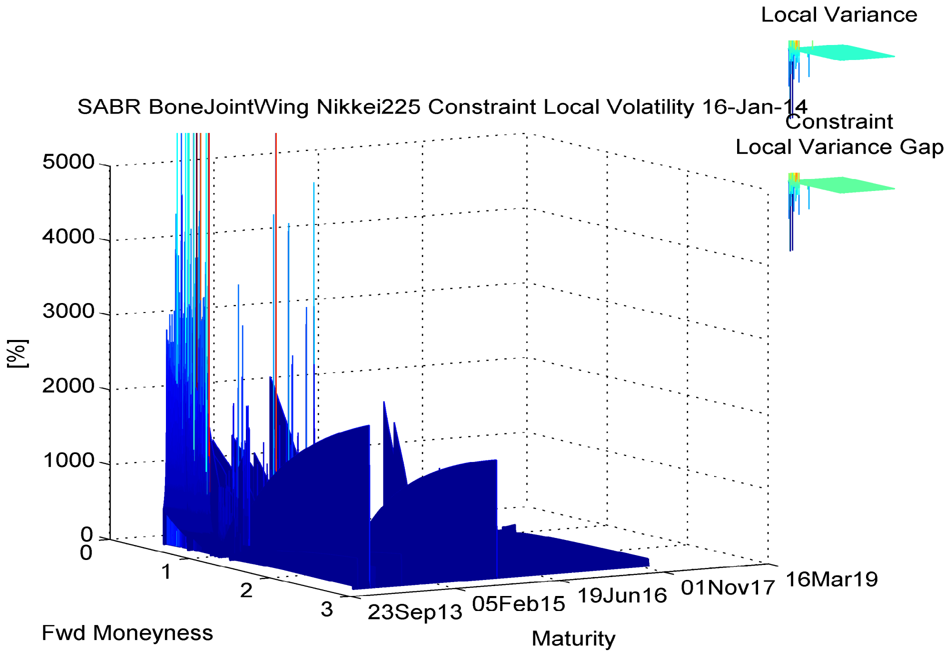


Figure B.159: SABR BoneWing Nikkei225 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



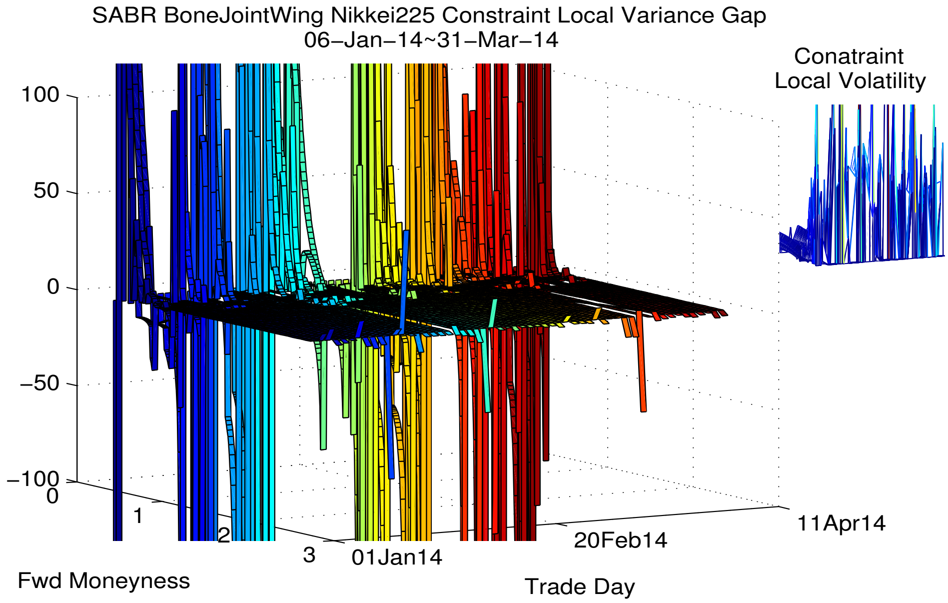
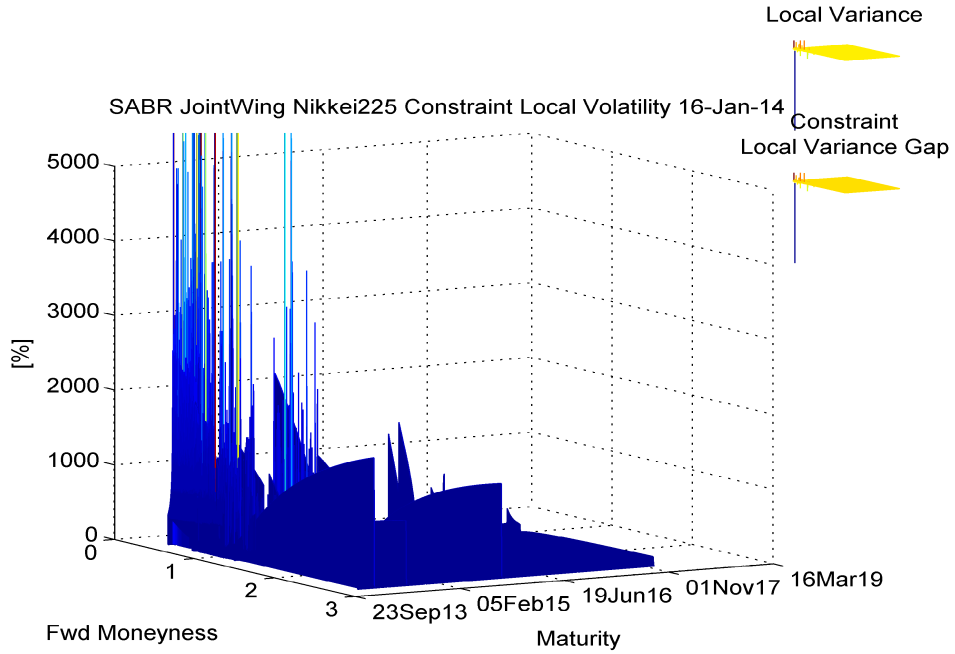


Figure B.160: SABR BoneJointWing Nikkei225 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)



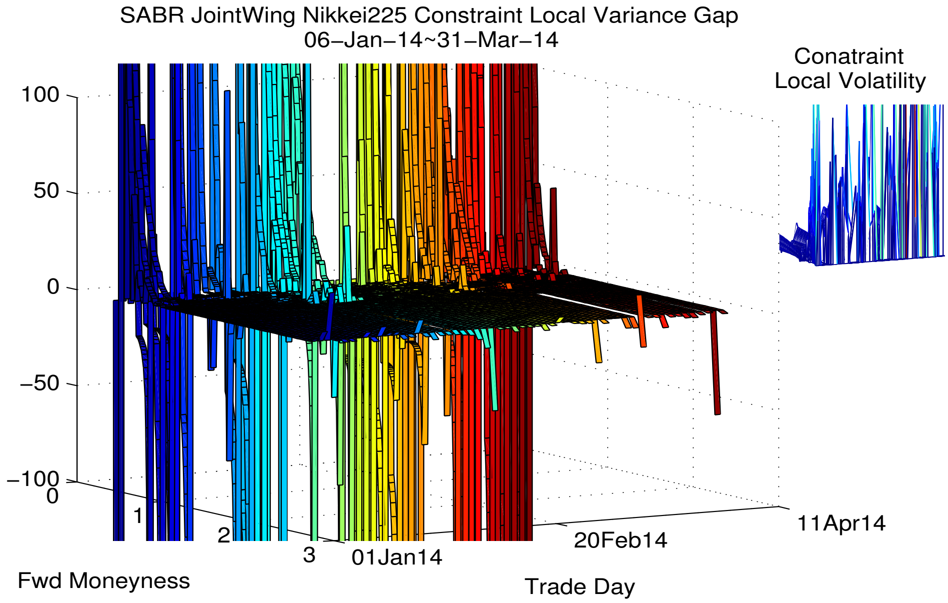
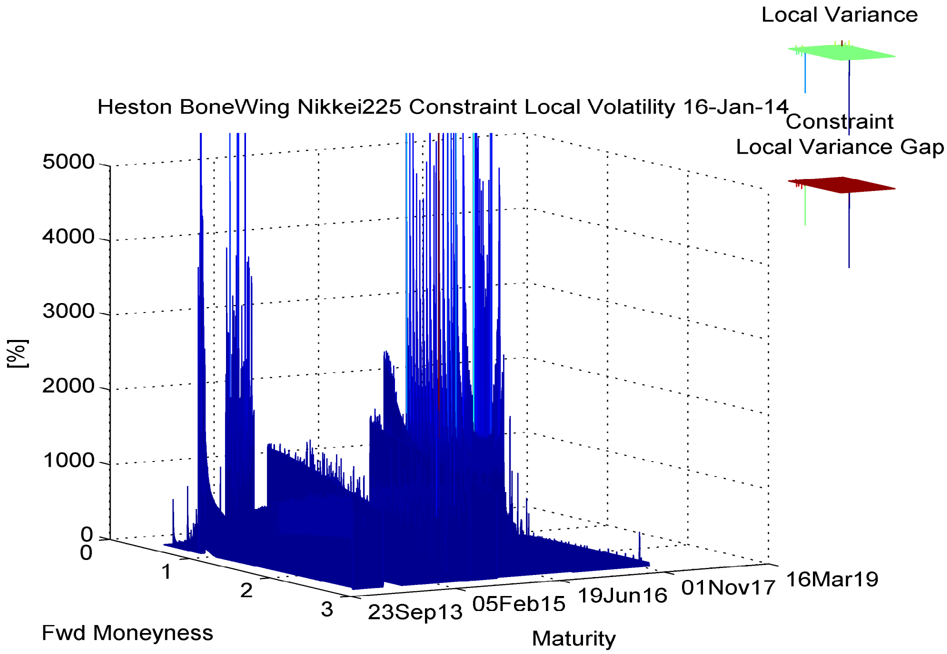


Figure B.161: SABR JointWing Nikkei225 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



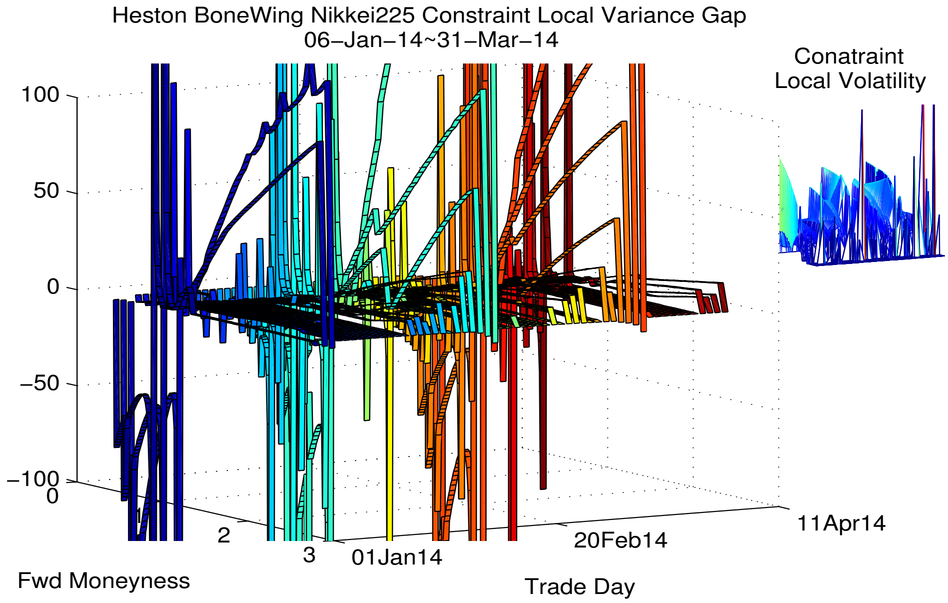
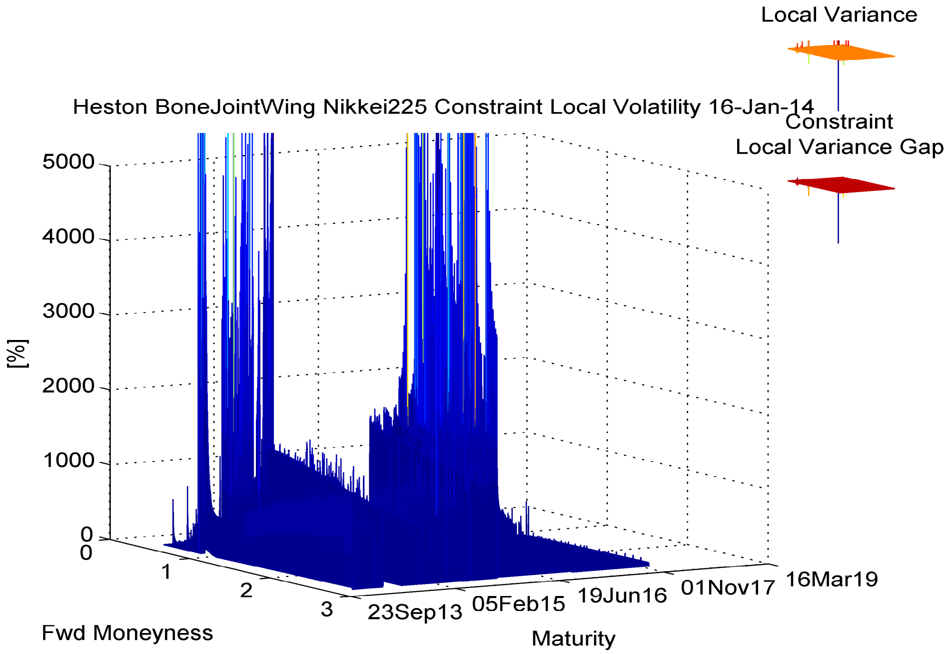


Figure B.162: Heston BoneWing Nikkei225 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)



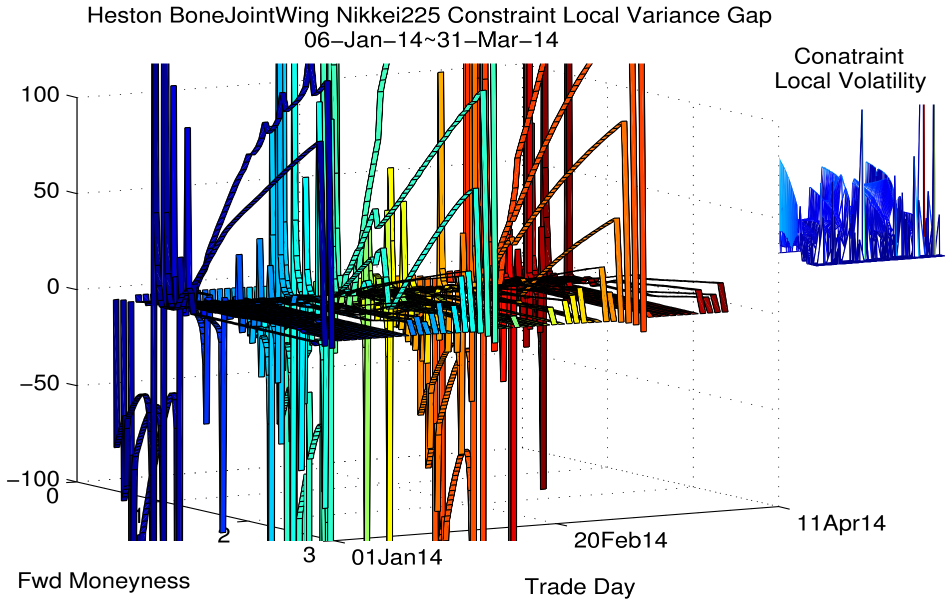
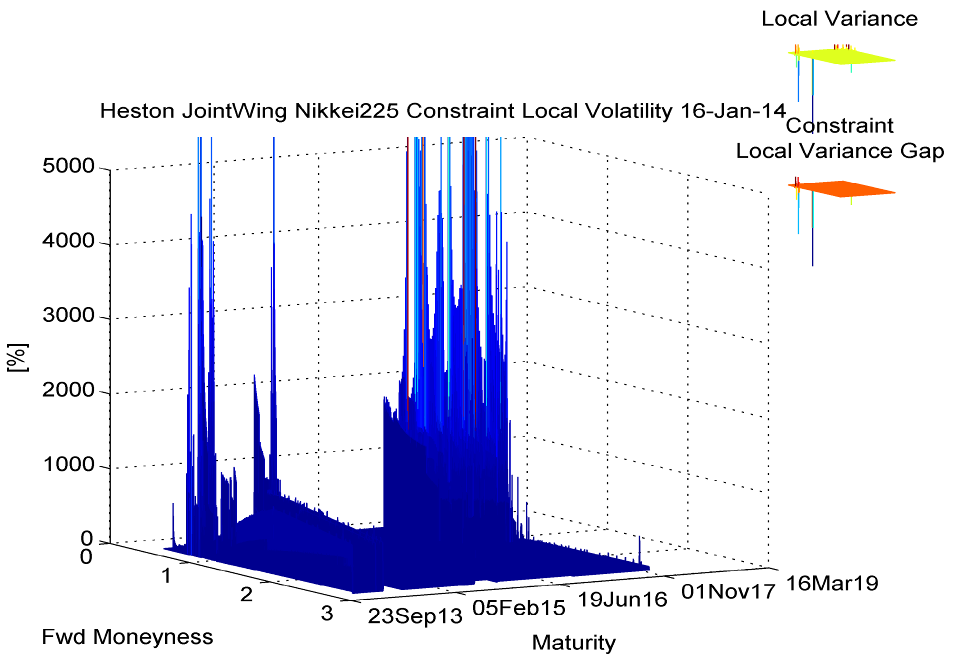


Figure B.163: Heston BoneJointWing Nikkei225 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)



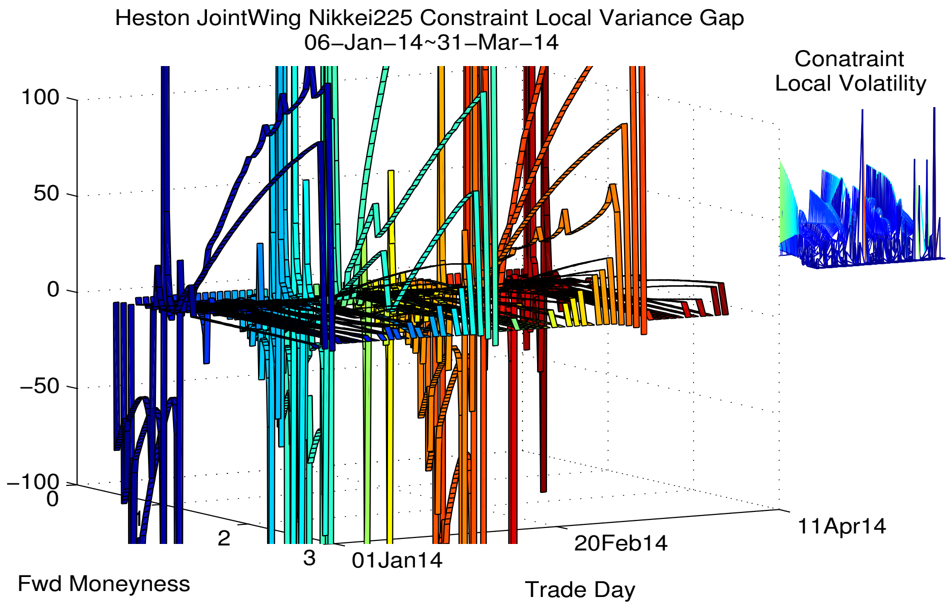


Figure B.164: Heston JointWing Nikkei225 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)

Table B.41: Root Mean Absolute Nikkei225 Constraint Local Variance Gap (06-Jan-14∼31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
1.867E+0	1.922E+0	2.520E+0	1.032E+0	1.121E+0	1.179E+0

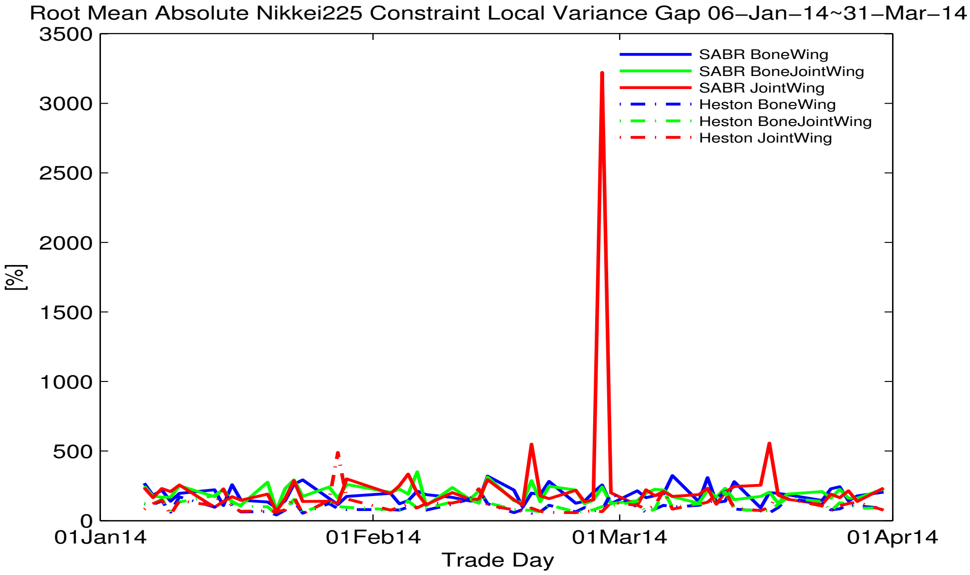
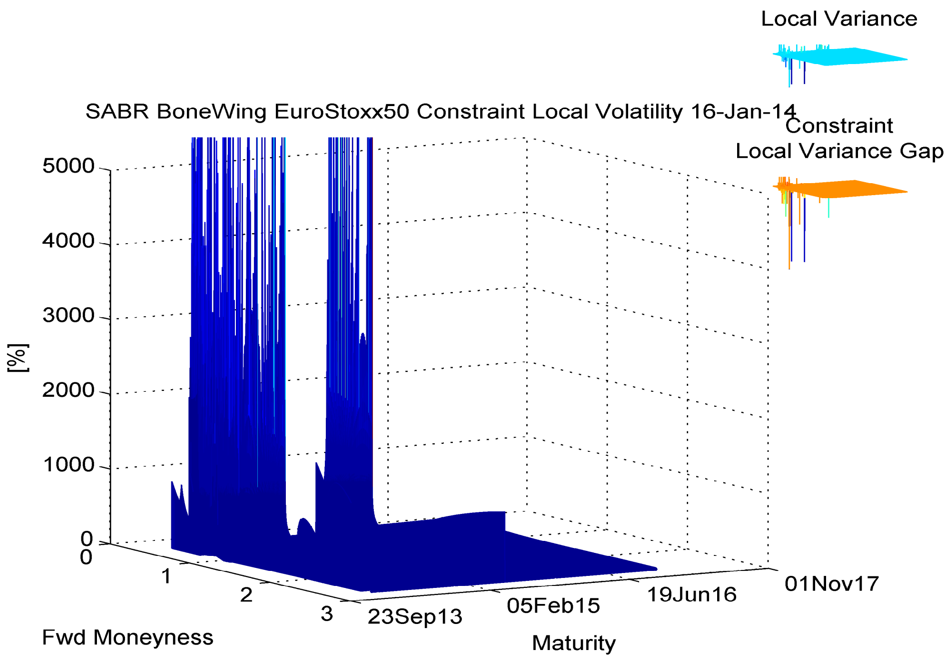


Figure B.165: Root Mean Absolute Nikkei225 Local Variance Gap (02-Jan-14∼31-Mar-14)



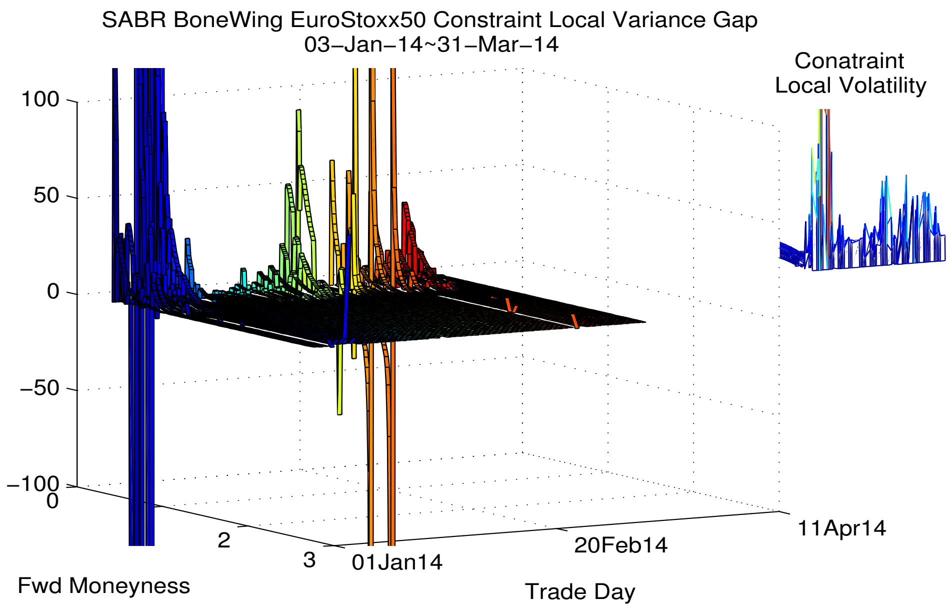
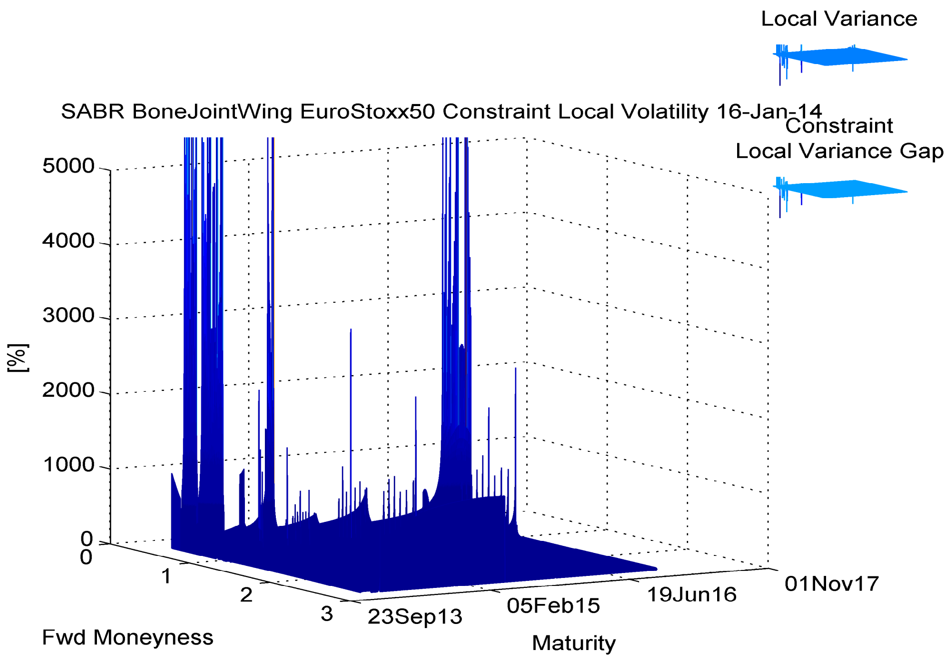


Figure B.166: SABR BoneWing EuroStoxx50 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



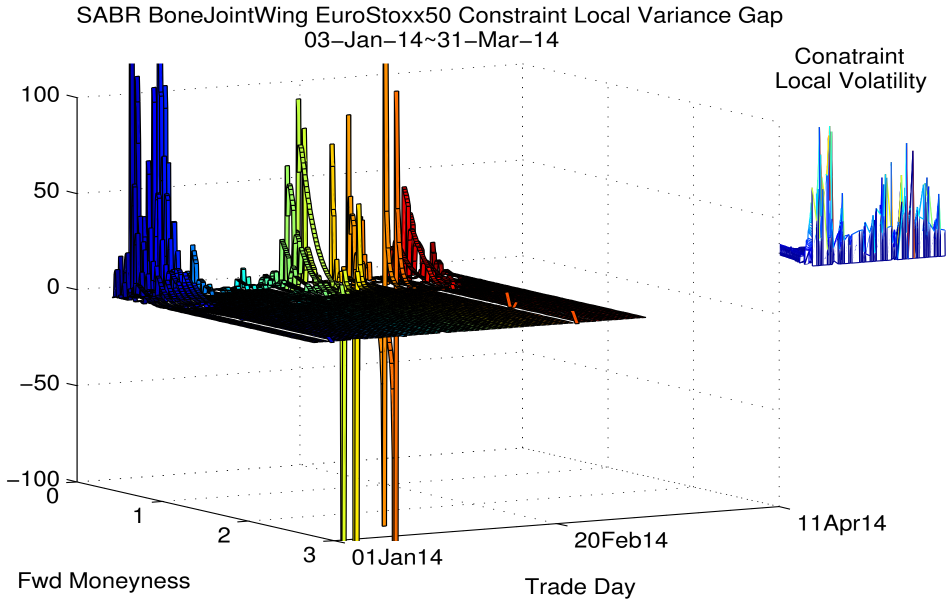
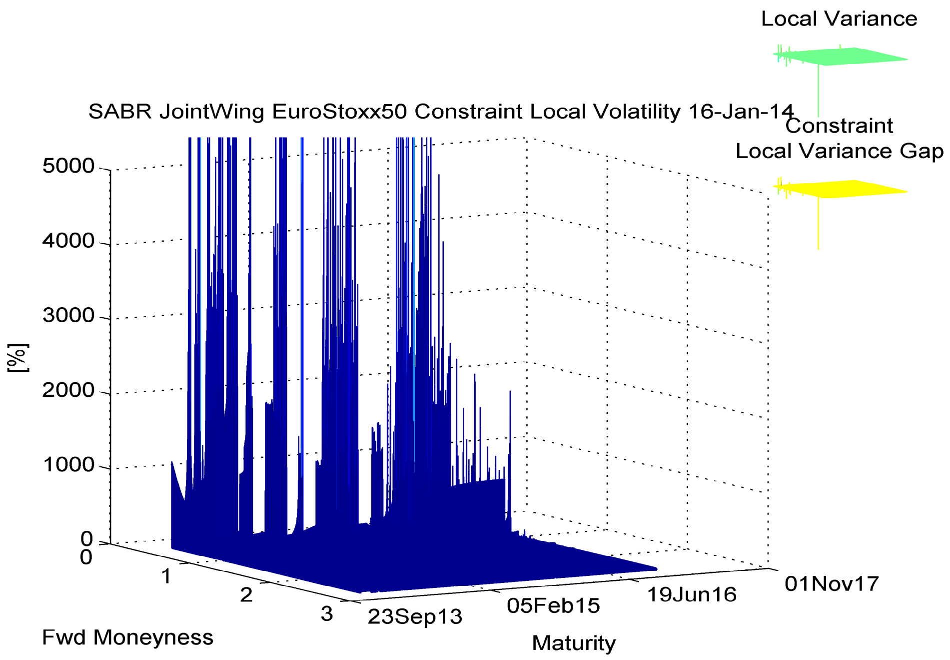


Figure B.167: SABR BoneJointWing EuroStoxx50 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)



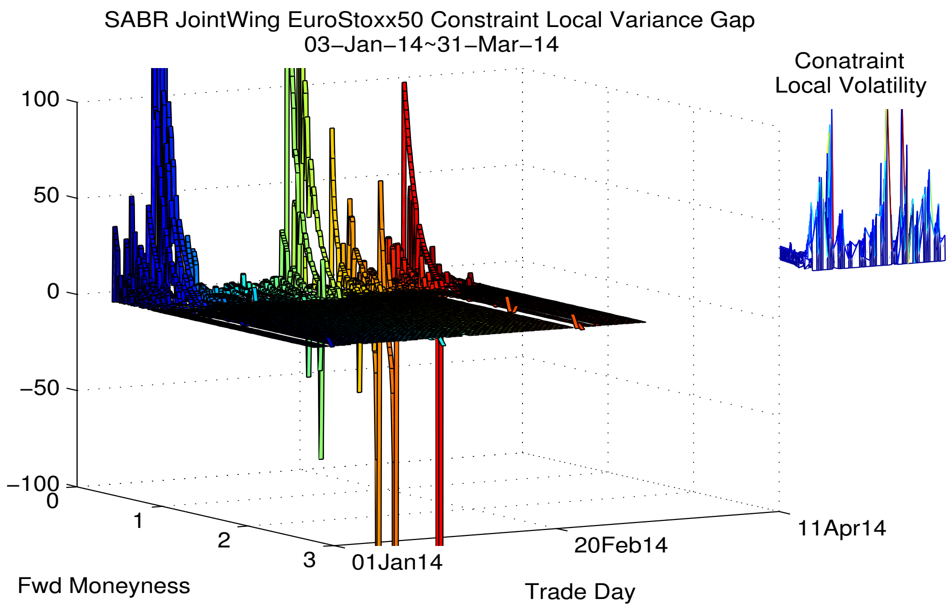
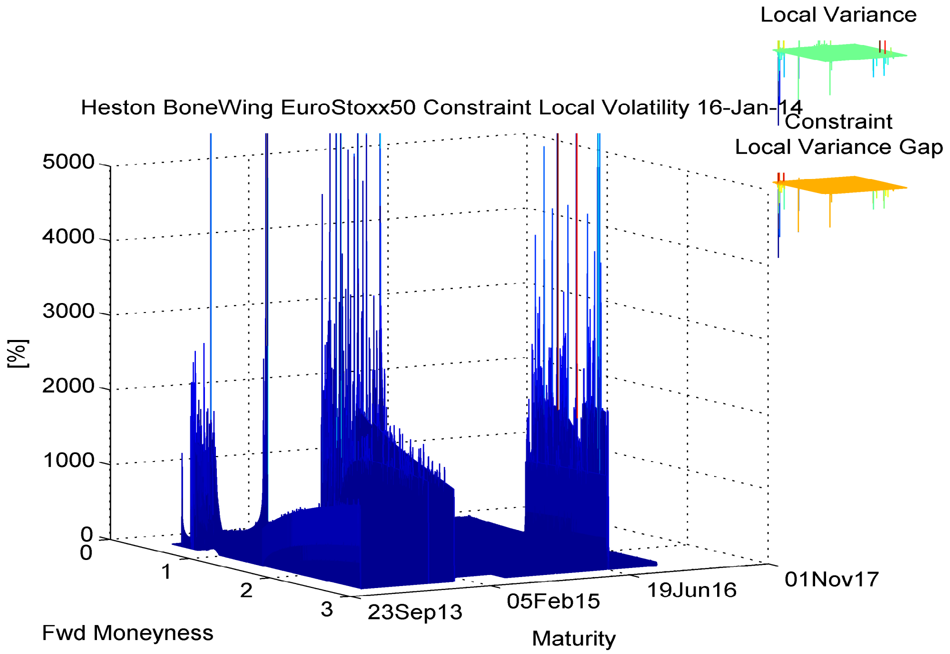


Figure B.168: SABR JointWing EuroStoxx50 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14~31-Mar-14)



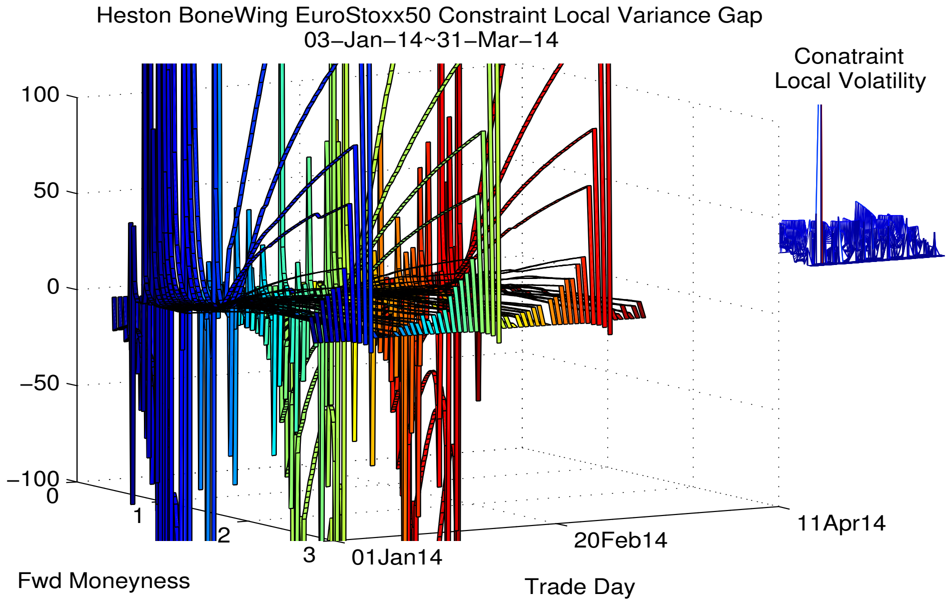


Figure B.169: Heston BoneWing EuroStoxx50 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)

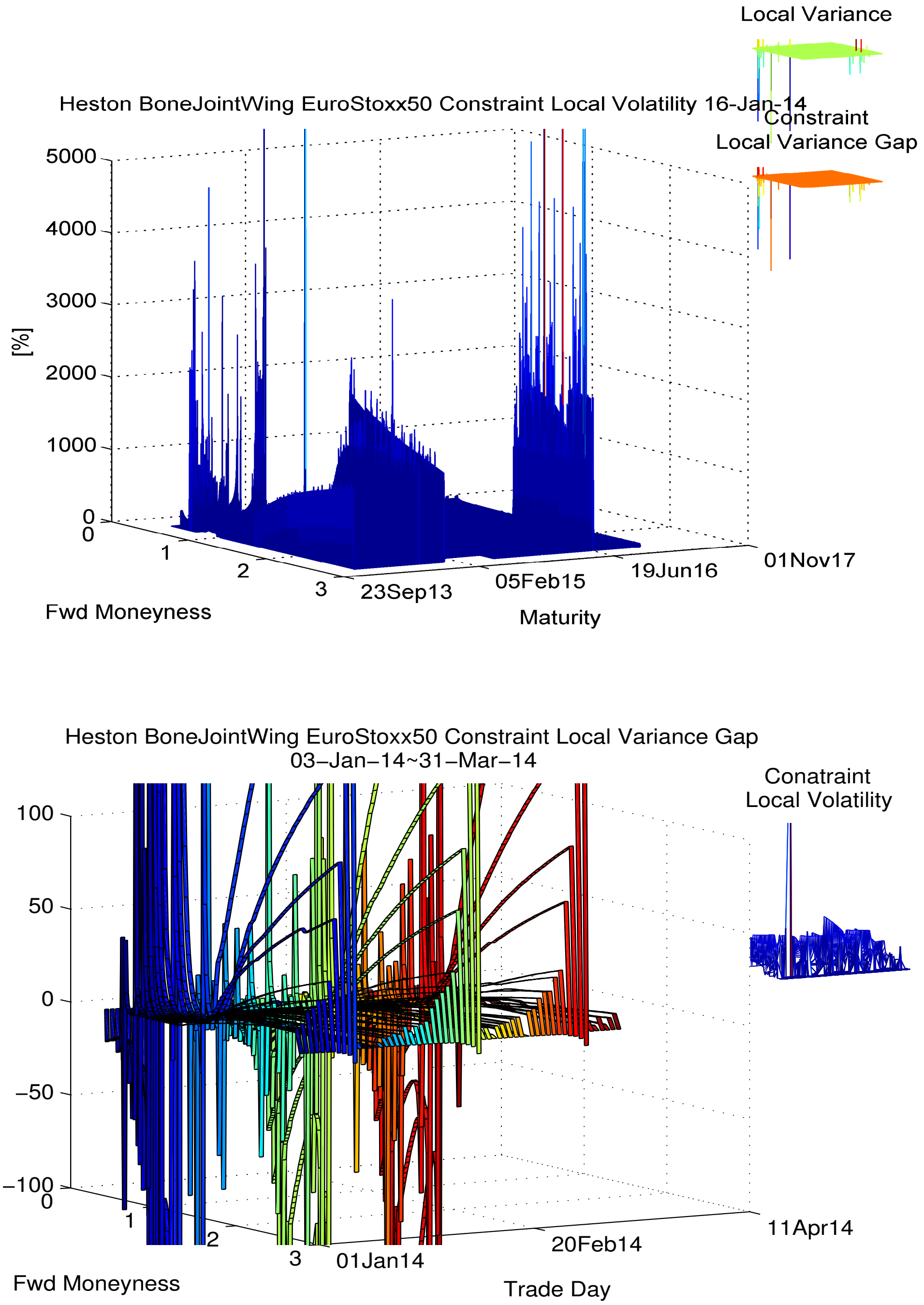
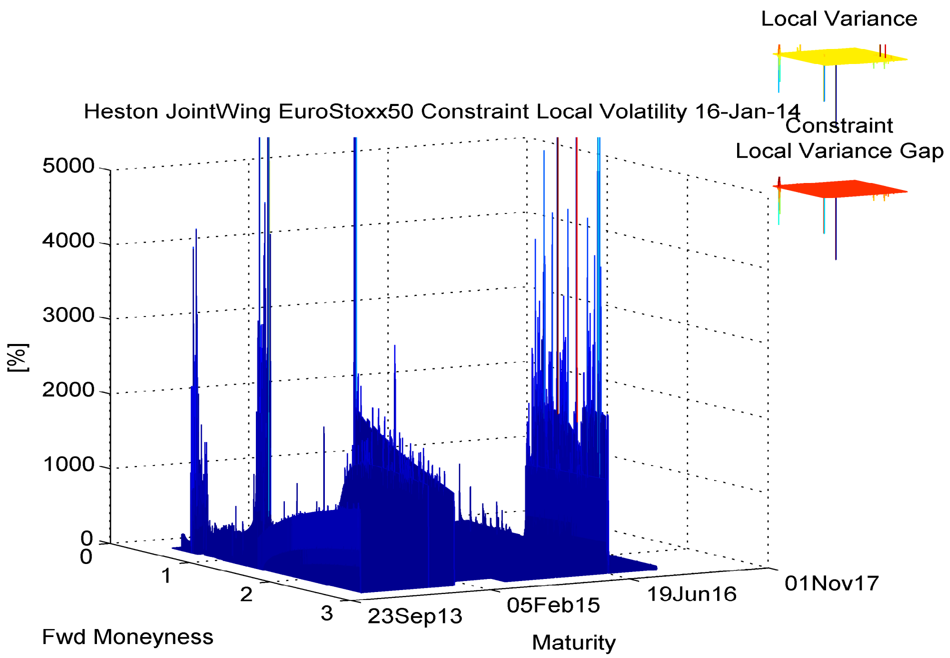


Figure B.170: Heston BoneJointWing EuroStoxx50 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)



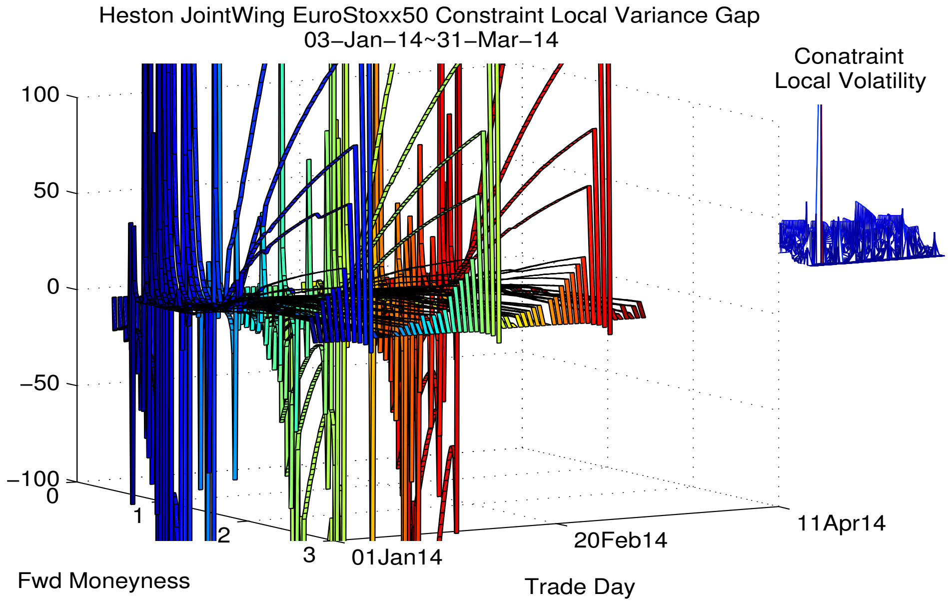


Figure B.171: Heston JointWing EuroStoxx50 Constraint Local Volatility, Variance Gap Nearest (16-Jan-14), (03-Jan-14∼31-Mar-14)

Table B.42: Root Mean Absolute EuroStoxx50 Constraint Local Variance Gap (03-Jan-14∼31-Mar-14)

SABR			Heston		
BoneWing	BoneJoint Wing	JointWing	BoneWing	BoneJoint Wing	JointWing
1.323E+0	1.362E+0	1.568E+0	1.379E+0	1.411E+0	1.358E+0

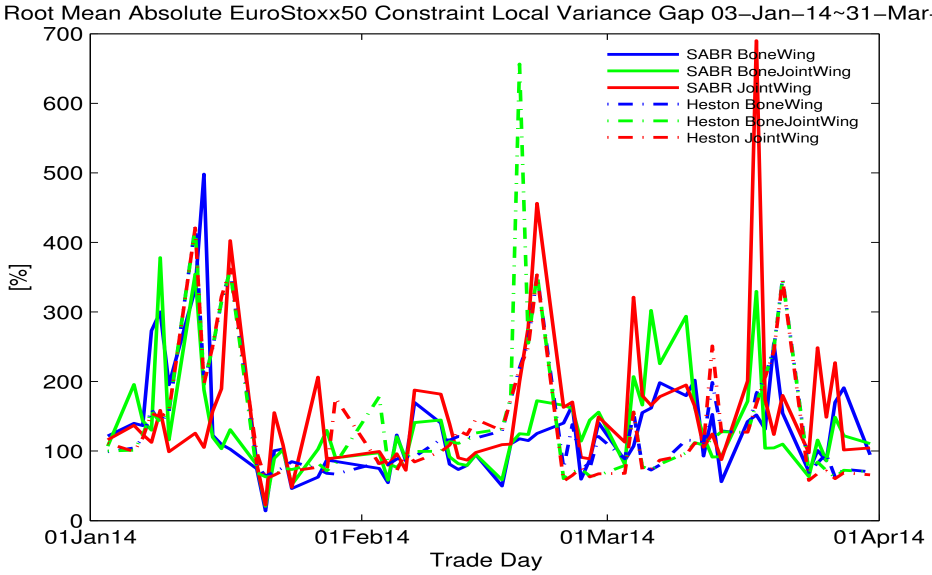


Figure B.172: Root Mean Absolute EuroStoxx50 Local Variance Gap (02-Jan-14∼31-Mar-14)

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국문초록

현대 변동성 모델들은 많은 개념을 담고 있지만 그 지향하는 바는 동일하다. 그래서 이 논문에서는 그 개념들을 서로 연결 시켜보려 한다. 특히, 내재 변동성 곡면을 그 중심에 놓고 국소 변동성, SABR, 헤스턴 모델을 분석한다. 비모수적 변동성 파트에서는 먼저 기존에 사용하고 있는 이론들을 종합하여 더욱 일관성 있는 선도 국소 변동성 모델을 제안하고, 델타 헤지 시뮬레이션을 통해 국소 변동성 모델을 제대로 사용하는 방법을 확인하는 것으로 파트를 마무리한다. 모수적 변동성 파트는 변동성 모델의 모든 입력 변수(배당, 이자율, 주가지수 선물)를 면밀히 살펴보는 것으로 시작하여 마지막으로 유용한 모델들을 더 유용하게 만들어주는 새로운 방법론 2가지를 제안한다. 합성 옵션을 붓스트래핑하는 것과 확률변동성 모델을 모자이크 형태로 구성하는 것이 바로 그 방법론들이다.

주요어: 내재 변동성 곡면; 국소 변동성; SABR 모델; Heston 모델; 합성옵션 붓스트래핑; Mosaic 모델

학번: 2011-20450